Unit 4

Lesson 4

Square + Cube Root Applications

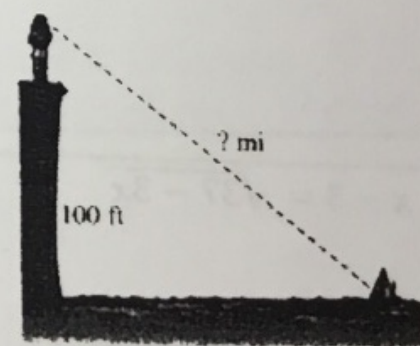
nit 4 – Radical & Rational Functions Date______________________ Pd______

sson 4 → Square Root & Cube Root Applications

1. Did you ever stand on a beach and wonder how far out into the ocean you could see? Or have you wondered how close a ship has to be to spot land? In either case, the function $d(h) = \sqrt{2h}$ can be used to estimate the distance to the horizon (in miles) from a given height (in feet). $d(h)=\sqrt{2h}$

   a. Cordelia stood on a cliff gazing out at the ocean. Her eyes were 100 ft above the ocean. She saw a ship on the horizon. Approximately how far was she from that ship?

   $d(h)=\sqrt{2(100)}$ $d(h)=\sqrt{200}$ ≈ 14.14 mi



   b. From a plane flying at 35,000 ft, how far away is the horizon?

   $d(h)=\sqrt{2(35{,}000)}$ ≈ 264.58 mi

   c. Given a distance, d, to the horizon, what altitude would allow you to see that far? Rewrite the formula and solve for h.

   $(d)^2=(\sqrt{2h})^2$ $\frac{d^2}{2}=\frac{2h}{2}$ $\frac{d^2}{2}=h$

A weight suspended on the end of a string is a pendulum.

2. A weight suspended on the end of a string is a *pendulum*. The most common example of a pendulum (this side of Edgar Allen Poe) is the kind found in many clocks. The regular back-and-forth motion of the pendulum is *periodic*, and one such cycle of motion is called a *period*. The time, in seconds, that it takes for one period is given by the radical equation $t = 2\pi\sqrt{\frac{l}{g}}$ in which g is the force of gravity $(10\ m/s^2)$ and l is the length of the pendulum $(meters)$.

a. Find the period (to the nearest hundredth of a second) if the pendulum is $0.9\ m$ long.

$t = 2\pi\sqrt{.9/10} \approx 1.88$ seconds

b. Find the period if the pendulum is $0.049\ m$ long.

$t = 2\pi\sqrt{\frac{.049}{10}} \approx .44$ seconds

c. Solve the equation for length l.

$t = 2\pi\sqrt{\frac{l}{10}}$ $\Rightarrow$ $\frac{5t^2}{2\pi^2} = l$

d. How long would the pendulum be if the period were exactly $1\ second$?

$\frac{5(1)^2}{2\pi^2} = l \approx .25$ meters

$$\frac{t}{2\pi} = \frac{\cancel{2\pi}\sqrt{\frac{l}{10}}}{\cancel{2\pi}}$$

$$\left(\frac{t}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{10}}\right)^2$$

$$10 \cdot \frac{t^2}{4\pi^2} = \frac{l}{10} \cdot 10$$

$$\frac{10t^2}{4\pi^2} = l$$

$$\frac{5t^2}{2\pi^2} = l$$

3. The current (in ampere) a machine needs to reach a certain power can be modeled by the equation, $I = 0.2\sqrt{P}$, where P is the power of machine in watts. If the current is 10 *amperes*, what is the power of machine in watts?

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$I = .2\sqrt{P}$

$\frac{10}{.2} = \frac{.2\sqrt{p}}{.2}$

$(50)^2 = (\sqrt{p})^2$

$2500 = p$

$10 = .2\sqrt{2500}$

4. When a car comes to a sudden stop, you can determine the skidding distance (in feet) for a given speed (in miles per hour) using the formula, $s(x) = 2\sqrt{5x}$, in which s is skidding distance and x is speed. Calculate the skidding distance for the following speeds (round to nearest tenth of a foot).

a. $55\ mph = 2\sqrt{5(55)} \approx 33.2$ ft

b. $65\ mph = 2\sqrt{5(65)} \approx 36.1$ ft

c. $75\ mph$ $2\sqrt{5(75)} \approx 38.7$ ft

d. $40\ mph$ $2\sqrt{5(40)} \approx 28.3$ ft

e. Given the skidding distance s, what formula would allow you to calculate the speed, x, in mph?

$\frac{S}{2} = \frac{2\sqrt{5x}}{2}$ $\left(\frac{S}{2}\right)^2 = \left(\sqrt{5x}\right)^2$ $\frac{1}{5} \cdot \frac{S^2}{4} = \cancel{5}x \cdot \cancel{\frac{1}{5}}$ $\frac{S^2}{20} = x$

f. Use the formula obtained in (e) to determine the speed of a car in miles per hour if the skid marks were 35 ft long.

$\frac{(35)^2}{20} = x \approx 61.25$ mph

$$\frac{S}{2} = \frac{2\sqrt{5x}}{2}$$

$$\left(\frac{S}{2}\right)^2 = \left(\sqrt{5x}\right)^2$$

$$\frac{1}{5}\cdot\frac{S^2}{4} = 5x\cdot\frac{1}{5}$$

$$\frac{S^2}{20} = x$$

$$\frac{5x}{5} = \frac{3}{5}$$

$$\frac{1}{\cancel{5}}\cdot\cancel{5}x = 3\cdot\frac{1}{5}$$

$$x = 3/5$$

Write an equation and then solve each of the following applications.

5. The sum of an integer and its square root is 12. Find the integer.

6. The difference between an integer and its square root is 12. What is the integer?

7. The sum of an integer and twice its square root is 24. What is the integer?

8. The sum of an integer and 3 times its square root is 40. Find the integer.

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5) $x + \sqrt{x} = 12$

$-x \qquad\qquad -x$

$\sqrt{x} = 12 - x$

$x = (12-x)(12-x)$

$x = 144 - 24x + x^2$

$-x \qquad\qquad -x$

$0 = 144 - 25x + x^2$

$\quad\quad\quad\quad\quad A \quad\quad M$

$0 = x^2 - 25x + 144$

$0 = (x-16)(x-9)$

$\cancel{x = 16}$

$x = 9$

6) $x - \sqrt{x} = 12$

$-x \qquad\qquad -x$

$-1(-\sqrt{x}) = (12 - x) \cdot -1$

$\sqrt{x} = x - 12$

$x = (x-12)^2$

$x = x^2 - 24x + 144$

$-x \qquad -x$

$0 = x^2 - 25x + 144$

$0 = (x-16)(x-9)$

$x = 16 \quad \cancel{x = 9}$

7) $x + 2\sqrt{x} = 24$

$(2\sqrt{x})^2 = (24-x)(24-x)$

$4x = 576 - 48x + x^2$

$-4x \qquad -4x$

$0 = x^2 - 52x + 576$

$0 = (x-36)(x-16)$

$\cancel{x = 36} \quad x = 16$

8) $x + 3\sqrt{x} = 40$

$-x \qquad\qquad -x$

$(3\sqrt{x})^2 = (40-x)^2$

$9x = 1600 - 80x + x^2$

$-9x \qquad\qquad -9x$

$0 = x^2 - 89x + 1600$

$0 = (x-25)(x-64)$

$x = 25 \quad \cancel{x = 64}$

HW: Pg 15-16 Quiz Review