

### 3.3 It All Adds Up

#### A Develop Understanding Task

Whenever we're thinking about algebra and working with variables, it is useful to consider how it relates to the number system and operations on numbers. Right now, polynomials are on our minds, so let's see if we can make some useful comparisons between whole numbers and polynomials.



CC BY Anders Sandberg  
<https://fflic.kr/p/3GWEh4>

Let's start by looking at the structure of numbers and polynomials. Consider the number 132. The way we write numbers is really a shortcut because:

$$132 = 100 + 30 + 2$$

1. Compare 132 to the polynomial  $x^2 + 3x + 2$ . How are they alike? How are they different?

$$\overline{1} \overline{3} \overline{2}$$

2. Write a polynomial that is analogous to the number 2,675.

$$2x^3 + 6x^2 + 7x + 5$$

When two numbers are to be added together, many people use a procedure like this:

$$\begin{array}{r} 132 \\ + 451 \\ \hline 583 \end{array}$$

3. Write an analogous addition problem for polynomials and find the sum of the two polynomials.

$$\begin{array}{r} x^2 + 3x + 2 \\ + 4x^2 + 5x + 1 \\ \hline \end{array}$$

4. How does adding polynomials compare to adding whole numbers?

See above

54-57

5. Use the polynomials below to find the specified sums in a-f.

$$f(x) = x^3 + 3x^2 - 2x + 10 \quad g(x) = 2x - 1 \quad h(x) = 2x^2 + 5x - 12 \quad + \quad k(x) = -x^2 - 3x + 4$$

$$l(x) = 4x^2 - 3y^2 + 5xy \quad n(x) = 4xy + 2x^2 \quad m(x) = 8xy + 3y^2 \quad p(x) = x^2 - 7xy + 4$$

$$\begin{array}{r} 2x^2 + 5x - 12 \\ + -x^2 - 3x + 4 \end{array}$$

a) h(x) + k(x)

b) g(x) (+) f(x)

c) f(x) + k(x)

$$\boxed{x^2 + 2x - 8}$$

d) l(x) + m(x)

e) m(x) + n(x)

f) l(x) + p(x)

6. What patterns do you see when polynomials are added?

Subtraction of whole numbers works similarly to addition. Some people line up subtraction vertically and subtract the bottom number from the top, like this:

$$\begin{array}{r} 368 \\ -157 \\ \hline 211 \end{array}$$

7. Write the analogous polynomials and subtract them.

8. Is your answer to #7 analogous to the whole number answer? If not, why not?

9. Subtracting polynomials can easily lead to errors if you don't carefully keep track of your positive and negative signs. One way that people avoid this problem is to simply change all the signs of the polynomial being subtracted and then add the two polynomials together. There are two common ways of writing this:

$$(x^3 + x^2 - 3x - 5) - (2x^3 - x^2 + 6x + 8)$$

Step 1:  $= (x^3 + x^2 - 3x - 5) + (-2x^3 + x^2 - 6x - 8)$

Step 2:  $= (-x^3 + 2x^2 - 9x - 13)$

Or, you can line up the polynomials vertically so that Step 1 looks like this:

Step 1: 
$$\begin{array}{r} x^3 + x^2 - 3x - 5 \\ +(-2x^3 + x^2 - 6x - 8) \\ \hline \end{array}$$

Step 2: 
$$-x^3 + 2x^2 - 9x - 13$$

The question for you is: Is it correct to change all the signs and add when subtracting? What mathematical property or relationship can justify this action?

10. Use the given polynomials to find the specified differences in a-d.

$$f(x) = x^3 + 2x^2 - 7x - 8 \quad g(x) = -4x - 7 \quad h(x) = 4x^2 - x - 15 \quad k(x) = -x^2 + 7x + 4$$

$$l(x) = 5x^2 - 7y^2 + 4xy$$

$$m(x) = -10x^2 + 9y^2 - 12xy + 4$$

Handwritten work for problem a):

$$4x^2 - x - 15$$

$$-(+x^2 - 7x - 4)$$

The above expressions are enclosed in a pink box. The minus sign in the second expression is written in red, and the terms  $+x^2$ ,  $-7x$ , and  $-4$  are also circled in red.

a)  $h(x) - k(x)$

b)  $f(x) - h(x)$

c)  $f(x) - g(x)$

Handwritten answer for problem d):

$$5x^2 - 8x - 19$$

The expression is enclosed in a purple box.

d)  $k(x) - f(x)$

e)  $l(x) - m(x)$

11. List three important things to remember when subtracting polynomials.

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Using the distributive property

**Multiply.**

1.  $2x(5x^2 + 7)$

2.  $9x(-x^2 - 3)$

3.  $5x^2(x^4 + 6x^3)$

4.  $-x(x^2 - x + 1)$   

$$\boxed{-x^3 + x^2 - x}$$

5.  $-3x^3(-2x^2 + x - 1)$   

$$\boxed{6x^5 - 3x^4 + 3x^3}$$

6.  $-1(x^2 - 4x + 8)$

**SET**

Topic: Adding and subtracting polynomials

**Add. Write your answers in descending order of the exponents. (Standard form)**

7.  $(3x^4 + 5x^2 - 1) + (2x^3 + x)$

8.  $(4x^2 + 7x - 4) + (x^2 - 7x + 14)$

9.  $(2x^3 + 6x^2 - 5x) + (x^5 + 3x^2 + 8x + 4)$

10.  $(-6x^5 - 2x + 13) + (4x^5 + 3x^2 + x - 9)$

$$\boxed{x^5 + 2x^3 + 9x^2 + 3x + 4}$$

**Subtract. Write your answers in descending order of the exponents. (Standard form)**

11.  $(5x^2 + 7x + 2) - (3x^2 + 6x - 2)$

12.  $(10x^4 + 2x^2 + 1) - (3x^4 + 3x + 11)$

$$\boxed{2x^2 + x}$$

13.  $(7x^3 - 3x + 7) - (4x^2 - 3x - 11)$

14.  $(x^4 - 1) - (x^4 + 1)$

**GO**

Topic: Using exponent rules to combine expressions

**Simplify.**

19.  $x^{-5} \cdot x^4 \cdot x^{-2}$

20.  $x^3 \cdot x^{-7} \cdot x^{-2}$

21.  $x^4 \cdot x^4 \cdot x^{-1}$

$$x^{-3} = \frac{1}{x^3}$$