Math 2
Unit 4 - Radical & Rational Functions
Lesson 5 → Types of Variations

Name_____
Date_____**Pd**_____

- **DIRECT VARIATION:** Linear function with a y-intercept of 0. In a direct variation, both of the quantities are either increasing or both are decreasing.
- There are two methods for solving a direct variation problem:
  1) Equation of Variation: $y = kx$ where k is called the **constant of variation**
  2) Proportion: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

#1: The distance that a body near Earth's surface will fall from rest varies directly as the **square** of the number of seconds it has been falling. If a boulder falls from a cliff a distance of $122.5\ m$ in $5\ seconds$, approximately how far will it fall in $8\ seconds$?

Method 1

$y = kx$
$d = ks^2$
$122.5 = k(5)^2$
$\frac{122.5}{25} = \frac{25k}{25}$
$k = 4.9$

Method 2

$d = (4.9)(8)^2$
313.6 meters

- **JOINT VARIATION**: more than two quantities in a **direct variation** relationship
- Equation of Variation: $y = kxz$ where k is called the **constant of variation**

#2: If y *varies jointly as* x *and* z, and $y = \frac{1}{2}$ when $x = 27$ and $z = \frac{-2}{3}$, find y when $x = 9$ and $z = 18$.

$y = kxz$
$\frac{1/2}{(27)(-2/3)} = \frac{k(27)(-2/3)}{(27)(-2/3)}$
$k = -\frac{1}{36}$
$y = (-1/36)(9)(18)$
$y = -9/2$ or -4.5

- **INVERSE VARIATION:** Rational function with vertical and horizontal asymptotes. In an inverse variation, one of the quantities is increasing while the second quantity is decreasing.
- Equation of Variation: $y = \frac{k}{x}$ where k is called the **constant of variation**

#3: The time of a trip varies inversely as the speed of the car. If a car being driven at $55\ mph$ takes $2\ hours$ to get from Wake Forest to Greensboro, how fast is the car traveling if the trip takes $2.5\ hours$?

$y = \frac{k}{x}$
$h = \frac{k}{m}$
$\frac{2}{1} = \frac{k}{55}$
$110 = k$
$\frac{2.5}{1} = \frac{110}{m}$
$2.5m = 110$
$m = 44\ mph$

➢ **COMPOUND VARIATION:** Both Inverse and Direct Variation in the same problem

➢ Equation of Variation: $y = \frac{kx}{z}$ where k is called the **constant of variation**

#4: The volume of gas varies directly with Kelvin temperature and inversely with pressure. If a certain gas has a volume of 342 $cubic\ meters$ at a temperature of 300 $Kelvin\ degrees$ under a pressure of 200 KPa $(kilopascals)$, what will be the volume of the same gas at a temperature of 320 $Kelvin\ degrees$ under a pressure of 400 kPA?

$y = \frac{kx}{z}$ $V = \frac{kt}{p}$ $\frac{342}{1} = \frac{k(300)}{200}$ $\frac{68400}{300} = \frac{300k}{300}$ $k = 228$ $V = \frac{228 \cdot 320}{400}$ $V = 182.4 m^3$

➢ **State whether each equation represents a direct, inverse, joint or compound variation. Then state the constant of variation.**

1. $y = \frac{9}{x}$	2. $z = 5xy$	3. $y = \frac{8x}{z}$	4. $y = 2x$	5. $xy = 12$
6. $z = \frac{xy}{15}$	7. $y = \frac{3}{4}xz$	8. $y = \frac{1}{3}x$	9. $z = \frac{x}{12y}$	10. $y = \frac{x}{5}$

➢ **Write a function for each variation relationship:**

11. W varies directly as the square of d. $W = kd^2$

12. V varies inversely as J. $V = \frac{k}{J}$

13. V varies inversely as p and directly as T. $V = \frac{kT}{p}$

14. F varies jointly as A and the square of v. $F = kAv^2$

15. L varies directly as the fourth power of d and inversely as the square root of h. $L = \frac{kd^4}{\sqrt{h}}$

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Write an equation for each statement and then solve:

1. If y *varies directly as* x and $y = 15$ when $x = 3$, find y when $x = 12$. $y = kx \quad k = 5$ $\frac{15}{3} = \frac{3k}{3}$ $y = 5(12)$ $y = 60$	2. If y *varies directly as* x and $x = 36$ when $y = 4$, find x when $y = 24$.	3. If y *varies directly as* x^2 and $y = 12$ when $x = 4$, find y when $x = 6$. $y = kx^2$
4. If y *varies inversely as* x and $y = 2$ when $x = 8$, find x when $y = 14$.	5. *If* y *varies inversely as* x and $x = 7$ when $y = 21$, find y when $x = 42$. $y = \frac{k}{x} \quad 21 = \frac{k}{7} \quad k = 147$ $y = \frac{147}{42}$ 3.5 / $\frac{7}{2}$	6. If y *varies inversely as* x^3 and $y = 6$ when $x = \frac{-3}{4}$, find y when $x = 3$.
7. Suppose y *varies jointly with* x *and* z. If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 14$ and $z = 8$.	8. Suppose z *varies jointly with* x *and* y. If $x = 3$ and $y = 2$ when $z = 12$, find z when $x = 4$ and $y = 5$.	9. Suppose m *varies jointly* as n *and* p. If $n = 4$ and $p = 5$ when $m = 60$, find m when $n = 12$ and $p = 2$.
10. Suppose that y *varies directly as* x *and inversely as* z. If $y = 5$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.	11. Suppose y *varies directly as* $\sqrt{x}$ and *inversely as* z. If $y = 10$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 10$.	12. Suppose x *varies directly as* y^3 *and inversely as* $\sqrt{z}$. If $x = 7$ when $y = 2$ and $z = 4$, find x when $y = 3$ and $z = 9$.