Math 2 – Honors
Unit 3 – Quadratic Functions Continued
Lesson 2 → Sets of Numbers

Name________________________________
Date__________________________ Pd______

In mathematics, the numbers we use can be **categorized into sets**. Our number system has two sets, the **real numbers** and the **complex numbers**. We will work with both the real numbers and the complex numbers in this course.

➢ **DEFINITIONS:**

- <u>**REAL NUMBERS**</u> is the set of rational numbers and irrational numbers.
- <u>**COUNTING NUMBERS OR NATURAL NUMBERS**</u> is the set of numbers defined by {1, 2, 3, 4, 5, ...}.
- <u>**WHOLE NUMBERS**</u> is the set of numbers defined by {0, 1, 2, 3, 4, 5, ...}.
- <u>**INTEGERS**</u> is the set of numbers defined by {..., -3, -2, -1, 0, 1, 2, 3, ...} or the set of all positive and negative whole numbers.
- <u>**RATIONAL NUMBERS**</u> is the set of numbers defined by $\{\frac{p}{q} \mid p \text{ and } q \text{ are integers}, q \neq 0\}$ or the set of numbers in which the decimal terminates or the decimal repeats.

$\frac{3}{\sqrt{2}}$ $\frac{p}{q}$

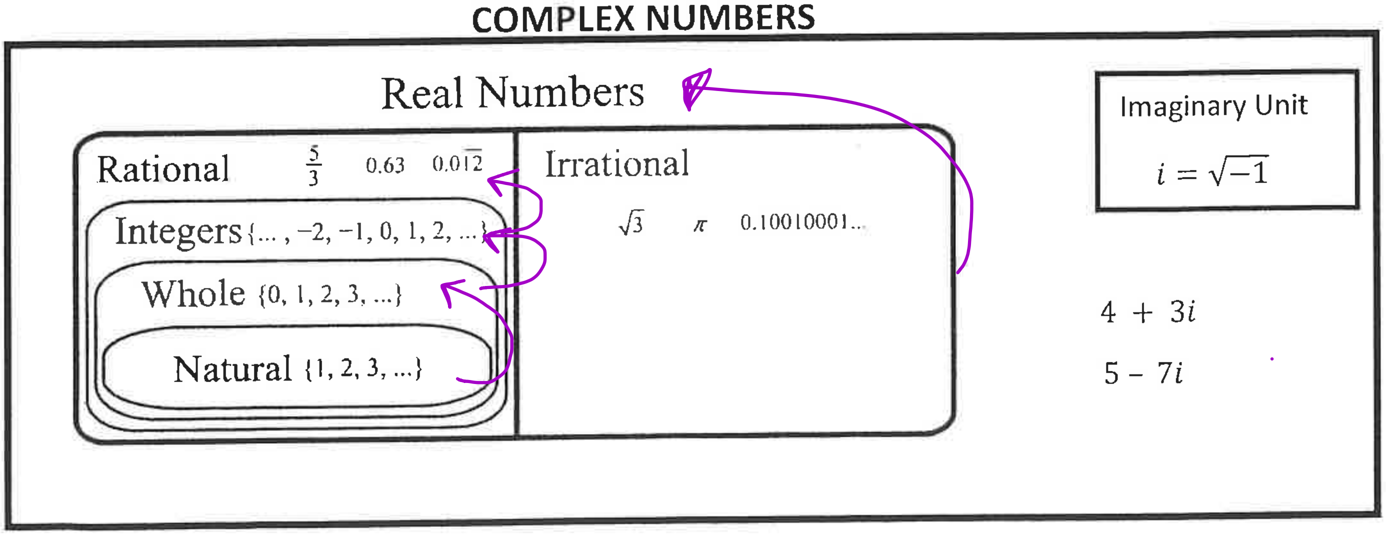
Examples: These are all **rational** numbers.

$\frac{1}{2} = 0.5$	*terminated decimal*	$5 = 5.0$	*terminated decimal*
$\frac{-2}{3} = -0.6666\ldots$	*repeating decimal*	$-\frac{12}{3} = 4.0$	*terminated decimal*
$\frac{2}{7} = 0.285714285\ldots$	*repeating decimal*	$\sqrt{4} = 2.0$	*terminated decimal*
$\frac{9}{4} = 2.25$	*terminated decimal*		

<u>**IRRATIONAL NUMBERS**</u> is the set of numbers in which the decimal does <u>not terminate</u> and does <u>not repeat</u>.
Examples: These are all **irrational** numbers.

$\sqrt{2} = 1.414213562\ldots$ *does not terminate nor repeats*
$\pi = 3.141592654\ldots$ *does not terminate nor repeats*
$\frac{\sqrt{3}}{5} = 0.3464101615\ldots$ *does not terminate nor repeats*

COMPLEX NUMBERS



- **COMPLEX NUMBERS:** the set of numbers including the Real Numbers and the imaginary unit, i. Complex number are written in the form $\boldsymbol{a} + \boldsymbol{bi}$ where a is the real part and bi is the imaginary part.

Conjugate $a - bi$

- **IMAGINARY UNIT:**

Some polynomial equations have complex (non-real) solutions, when a negative number is under the radical symbol.

For example: there is no real solution to $\sqrt{-16}$ or $\sqrt{-36}$.

Mathematicians created a new system of numbers using the imaginary unit, i, defined as $i = \sqrt{-1}$. With this new system of numbers, radicals of negative numbers can now be simplified!

Therefore: $\boldsymbol{i = \sqrt{-1}}$

Simplify:

$\sqrt{-16} =$ $4i$ $\sqrt{-1}\cdot\sqrt{16}$

$\sqrt{-20} =$ $\sqrt{20}\cdot\sqrt{-1} = 2i\sqrt{5}$

$\sqrt{-45} =$ $3i\sqrt{5}$

$\sqrt{-36} =$ $\sqrt{36}\cdot\sqrt{-1} = 6i$

$\sqrt{-27} =$ $3i\sqrt{3}$

$\sqrt{-75} =$ $5i\sqrt{3}$

Always, Sometimes or Never True:

________ 1.The sum of a rational number and an irrational number is irrational.

________ 2. The circumference of a circle is irrational.

________ 3. The diagonal of a square is irrational.

________ 4. The sum of two rational numbers is rational.

________ 5. The product of a rational number and an irrational number is irrational.

________ 6. The sum of two irrational numbers is irrational.

________ 7. The product of two rational numbers is irrational.

________ 8. The product of two irrational numbers is irrational.

________ 9. An expression containing both 6 and π is irrational.

________ 10. Between two rational numbers there is an irrational number.

________ 11. Between two irrational numbers there is an irrational number.

________ 12. The circumference of a circle is irrational.

________ 13. A real number is a complex number.

________ 14. A complex number can also a real number.

________ 15. A complex number can be only imaginary.

Math 2 – Honors
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Lesson 2 → Sets of Numbers HOMEWORK

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1. Determine whether each number is **rational** or **irrational:**

6 r	$\frac{5}{6}$ r	$\sqrt{6}+\sqrt{3}$ i	$1-\pi$ i	$5+\sqrt{6}$ i
$0.\overline{6}$ 2/3 r	π i	$\frac{\pi}{2}$ i	$\frac{\sqrt{6}}{\sqrt{3}}$ i	0.45 r 9/20
-6 r	$0.456789\ldots$ i	$4+\sqrt{3}$ i	0 rat	$0.\overline{273}$ rat

➢ Find a **rational number** and an **irrational number** between each pair of numbers:

2. 1.3 *and* 1.4

Rational: 1.35

Irrational: 1.3728...

3. .625 .7
$\frac{5}{8}$ *and* $\frac{7}{10}$

Rational: .65

Irrational: .6514983...

4. $\frac{7}{9}$ *and* 1.4
.77 1.4

Rational: .8

Irrational: .845317

5. $0.\overline{13}$ *and* $0.1\overline{3}$

Rational: ___________

Irrational: ___________

➢ **Always, Sometimes or Never True:**

_______ 6. The sum of a rational number and a rational number is rational.

_______ 7. The sum of a rational number and an irrational number is irrational.

_______ 8. The sum of an irrational number and an irrational number is irrational.

_______ 9. The product of a rational number and a rational number is rational.

_______ 10. The product of a rational number and an irrational number is irrational.

_______ 11. The product of an irrational number and an irrational number is irrational.

➢ Express each number in terms of i and then simplify:

12. $\sqrt{-36}$ $6i$	13. $\sqrt{-100}$ $10i$	14. $-\sqrt{-81}$ $-9i$	15. $2\sqrt{-49}$ $2 \cdot 7i$ $14i$
16. $\frac{1}{8}\sqrt{-64}$ $\frac{1}{8} \cdot 8i$ $\boxed{i}$	17. $\frac{-2}{3}\sqrt{-9}$ $\frac{-2}{3} \cdot i \cdot 3$ $-2i$	18. $\frac{3}{4}\sqrt{-144}$ $\frac{3}{4} \cdot i \cdot 12$ $9i$	19. $\frac{1}{3}\sqrt{-25}$ $\frac{1}{3} \cdot 5 \cdot i$ $\boxed{\frac{5}{3}i}$
20. $\sqrt{-\frac{1}{4}}$ $\frac{1}{2}i$	21. $\sqrt{-\frac{16}{25}}$ $\frac{4}{5}i$	22. $4\sqrt{-\frac{49}{64}}$ $4 \cdot i \cdot \frac{7}{8}$ $7/2\,i$	23. $\frac{3}{5}\sqrt{-\frac{100}{9}}$ $\frac{3}{5} \cdot \frac{10}{3} \cdot i$ $\frac{30}{15}i = 2i$
24. $\sqrt{-3}$ $i\sqrt{3}$	25. $\sqrt{-29}$ $i\sqrt{29}$	26. $3\sqrt{-11}$ $3i\sqrt{11}$	27. $-\sqrt{-10}$ $-i\sqrt{10}$
28. $\sqrt{-20}$ $i \cdot \sqrt{20}$ $2i\sqrt{5}$	29. $-\sqrt{-28}$ $- \cdot i \cdot \sqrt{28}$ $-1 \cdot i \cdot 2\sqrt{7}$ $\boxed{-2i\sqrt{7}}$	30. $2\sqrt{-75}$ $2 \cdot i \cdot \sqrt{75}$ $10i\sqrt{3}$	31. $5\sqrt{-8}$ $5i\sqrt{8}$ $\boxed{10i\sqrt{2}}$
32. $3\sqrt{-98}$ $21i\sqrt{2}$	33. $-2\sqrt{-75}$ $-2 \cdot 5 \cdot \sqrt{3} \cdot i$ $-10i\sqrt{3}$	34. $\pm\sqrt{-45}$ $\pm 3i\sqrt{5}$	35. $\frac{3\sqrt{7}}{\sqrt{-28}}$ $\frac{3}{2i}$ $\frac{3\sqrt{7}}{2i\sqrt{7}}$

Math 2 – Honors
Unit 3 – Quadratic Functions Continued
Lesson 3 → Completing the Square

Name________________________________
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Ways to Graph a Parabola: $y = a(x-h)^2 + k$ and $y = a(x - int.)(x - int.)$

- What if a quadratic equation is in standard form? $y = ax^2 + bx + c$
- Recall from Math I: The vertex can be found using $\left(\frac{-b}{2a}, y\right)$ and the axis of symmetry is $x = \frac{-b}{2a}$.

✓ Complete the information for each parabola. Graph on the calculator to verify your vertex.

$y = -2x^2 - 12x - 16$	$y = 3x^2 + 10x - 2$	$y = 2x^2 + 15x + 29$
1. Vertex:	1. Vertex:	1. Vertex:
2. Maximum or Minimum	2. Maximum or Minimum	2. Maximum or Minimum
3. Axis of Symmetry:	3. Axis of Symmetry:	3. Axis of Symmetry:
4. y – intercept:	4. y – intercept:	4. y – intercept:
5. x – intercepts:	5. x – intercepts:	5. x – intercepts:
6. Domain:	6. Domain:	6. Domain:
7. Range:	7. Range:	7. Range:

- How can we solve a quadratic equation that has **irrational** or **complex** solutions?

❖ **COMPLETING THE SQUARE** will allow us to find **ALL** solutions (rational, irrational & imaginary).

1) **REWRITE** as $x^2 + bx + c = 0$ as $x^2 + bx = -c$
2) $x^2 + bx +$ _____ $= -c +$ _____
3) **COMPLETE THE SQUARE** by taking half of b; square it and ADD IT TO BOTH SIDES of the equation in the blanks.
4) **FACTOR** the perfect square trinomial.
5) Take the **SQUARE ROOT** of both sides. Don't forget to include a $\pm$ to create 2 solutions.
6) **SOLVE** both equations. **SIMPLIFY** all irrational and complex solutions.

1. $x^2 - 6x + 8 = 0$	2. $x^2 + 16x - 16 = 0$
$(x-4)(x-2) = 0$ $\boxed{x = +4, +2}$	A B C $x^2 + 16x + \underline{64} = +16 + \underline{64}$ $\sqrt{(x+8)^2} = \sqrt{80}$ $x + 8 = \pm\sqrt{80}$ $x + 8 = \pm 4\sqrt{5}$ $-8 \qquad -8$ 2 80 2 40 2 20 2 10 5

$\boxed{x = \pm 4\sqrt{5} - 8}$ $3(x^2-2x$

3. $x^2 + 12x + 43 = 0$ $x^2+12x+36=-43+36$ $\sqrt{(x+6)^2}=\sqrt{-7}$ $x+6 = \pm i\sqrt{7}$ $-6 \qquad -6$ $\boxed{-6 \pm i\sqrt{7}}$	4. $3x^2 - 6x - 45 = 0$ $3(x-5)(x+3)$ $x=5, -3$ $3x^2-6x+\_\_\_ = 45+\_\_\_$ $3(x^2-2x+1) = 45+3\cdot 1$ $\frac{3(x-1)^2}{3} = \frac{48}{3}$ $\sqrt{(x-1)^2} = \sqrt{16}$ $x-1=\pm 4$ $x = 1 \pm 4$

$x = \pm i\sqrt{7} - 6$

$x = 5$ or -3

1) **BEGIN** with $ax^2 + bx + c = 0$ and **MULTIPLY** "a" to "c"
2) **REWRITE** $x^2 + bx = -c \cdot a$
3) $x^2 + bx + \_\_\_\_\_ = -c \cdot a + \_\_\_\_\_$
4) **COMPLETE THE SQUARE** by taking half of b; square it and ADD IT TO BOTH SIDES of the equation in the blanks.
5) **FACTOR** the perfect square trinomial.
6) Take the **SQUARE ROOT** of both sides. Don't forget to include a $\pm$ to create 2 solutions.
7) **SOLVE** both equations. **SIMPLIFY** all irrational and complex solutions.
8) **DIVIDE** by "a" and **REDUCE** all final solutions.

5. $3x^2 + 10x - 8 = 0$	6. $4x^2 - 8x + 3 = 0$
7. $4x^2 - 16x + 71 = 0$	8. $3x^2 + 6x - 4 = 0$