

**Math 2**  
**Unit 4 –Radical & Rational Functions**  
**Lesson 3 → Graphs of Rational Functions**

Name \_\_\_\_\_  
 Date \_\_\_\_\_ Pd \_\_\_\_\_

➤ A rational function is a function that can be written as the ratio of two polynomials where the denominator does not equal zero.

➤  $f(x) = \frac{p(x)}{q(x)}$  where  $q(x) \neq 0$

$y = (x+3)^2 + 2$   
 L2      U2

$y = \frac{4x+8}{x-5}$   
 L/R      U/D

❖ Steps to graph a rational function:

$y = \frac{n}{x-h} + k$

1) Determine the location of the asymptotes based on the transformations:

A) Vertical asymptotes are placed based on the **horizontal translation**:  $x = h$

B) Horizontal asymptotes are placed based on the **vertical translation**:  $y = k$

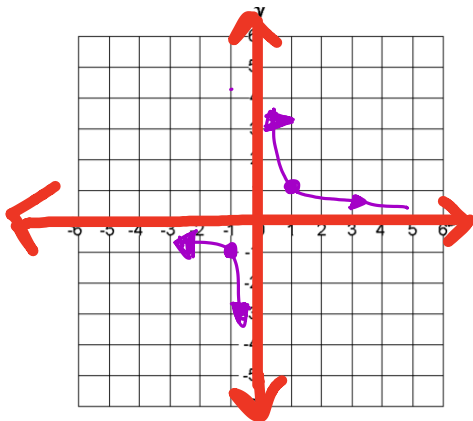
2) **Vertical Stretch or Compression**:  $n$  tells us how far the branches have been stretched from the asymptotes. We can use it to help us find out corner points to start our branches.

**Distance from asymptotes** =  $\sqrt{n}$

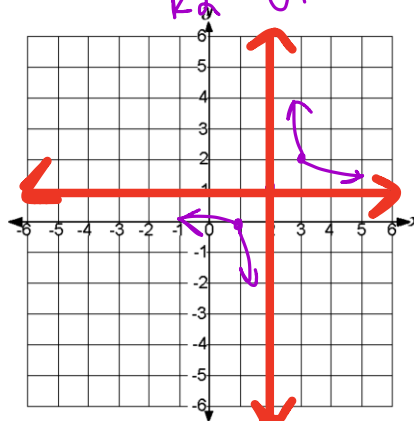
3) Look at the table on the calculator for other points  $k$  and then sketch the two branches.

❖ Graph each of the following examples:

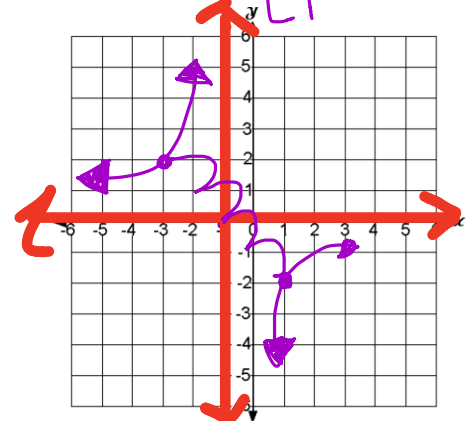
1.  $y = \frac{1}{x}$



2.  $y = \frac{1}{x-2} + 1$



3.  $y = -\frac{4}{x+1}$



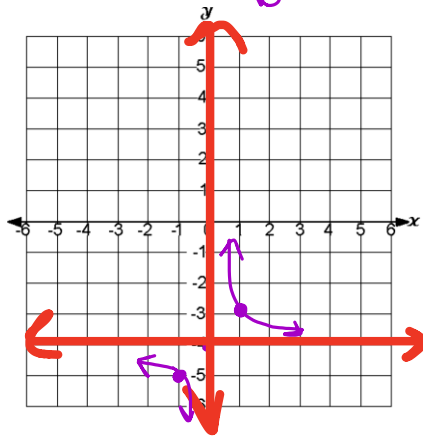
Rx-axis  
 S by 4

Equation of VA:  $x=0$   
 Equation of HA:  $y=0$   
 Describe translations: ~~N/A~~  
 Domain:  $x \neq 0$   
 Range:  $y \neq 0$

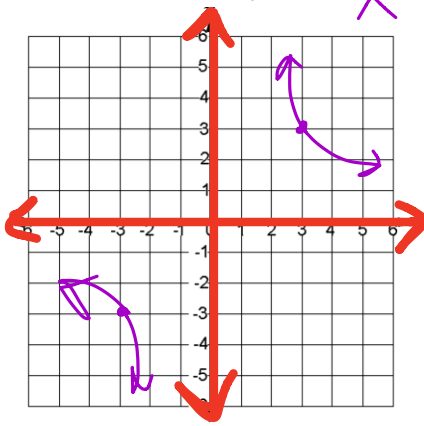
Equation of VA:  $x=2$   
 Equation of HA:  $y=1$   
 Describe translations: R2, U1  
 Domain:  $x \neq 2$   
 Range:  $y \neq 1$

Equation of VA:  $x=-1$   
 Equation of HA:  $y=0$   
 Describe translations: Rx-axis, L1  
 Domain:  $x \neq -1$   
 Range:  $y \neq 0$

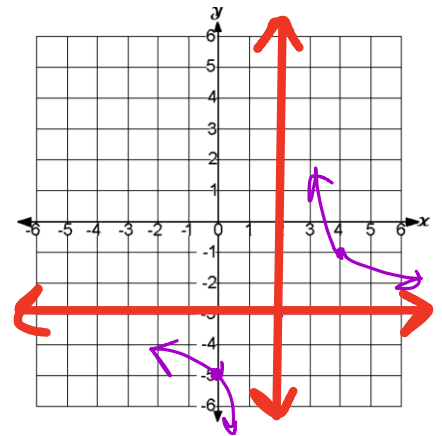
4.  $y = \frac{1}{x} - 4$  04



5.  $xy = 9$   $y = \frac{9}{x}$



6.  $y = \frac{4}{x-2} - 3$



Equation of VA:  $x=0$   
 Equation of HA:  $y=-4$   
 Describe translations: Down 4  
 Domain:  $x \neq 0$   
 Range:  $y \neq -4$

Equation of VA:  $x=0$   
 Equation of HA:  $y=0$   
 Describe translations: S by 9  
 Domain:  $x \neq 0$   
 Range:  $y \neq 0$

Equation of VA:  $x=2$   
 Equation of HA:  $y=-3$   
 Describe translations: S by 4  
 Domain:  $x \neq 2$   
 Range:  $y \neq -3$

7. Describe each graph as compared to the parent graph  $y = \frac{1}{x}$ .

$y = \frac{-2}{x-7} + 5$

The graph of this \_\_\_\_\_ function has been translated \_\_\_\_\_ seven units and translated \_\_\_\_\_ units \_\_\_\_\_. It has been vertically stretched by a factor of \_\_\_\_\_ and \_\_\_\_\_ across the x-axis. The graph is increasing from \_\_\_\_\_ to \_\_\_\_\_. The function has a domain of \_\_\_\_\_ and a range of \_\_\_\_\_.

$y = \frac{7}{x+2} - 4$

The graph of this \_\_\_\_\_ function has been translated \_\_\_\_\_ two units and translated \_\_\_\_\_ units \_\_\_\_\_. It has been vertically stretched by a factor of \_\_\_\_\_. The graph is \_\_\_\_\_ from left to right. The function has a domain of \_\_\_\_\_ and a range of \_\_\_\_\_.

8. Write the equation of a rational function  $y = \frac{1}{x}$  with following transformations:

A. Right 4 and Down 5

$\frac{1}{x-4} - 5 = y$

B. Left 3 and Up 2 and Reflected across  $x$ -axis.

$-\frac{1}{x+3} + 2 = y$

C. Left 6 and Vertically Stretched by a factor of 4.

$\frac{4}{x+6} = y$

D. Right 5 and graph will be in II & IV quadrants

~~$\frac{1}{x-5} = y$~~