

QUIZ DATE: _____

TEST DATE: _____

Math 2

Name _____

Unit 3 – Rational Exponents and Solving Quadratics

Date _____ Pd _____

Lesson 1 → Rational Exponents

❖ Rational or fractional exponents can be rewritten in radical form:

Converting from rational exponent to radical form:

$$x^{a/b} = \sqrt[b]{x^a}$$

The **numerator** of the exponent becomes the **exponent** of the radicand.

The **denominator** of the exponent becomes the **index** of the radical.

➤ EXAMPLES:

| | |
|-----------------|--|
| 1. $9^{1/2} =$ | 2. $64^{1/3} =$ |
| 3. $x^{2/3} =$ | 4. $16^{-1/2} =$ *** Negative exponents become fractions |
| 5. $4x^{1/7} =$ | 6. $(3x)^{3/4} =$ |

➤ You Try: Write each expression in **simplest** radical form:

| | | | | |
|-----------------|------------------|----------------------|----------------|------------------|
| 1. $2^{1/2}$ | 2. $3^{1/2}$ | 3. $9^{-1/2}$ | 4. $25^{1/2}$ | 5. $7^{1/3}$ |
| 6. $x^{4/7}$ | 7. $15^{-1/4}$ | 8. $x^{1/2}$ | 9. $y^{-1/2}$ | 10. $4x^{2/3}$ |
| 11. $3x^{-1/2}$ | 12. $(9a)^{1/2}$ | 13. $(16x^5)^{-1/2}$ | 14. $27^{5/3}$ | 15. $(5x)^{1/6}$ |

❖ Radicals can be rewritten in rational exponent form:

Converting from radical to rational exponent form:

$$\sqrt[b]{x^a} = x^{a/b}$$

The **exponent** of the radicand becomes the **numerator** of the fraction.

The **index** of the radical becomes the **denominator** of the fraction.

➤ **EXAMPLES:**

| | |
|---------------------|--------------------------------|
| 1. $\sqrt{5} =$ | 2. $\sqrt[3]{7^2} =$ |
| 3. $\sqrt[4]{x} =$ | 4. $\frac{1}{\sqrt[3]{x^2}} =$ |
| 5. $5\sqrt[3]{x} =$ | 6. $\sqrt[5]{3x^2} =$ |

➤ You Try: Write each expression in **exponential** form:

| | | | | |
|------------------------|-----------------------------|---------------------|---------------------|------------------------|
| 16. $\sqrt{7}$ | 17. $\sqrt{6}$ | 18. $\sqrt[4]{8}$ | 19. $\sqrt[5]{18}$ | 20. $\sqrt[3]{x^2}$ |
| 21. $\sqrt[3]{(2x^2)}$ | 22. $\frac{1}{\sqrt[3]{5}}$ | 23. $2\sqrt[4]{15}$ | 24. $\sqrt{(3x)^7}$ | 25. $(\sqrt[3]{3v})^2$ |

Lesson 1 → Rational Exponents HOMEWORK

➤ Rewrite each expression in radical form and then simplify completely:

| | | | | |
|----------------|------------------|------------------|--------------------|------------------|
| 1. $100^{1/2}$ | 2. $125^{1/3}$ | 3. $(17x)^{1/2}$ | 4. $64^{1/3}$ | 5. $16^{1/4}$ |
| 6. $16^{3/4}$ | 7. $(8^{1/2})^2$ | 8. $(8^{1/3})^3$ | 9. $(16x^4)^{1/4}$ | 10. $125^{-1/3}$ |

➤ Rewrite each expression in exponential form and then simplify completely:

| | | | | |
|------------------------|---------------------|------------------------|------------------------|---------------------------------|
| 11. $\sqrt{81}$ | 12. $\sqrt[3]{125}$ | 13. $\sqrt[4]{20x^3}$ | 14. $\sqrt[3]{-64}$ | 15. $\sqrt[3]{8}$ |
| 16. $(\sqrt[3]{8x})^3$ | 17. $(\sqrt{98})^2$ | 18. $(\sqrt[3]{98})^3$ | 19. $(\sqrt[4]{98})^4$ | 20. $(\frac{1}{\sqrt{x}})^{-4}$ |

➤ Evaluate each of the following expressions. Give exact answers.

| | | | | |
|----------------------------|------------------------|----------------------------|---------------------------|-------------------------------|
| 21. $27^{2/3}$ | 22. $1^{3.5}$ | 23. $(\frac{1}{32})^{1/5}$ | 24. $(-27)^{-2/3}$ | 25. $4^{2.5}$ |
| 26. $(\frac{1}{16})^{3/4}$ | 27. $216^{1/3}$ | 28. $16^{-1/4}$ | 29. $25^{3/2}$ | 30. $(x^6)^{1/2}$ |
| 31. $(9x^2)^{1/2}$ | 32. $(4x^{1/2})^{1/2}$ | 33. $((8x^3)^2)^{1/3}$ | 34. $(9x^{-5}y^2)^{-1/2}$ | 35. $((-4x^3y^{-2})^3)^{1/2}$ |

What Happens When the King of Beasts Runs in Front of a Train?



Match each expression with its equivalent expression. Write the corresponding letter in the box at the bottom of the page.

| | | | | |
|-----|--------------------|--|---|----------------------|
| 1. | $7^{1/2}$ | | N | $10^{3/2}$ |
| 2. | $\sqrt[3]{2}$ | | N | 216 |
| 3. | $a^{6/5}$ | | I | 3 |
| 4. | $(\sqrt[3]{3a})^4$ | | I | $\sqrt{7}$ |
| 5. | $(6v)^{1.5}$ | | T | $3n^2$ |
| 6. | $\sqrt[6]{2}$ | | F | $10^{1/6}$ |
| 7. | $4^{4/3}$ | | S | $(\sqrt[5]{a})^6$ |
| 8. | $(\sqrt{10})^3$ | | O | $5^{5/4}$ |
| 9. | $(x^6)^{1/2}$ | | H | $\frac{1}{\sqrt{m}}$ |
| 10. | $\sqrt{6p}$ | | O | $(6p)^{1/2}$ |
| 11. | $\sqrt[6]{10}$ | | L | $(\sqrt{10n})^3$ |
| 12. | $(9n^4)^{1/2}$ | | T | $(3a)^{4/3}$ |
| 13. | $m^{-1/2}$ | | E | $2^{5/3}$ |
| 14. | $(\sqrt[3]{2})^5$ | | E | $2^{1/6}$ |
| 15. | $(10n)^{3/2}$ | | D | x^3 |
| 16. | $9^{1/2}$ | | H | $(\sqrt{6v})^3$ |
| 17. | $(\sqrt[4]{5})^5$ | | E | $(\sqrt[3]{4})^4$ |
| 18. | $36^{1.5}$ | | T | $2^{1/3}$ |

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|--|---|---|---|--|---|---|---|--|----|----|--|----|----|----|--|----|----|----|----|
| 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 | | 10 | 11 | | 12 | 13 | 14 | | 15 | 16 | 17 | 18 |
|---|---|---|--|---|---|---|--|---|---|---|--|----|----|--|----|----|----|--|----|----|----|----|

QUIZ DATES: _____ & _____

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Math 2

Name _____

Unit 3 – Quadratic Functions Continued

Date _____ Pd _____

Lesson 2 → Simplifying Square Roots

| PERFECT SQUARES | | | | | | | |
|-------------------|-----------------|-------------------|-----------------|-------------------|-----------------|-------------------|-----------------|
| NUMBER MULTIPLIED | PERFECT SQUARES | NUMBER MULTIPLIED | PERFECT SQUARES | NUMBER MULTIPLIED | PERFECT SQUARES | NUMBER MULTIPLIED | PERFECT SQUARES |
| 1 X 1 = | | 6 X 6 = | | 11 X 11 = | | 16 X 16 = | |
| 2 X 2 = | | 7 X 7 = | | 12 X 12 = | | 17 X 17 = | |
| 3 X 3 = | | 8 X 8 = | | 13 X 13 = | | 18 X 18 = | |
| 4 X 4 = | | 9 X 9 = | | 14 X 14 = | | 19 X 19 = | |
| 5 X 5 = | | 10 X 10 = | | 15 X 15 = | | 20 X 20 = | |

Taking the square root of a number is the **inverse** of raising the number to the second power.

SQUARE ROOTS and CUBE ROOTS

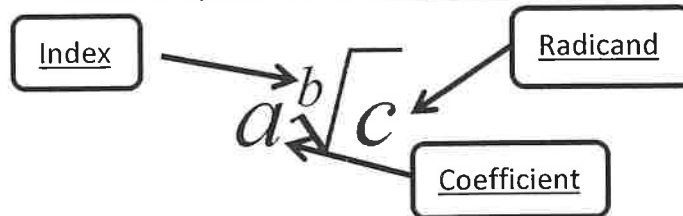
For example: If $3^2 = \underline{\hspace{2cm}}$, then $\sqrt{9} = \underline{\hspace{2cm}}$. For example: If $7^2 = \underline{\hspace{2cm}}$, then $\sqrt{49} = \underline{\hspace{2cm}}$.

Taking the cube root of a number is the inverse of raising the number to the third power.

For example: If $3^3 = \underline{\hspace{2cm}}$, then $\sqrt[3]{27} = \underline{\hspace{2cm}}$. For example: If $7^3 = \underline{\hspace{2cm}}$, then $\sqrt[3]{343} = \underline{\hspace{2cm}}$.

PARTS OF A RADICAL

An expression that contains a square root is a radical. It can have three parts.



➤ Simplify the following radical expressions.

$\sqrt{100} = \underline{\hspace{2cm}}$ $3\sqrt{121} = \underline{\hspace{2cm}}$ $-\sqrt{225} = \underline{\hspace{2cm}}$ $-2\sqrt{144} = \underline{\hspace{2cm}}$

$\sqrt{25} = \underline{\hspace{2cm}}$ $7\sqrt{81} = \underline{\hspace{2cm}}$ $\pm\sqrt{49} = \underline{\hspace{2cm}}$ $\pm 9\sqrt{9} = \underline{\hspace{2cm}}$

➤ What is the radicand is not a perfect square but has a factor that is a perfect square?

- Simplify: $\sqrt{24} =$

What is the highest factor of 24 that is also a perfect square? _____. Therefore, $24 = \text{_____} \cdot \text{_____}$

- Simplify: $\sqrt{32} =$

What is the highest factor of 32 that is also a perfect square? _____. Therefore, $32 = \text{_____} \cdot \text{_____}$

- Simplify: $\sqrt{54}$

What is the highest factor of 54 that is also a perfect square? _____. Therefore, $54 = \text{_____} \cdot \text{_____}$

➤ Classwork:

| | | | | |
|-------------------|-----------------------------|----------------------|-----------------------------|-----------------------|
| 1. $\sqrt{18}$ | 2. $\sqrt{20}$ | 3. $\sqrt{40}$ | 4. $\sqrt{50}$ | 5. $\sqrt{63}$ |
| 6. $\pm\sqrt{63}$ | 7. $\sqrt{48}$ | 8. $\sqrt{98}$ | 9. $\sqrt{75}$ | 10. $\sqrt{256}$ |
| 11. $2\sqrt{18}$ | 12. $-4\sqrt{12}$ | 13. $5\sqrt{24}$ | 14. $\frac{-1}{2}\sqrt{20}$ | 15. $5\sqrt{500}$ |
| 16. $-\sqrt{44}$ | 17. $12\sqrt{60}$ | 18. $-10\sqrt{80}$ | 19. $\frac{1}{2}\sqrt{8}$ | 20. $\pm\sqrt{12}$ |
| 21. $3\sqrt{250}$ | 22. $-\frac{4}{5}\sqrt{50}$ | 23. $\pm 7\sqrt{90}$ | 24. $3\sqrt{10}$ | 25. $\pm 2\sqrt{117}$ |
| 26. $\sqrt{x^2}$ | 27. $\sqrt{16x^2}$ | 28. $\sqrt{9x^3}$ | 29. $\sqrt{27x^4}$ | 30. $\sqrt{48x^3}$ |

Math 2

Unit 3 – Rational Exponents and Solving Quadratics

Lesson 2 → Simplifying Square Roots HOMEWORK

Name _____

Date _____ Pd _____

| | | |
|----------------------------|----------------------------|---------------------------|
| 1. $\sqrt{125n}$ | 2. $\sqrt{216v}$ | 3. $\sqrt{512k^2}$ |
| 4. $\sqrt{512m^3}$ | 5. $\sqrt{216k^4}$ | 6. $\sqrt{100v^3}$ |
| 7. $\sqrt{80p^3}$ | 8. $\sqrt{45p^2}$ | 9. $\sqrt{147m^3n^3}$ |
| 10. $\sqrt{200m^4n}$ | 11. $\sqrt{75x^2y}$ | 12. $\sqrt{64m^3n^3}$ |
| 13. $\sqrt{16u^4v^3}$ | 14. $\sqrt{28x^3y^3}$ | 15. $\sqrt{36x^2y^3}$ |
| 16. $\sqrt{384x^4y^3}$ | 17. $7\sqrt{96m^3}$ | 18. $6\sqrt{72x^2}$ |
| 19. $-6\sqrt{150r}$ | 20. $5\sqrt{80a^3}$ | 21. $2\sqrt{125v}$ |
| 22. $-8\sqrt{24k^3}$ | 23. $-4\sqrt{192x}$ | 24. $2\sqrt{8p^2q^3r}$ |
| 25. $-4\sqrt{216x^2y^2z}$ | 26. $-3\sqrt{24a^4b^2c^3}$ | 27. $3\sqrt{16x^4y^4z}$ |
| 28. $-2\sqrt{48a^3b^4c^2}$ | 29. $6\sqrt{75mp^2q^3}$ | 30. $4\sqrt{36x^2y^3z^4}$ |

In mathematics, the numbers we use can be **categorized into sets**. Our number system has two sets, the **real numbers** and the **complex numbers**. We will work with both the real numbers and the complex numbers in this course.

➤ **DEFINITIONS:**

- **REAL NUMBERS** is the set of rational numbers and irrational numbers.
- **COUNTING NUMBERS OR NATURAL NUMBERS** is the set of numbers defined by $\{1, 2, 3, 4, 5, \dots\}$.
- **WHOLE NUMBERS** is the set of numbers defined by $\{0, 1, 2, 3, 4, 5, \dots\}$.
- **INTEGERS** is the set of numbers defined by $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ or the set of all positive and negative whole numbers.
- **RATIONAL NUMBERS** is the set of numbers defined by $\{\frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0\}$ or the set of numbers in which the decimal terminates or the decimal repeats.

Examples: These are all **rational** numbers.

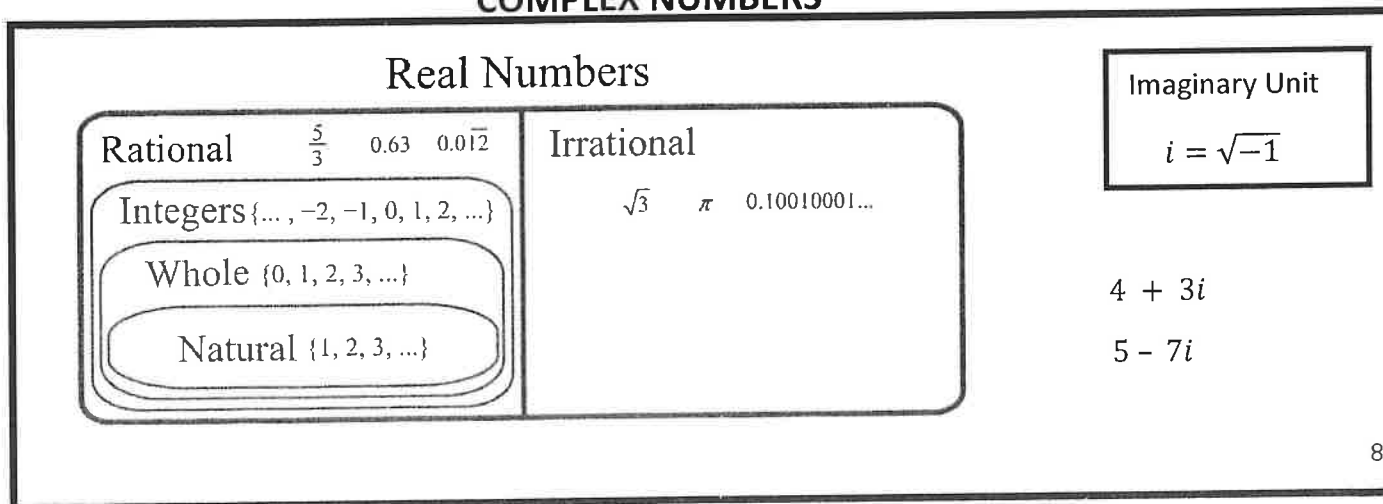
| | | | |
|-----------------------------------|---------------------------|-----------------------|---------------------------|
| $\frac{1}{2} = 0.5$ | <i>terminated decimal</i> | $5 = 5.0$ | <i>terminated decimal</i> |
| $\frac{-2}{3} = -0.6666 \dots$ | <i>repeating decimal</i> | $-\frac{12}{3} = 4.0$ | <i>terminated decimal</i> |
| $\frac{2}{7} = 0.285714285 \dots$ | <i>repeating decimal</i> | $\sqrt{4} = 2.0$ | <i>terminated decimal</i> |
| $\frac{9}{4} = 2.25$ | <i>terminated decimal</i> | | |

IRRATIONAL NUMBERS is the set of numbers in which the decimal does not terminate and does not repeat.

Examples: These are all **irrational** numbers.

| | |
|--|---------------------------------------|
| $\sqrt{2} = 1.414213562\dots$ | <i>does not terminate nor repeats</i> |
| $\pi = 3.141592654\dots$ | <i>does not terminate nor repeats</i> |
| $\frac{\sqrt{3}}{5} = 0.3464101615\dots$ | <i>does not terminate nor repeats</i> |

COMPLEX NUMBERS



➤ **COMPLEX NUMBERS:** the set of numbers including the Real Numbers and the imaginary unit, i . Complex numbers are written in the form $a + bi$ where a is the real part and bi is the imaginary part.

➤ **IMAGINARY UNIT:**

Some polynomial equations have complex (non-real) solutions, when a negative number is under the radical symbol.

For example: there is no real solution to $\sqrt{-16}$ or $\sqrt{-36}$.

Mathematicians created a new system of numbers using the imaginary unit, i , defined as $i = \sqrt{-1}$. With this new system of numbers, radicals of negative numbers can now be simplified!

Therefore: $i = \sqrt{-1}$

Simplify: $\sqrt{-16} =$ _____ $\sqrt{-36} =$ _____
 $\sqrt{-20} =$ _____ $\sqrt{-27} =$ _____
 $\sqrt{-45} =$ _____ $\sqrt{-75} =$ _____

Always, Sometimes or Never True:

- _____ 1. The sum of a rational number and an irrational number is irrational.
- _____ 2. The circumference of a circle is irrational.
- _____ 3. The diagonal of a square is irrational.
- _____ 4. The sum of two rational numbers is rational.
- _____ 5. The product of a rational number and an irrational number is irrational.
- _____ 6. The sum of two irrational numbers is irrational.
- _____ 7. The product of two rational numbers is irrational.
- _____ 8. The product of two irrational numbers is irrational.
- _____ 9. An expression containing both 6 and π is irrational.
- _____ 10. Between two rational numbers there is an irrational number.
- _____ 11. Between two irrational numbers there is an irrational number.
- _____ 12. The circumference of a circle is irrational.
- _____ 13. A real number is a complex number.
- _____ 14. A complex number can also be a real number.
- _____ 15. A complex number can be only imaginary.

1. Determine whether each number is **rational** or **irrational**:

| | | | | |
|-------------|---------------|-----------------------|-----------------------------|--------------------|
| 6 | $\frac{5}{6}$ | $\sqrt{6} + \sqrt{3}$ | $1 - \pi$ | $5 + \sqrt{6}$ |
| $0.\bar{6}$ | π | $\frac{\pi}{2}$ | $\frac{\sqrt{6}}{\sqrt{3}}$ | 0.45 |
| -6 | 0.456789 ... | $4 + \sqrt{3}$ | 0 | $0.\overline{273}$ |

➤ Find a **rational number** and an **irrational number** between each pair of numbers:

2. 1.3 and 1.4

Rational: _____

Irrational: _____

3. $\frac{5}{8}$ and $\frac{7}{10}$

Rational: _____

Irrational: _____

4. $\frac{7}{9}$ and 1.4

Rational: _____

Irrational: _____

5. $0.\overline{13}$ and $0.1\bar{3}$

Rational: _____

Irrational: _____

➤ **Always, Sometimes or Never True:**

_____ 6. The sum of a rational number and a rational number is rational.

_____ 7. The sum of a rational number and an irrational number is irrational.

_____ 8. The sum of an irrational number and an irrational number is irrational.

_____ 9. The product of a rational number and a rational number is rational.

_____ 10. The product of a rational number and an irrational number is irrational.

_____ 11. The product of an irrational number and an irrational number is irrational.

➤ Express each number in terms of i and then simplify:

| | | | |
|-----------------------------|-----------------------------|------------------------------|--|
| 12. $\sqrt{-36}$ | 13. $\sqrt{-100}$ | 14. $-\sqrt{-81}$ | 15. $2\sqrt{-49}$ |
| 16. $\frac{1}{8}\sqrt{-64}$ | 17. $\frac{-2}{3}\sqrt{-9}$ | 18. $\frac{3}{4}\sqrt{-144}$ | 19. $\frac{1}{3}\sqrt{-25}$ |
| 20. $\sqrt{-\frac{1}{4}}$ | 21. $\sqrt{-\frac{16}{25}}$ | 22. $4\sqrt{-\frac{49}{64}}$ | 23. $\frac{3}{5}\sqrt{-\frac{100}{9}}$ |
| 24. $\sqrt{-3}$ | 25. $\sqrt{-29}$ | 26. $3\sqrt{-11}$ | 27. $-\sqrt{-10}$ |
| 28. $\sqrt{-20}$ | 29. $-\sqrt{-28}$ | 30. $2\sqrt{-75}$ | 31. $5\sqrt{-8}$ |
| 32. $3\sqrt{-98}$ | 33. $-2\sqrt{-75}$ | 34. $\pm\sqrt{-45}$ | 35. $\frac{3\sqrt{7}}{\sqrt{-28}}$ |

Ways to Graph a Parabola: $y = a(x - h)^2 + k$ and $y = a(x - int.)(x - int.)$

- What if a quadratic equation is in standard form? $y = ax^2 + bx + c$
- Recall from Math I: The vertex can be found using $\left(\frac{-b}{2a}, y\right)$ and the axis of symmetry is $x = \frac{-b}{2a}$.

✓ Complete the information for each parabola. Graph on the calculator to verify your vertex.

| $y = -2x^2 - 12x - 16$ | $y = 3x^2 + 10x - 2$ | $y = 2x^2 + 15x + 29$ |
|------------------------|-----------------------|-----------------------|
| 1. Vertex: | 1. Vertex: | 1. Vertex: |
| 2. Maximum or Minimum | 2. Maximum or Minimum | 2. Maximum or Minimum |
| 3. Axis of Symmetry: | 3. Axis of Symmetry: | 3. Axis of Symmetry: |
| 4. y – intercept: | 4. y – intercept: | 4. y – intercept: |
| 5. x – intercepts: | 5. x – intercepts: | 5. x – intercepts: |
| 6. Domain: | 6. Domain: | 6. Domain: |
| 7. Range: | 7. Range: | 7. Range: |

- How can we solve a quadratic equation that has irrational or complex solutions?
- ❖ **COMPLETING THE SQUARE** will allow us to find all solutions (rational, irrational & imaginary).
 - 1) **REWRITE** as $x^2 + bx + c = 0$ as $x^2 + bx = -c$
 - 2) $x^2 + bx + \underline{\hspace{2cm}} = -c + \underline{\hspace{2cm}}$
 - 3) **COMPLETE THE SQUARE** by taking half of b ; square it and **ADD IT TO BOTH SIDES** of the equation in the blanks.
 - 4) **FACTOR** the perfect square trinomial.
 - 5) Take the **SQUARE ROOT** of both sides. Don't forget to include a \pm to create 2 solutions.
 - 6) **SOLVE** both equations. **SIMPLIFY** all irrational and complex solutions.

| | |
|-----------------------|-------------------------|
| 1. $x^2 - 6x + 8 = 0$ | 2. $x^2 + 16x - 16 = 0$ |
|-----------------------|-------------------------|

3. $x^2 + 12x + 43 = 0$

4. $3x^2 - 6x - 45 = 0$

- 1) **BEGIN** with $ax^2 + bx + c = 0$ and **MULTIPLY** "a" to "c"
- 2) **REWRITE** $x^2 + bx = -c \cdot a$
- 3) $x^2 + bx + \underline{\hspace{2cm}} = -c \cdot a + \underline{\hspace{2cm}}$
- 4) **COMPLETE THE SQUARE** by taking half of b ; square it and **ADD IT TO BOTH SIDES** of the equation in the blanks.
- 5) **FACTOR** the perfect square trinomial.
- 6) Take the **SQUARE ROOT** of both sides. Don't forget to include a \pm to create 2 solutions.
- 7) **SOLVE** both equations. **SIMPLIFY** all irrational and complex solutions.
- 8) **DIVIDE** by "a" and **REDUCE** all final solutions.

5. $3x^2 + 10x - 8 = 0$

6. $4x^2 - 8x + 3 = 0$

7. $4x^2 - 16x + 71 = 0$

8. $3x^2 + 6x - 4 = 0$

SOLVE BY COMPLETING THE SQUARE:

1. $x^2 + 14x - 51 = 0$

2. $x^2 - 12x + 23 = 0$

3. $x^2 - 4x + 6 = 0$

4. $x^2 - 10x + 18 = 0$

5. $x^2 + 18x - 40 = 0$

6. $4x^2 + 4x + 36 = 0$

7. $x^2 + 2x + 20 = 0$

8. $3x^2 + 12x + 21 = 0$

9. $3x^2 - 8x + 4 = 0$

10. $3x^2 - 2x - 5 = 0$

11. $2x^2 - 2x - 5 = 0$

12. $10x^2 + 4x + 68 = 0$

Math 2
Unit 3 – Rational Exponents and Solving Quadratics
Lesson 5 → Discriminant & Quadratic Formula

Name _____
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- ❖ Solve the following equations by factoring.
- ❖ Graph the equation.

| | | |
|----------------------------|----------------------------|----------------------------|
| 1. $x^2 + x - 6 = 0$ | 2. $x^2 + 6x + 9 = 0$ | 3. $x^2 + 4 = 0$ |
| | | |
| Number of Solutions: _____ | Number of Solutions: _____ | Number of Solutions: _____ |

➤ **Quadratic Equation:** $ax^2 + bx + c = 0$

➤ **The Discriminant:** $b^2 - 4ac$

✓ The discriminant is used to determine the **number** and **type** of solutions (roots) of a quadratic equation.

❖ Using the same three examples from above, find the value of the discriminant and describe the roots.

| | | |
|-----------------------------|-----------------------------|-----------------------------|
| 1. $x^2 + x - 6 = 0$ | 2. $x^2 + 6x + 9 = 0$ | 3. $x^2 + 4 = 0$ |
| D = _____ # of Roots: _____ | D = _____ # of Roots: _____ | D = _____ # of Roots: _____ |
| Type of Roots: _____ | Type of Roots: _____ | Type of Roots: _____ |

➤ **Discriminant Conclusions:**

| Value of the Discriminant: $b^2 - 4ac$ | Number and Type of Roots | What does the graph look like? |
|--|--------------------------|---------------------------------|
| $b^2 - 4ac$ is POSITIVE and a PERFECT SQUARE $b^2 - 4ac > 0$ | | Intersects the x-axis twice |
| $b^2 - 4ac$ is POSITIVE and NOT a PERFECT SQUARE $b^2 - 4ac > 0$ | | Intersects the x-axis twice |
| $b^2 - 4ac = 0$ | | Intersects the x-axis once |
| $b^2 - 4ac$ is NEGATIVE $b^2 - 4ac < 0$ | | Never Intersects the x-axis |

❖ **Classwork:** Find the value of the discriminant and state the number and type of roots.

| Equation | Discriminant | Number and Type of Roots | Rational or Irrational |
|-------------------------|--------------|--------------------------|------------------------|
| 1. $8x^2 + 2x - 1 = 0$ | | | |
| 2. $x^2 + x + 1 = 0$ | | | |
| 3. $x^2 - 27 = 0$ | | | |
| 4. $x^2 - 8x = -16$ | | | |
| 5. $x^2 + 4x + 9 = 10$ | | | |
| 6. $3x^2 + 5x - 12 = 0$ | | | |

➤ Solving Quadratic Equations using the Quadratic Formula

- $ax^2 + bx + c = 0$

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- The Quadratic Formula is used to solve any quadratic equation, especially those that will not factor.

- Examples: Solve using the Quadratic Formula $\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. $x^2 - 5x - 24 = 0$

2. $x^2 + 5x + 5 = 0$

$$3. 4x^2 + 8x - 1 = 0$$

$$4. 4x^2 = -11x + 20$$

$$5. x^2 + 25 = 10x$$

$$6. x^2 + 2x + 4 = 0$$

Math 2

Unit 3 – Rational Exponents and Solving Quadratics

Lesson 5 → Discriminant & Quadratic Formula **HOMEWORK**

Name _____

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❖ Solve using the Quadratic Formula → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

❖ Express answers in simplest radical form or complex form. **NO DECIMALS!!**

| | |
|--------------------------|------------------------|
| 1. $4x^2 + 11x - 20 = 0$ | 2. $x^2 - 5x - 24 = 0$ |
| 3. $x^2 - 3x - 3 = 0$ | 4. $x^2 + 5x + 5 = 0$ |
| 5. $x^2 = -x + 1$ | 6. $4x^2 + 8x = 1$ |

7. $4x^2 + 7x - 15 = 0$

8. $x^2 + 3x = 10$

9. $x^2 - x + 3 = 0$

10. $2x^2 - 14x = -23$

11. $x^2 = 2x + 48$

12. $2x^2 + 39 = -18x$

13. $5x^2 + 3x + 1 = 0$

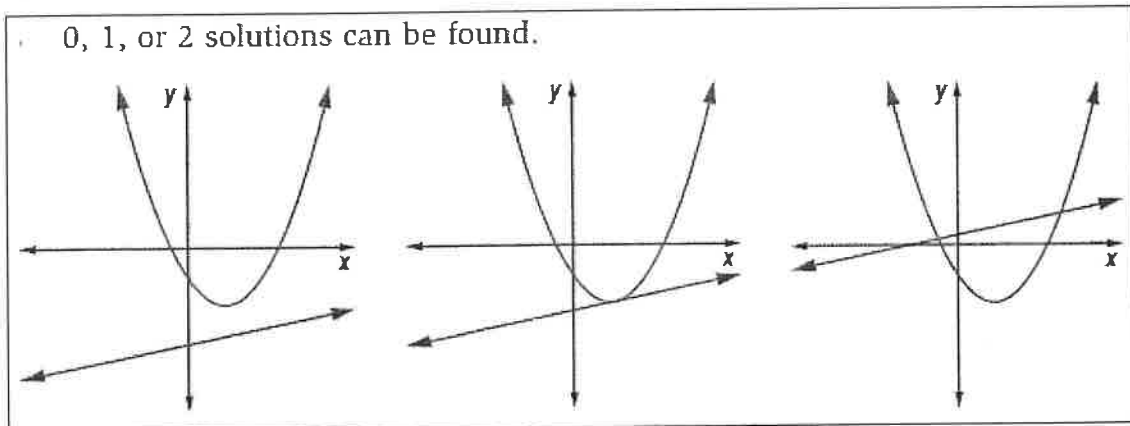
14. $5x^2 + 50x + 125 = 0$

Solve using the best method: Factoring, Completing the Square or Quadratic Formula

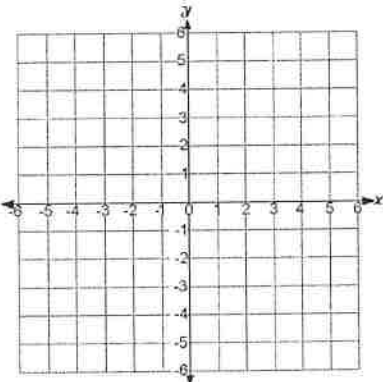
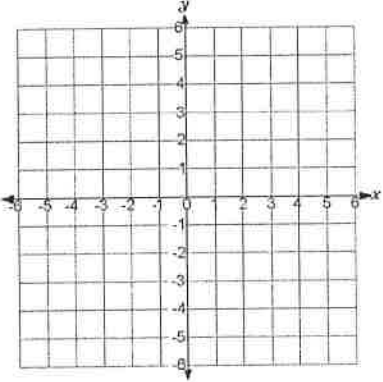
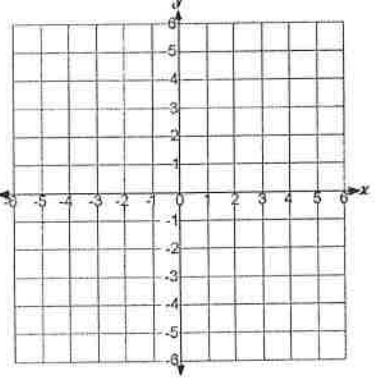
Express all solutions in simplest form.

| | |
|--------------------------|--------------------------|
| 1. $x^2 + 4x - 9 = 13$ | 2. $x^2 + 7x + 12 = 0$ |
| 3. $7(x - 3)^2 = 35$ | 4. $4x^2 = 36$ |
| 5. $x^2 = 81$ | 6. $x^2 + 9x + 38 = 13$ |
| 7. $3x^2 - 6x = 13$ | 8. $x^2 + 6x - 8 = 0$ |
| 9. $x^2 = 3x + 8$ | 10. $x^2 - 121 = 0$ |
| 11. $(x + 2)^2 - 6 = 11$ | 12. $5x^2 - 7x + 13 = 0$ |

- When a linear function and a quadratic function are graphed on the same coordinate plane, the graphs below represent the possible number of solutions for the system of equations.



- Solve each system of equations **graphically**:

| | | |
|---|--|--|
| $y = x^2 - x + 3$ $y = 2x - 1$  <p>$(x, y) = \underline{\hspace{2cm}}$</p> | $y = x^2 - 3x + 2$ $y = x - 2$  <p>$(x, y) = \underline{\hspace{2cm}}$</p> | $y = 10x^2 - 28x - 39$ $y = 2x + 1$  <p>$(x, y) = \underline{\hspace{2cm}}$</p> |
|---|--|--|

- Solve each system of equations **algebraically**:

| | | |
|--|--|---|
| $y = x^2 - x + 3$ $y = 2x - 1$ <p>$(x, y) = \underline{\hspace{2cm}}$</p> | $y = x^2 - 3x + 2$ $y = x - 2$ <p>$(x, y) = \underline{\hspace{2cm}}$</p> | $y = 10x^2 - 28x - 39$ $y = 2x + 1$ <p>$(x, y) = \underline{\hspace{2cm}}$</p> |
|--|--|---|

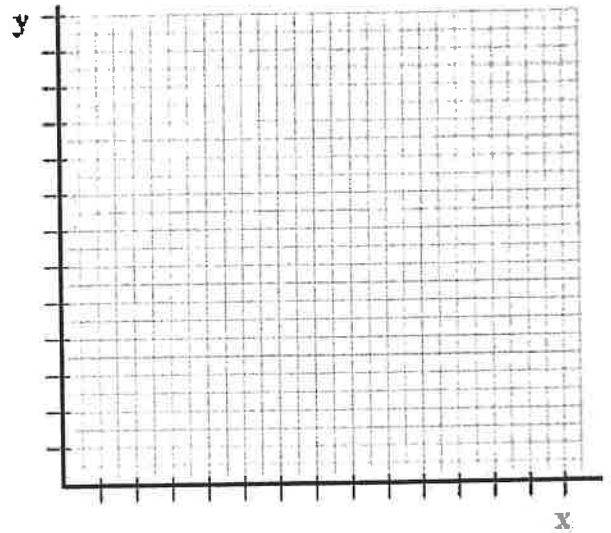
➤ Applications of Linear/Quadratic Systems:

Example #1: A ball thrown is modeled by the function: $y = -16x^2 + 22x + 3$.
Using what you know about quadratic functions, answer the following questions.

- 1) Sketch the graph :
- 2) Given the context of the problem, what is an appropriate domain and range for the graph?

D: _____ R: _____

- 3) Write an equation to show when the ball will be **exactly** 10 feet in the air and then solve.
- 4) Write an inequality to show when the ball will be at a height **less than** 10 feet in the air and then solve.



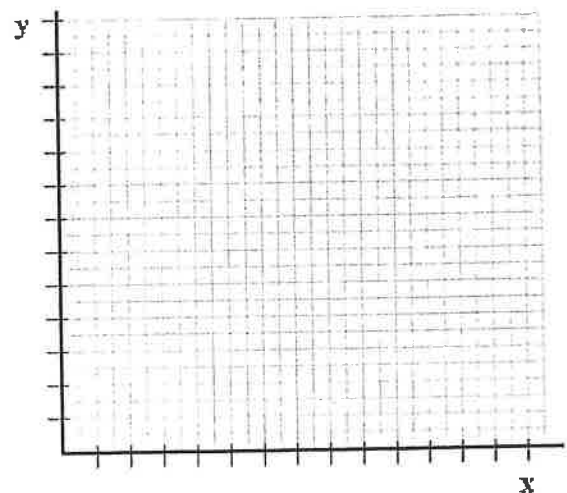
- 5) Write an inequality to show when the ball will be at a height **higher than** 10 feet in the air and then solve.

Example #2: The student council decides to put on a concert to raise money for an after school program. They have determined that the price of the ticket will affect their profit. The functions shown below represent their potential income and cost of putting on the concert, where t represents ticket price.

Income: $I(t) = -30t^2 + 330t$

Cost: $C(t) = -30t + 330$

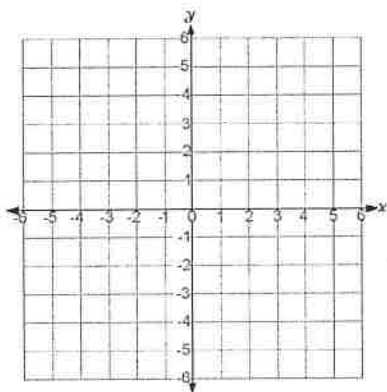
- 1) Sketch the graph of each function:
- 2) Find algebraically and graphically the **break-even** point. (Hint: $Income = Cost$)
- 3) Write an inequality to show where the cost is greater than the income and then solve.
- 4) Write an inequality to show where the income is greater than the cost and then solve.



- 5) Which ticket price would you use in order to maximize your profit? Where is this shown on the graph?

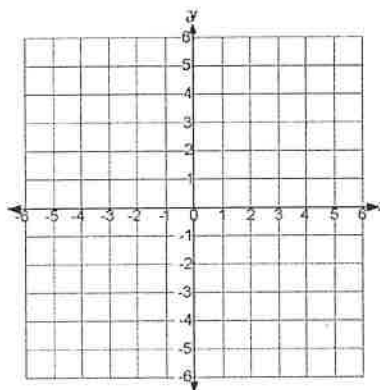
Solve each of these equations, sketch graphs showing the functions involved, and label points corresponding to solutions with their coordinates.

1. $y = x + 2$
 $y = x^2 + 3x - 6$



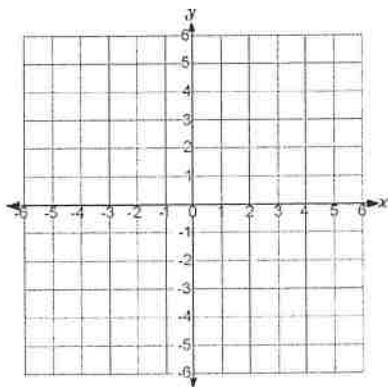
$(x, y) = \underline{\hspace{2cm}}$

2. $y = -x + 2$
 $y = x^2 + x - 6$



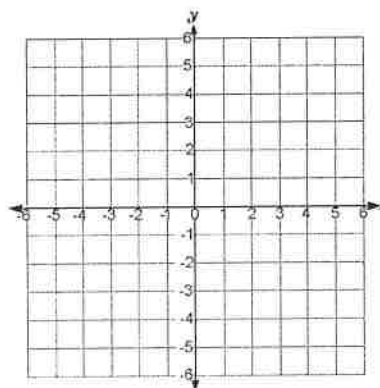
$(x, y) = \underline{\hspace{2cm}}$

3. $y = 2x + 3$
 $y = 4 + x^2$



$(x, y) = \underline{\hspace{2cm}}$

4. $y = x^2 - x$
 $y = 2x + 4$



$(x, y) = \underline{\hspace{2cm}}$

Solve each system algebraically:

5. $y = x^2 - 6x + 10$
 $y = -x + 4$

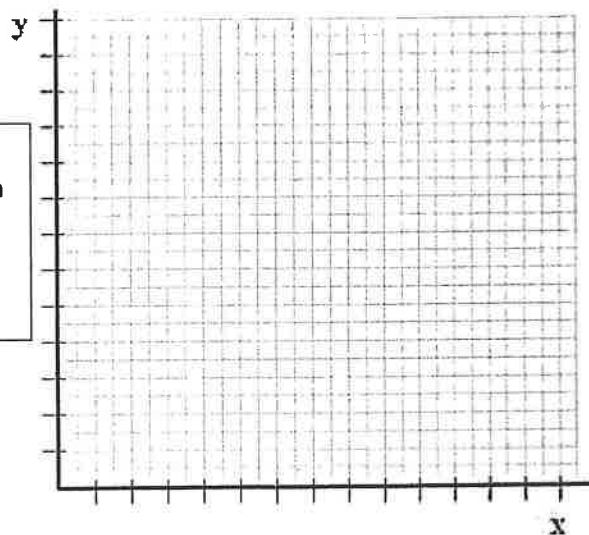
$(x, y) = \underline{\hspace{2cm}}$

6. $y = -x + 2$
 $y = x^2$

$(x, y) = \underline{\hspace{2cm}}$

➤ Application of Quadratic and Linear Inequalities

7. Each year the 'Rock the Vote' committee organizes a public rally. Based on previous years, the organizers decided that the income from ticket sales, $I(t)$, is related to ticket price (t) by the equation $I(t) = -40t^2 + 400t$. Cost, $C(t)$, of operating the public event is also related to ticket price (t) by the equation $C(t) = -40t + 400$.



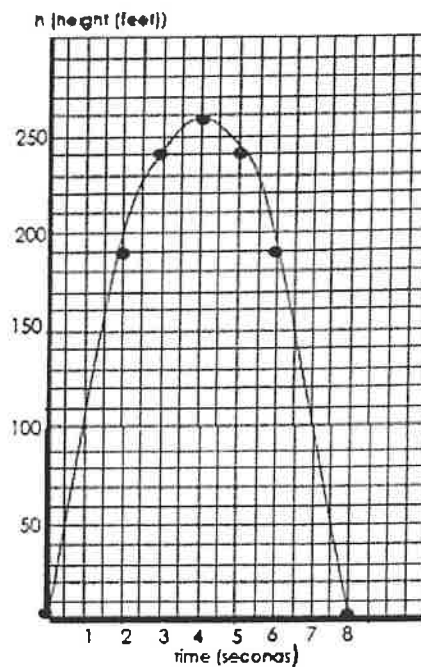
A) What ticket price would generate the maximum income? Where is this shown on the graph?

B) For what ticket price would the operating cost be equal to the income from ticket sales?

C) Write and solve an inequality to show where the operating cost is greater than the income from ticket sales.

D) Write and solve an inequality to show where the income from ticket sales is greater than the operating cost.

1.) Using the graph at the right, it shows the **height h** in feet of a small rocket **t seconds** after it is launched. The path of the rocket is given by the equation:
 $h = -16t^2 + 128t$.



a.) How long is the rocket in the air? _____

b.) What is the greatest height the rocket reaches? _____

c.) What does $f(1)$ mean in this context?

d.) Find $f(1)$ _____

e.) What would $f(x) = 0$ mean in this context?

f.) Find $f(x) = 0$

g.) Find $f(2)$. Is it going up or down (increasing or decreasing)?

h.) Find $f(6)$. Is it going up or down (increasing or decreasing)?

i.) What is the domain?

j.) What is the range?

Review:

➤ Steps to Graph an Inequality:

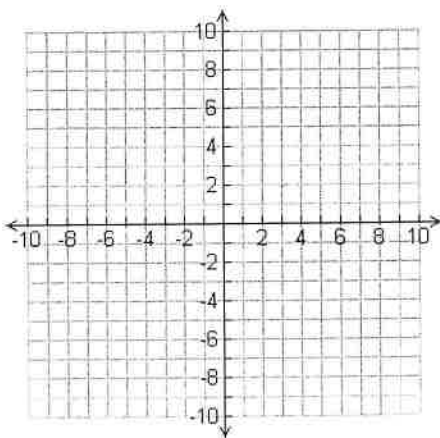
- ✓ Graph the boundary line
 - ➔ If the symbol is $<$ or $>$ use a dotted line
 - ➔ If the symbol is \leq or \geq use a solid line

- ✓ Determine the shading
 - ➔ If the symbol is $>$ or \geq then shade above the line or curve
 - ➔ If the symbol is $<$ or \leq then shade below the line or curve

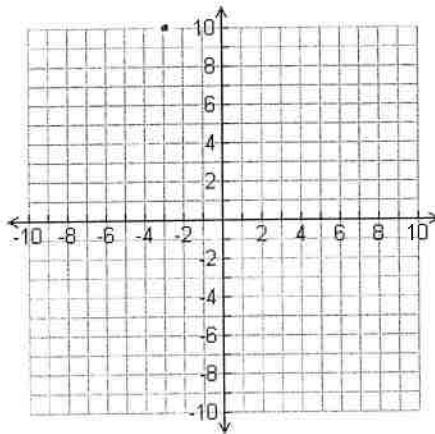
- ✓ You can check your shading by picking a point on the graph and plugging it into the inequality. If it is a solution then shade that way. If it is not a solution, then shade the other way.

➤ EXAMPLES: Graph each linear or quadratic inequality

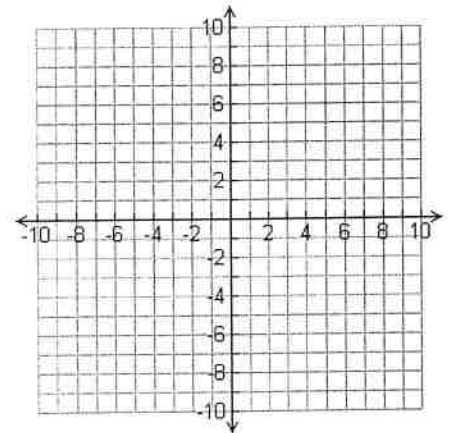
1) $y > x - 2$



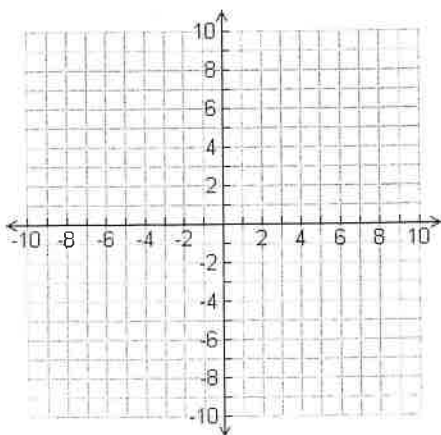
2) $y \leq -2x + 1$



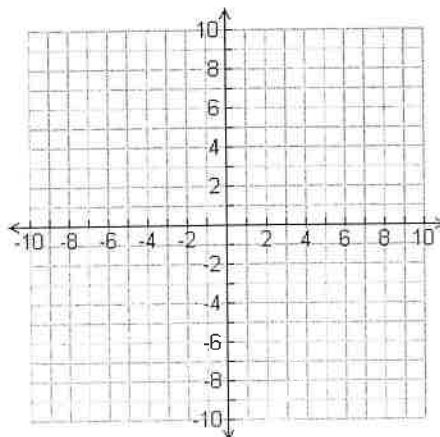
3) $y \geq \frac{-2}{3}x - 1$



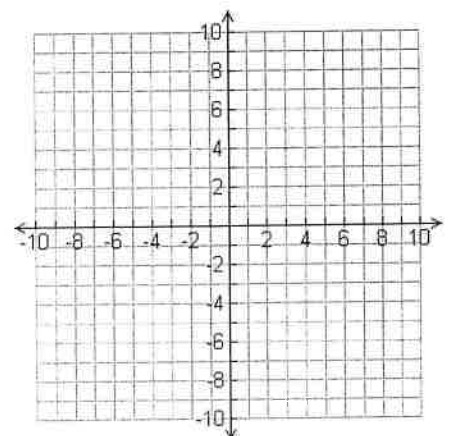
4) $y > x^2 + 4x + 4$



5) $y \geq -x^2 - 2x - 3$

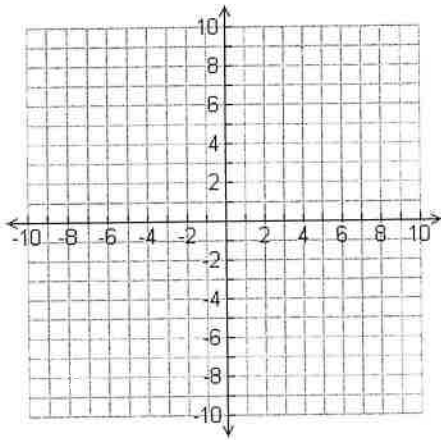


6) $y < x^2 - 7x + 10$

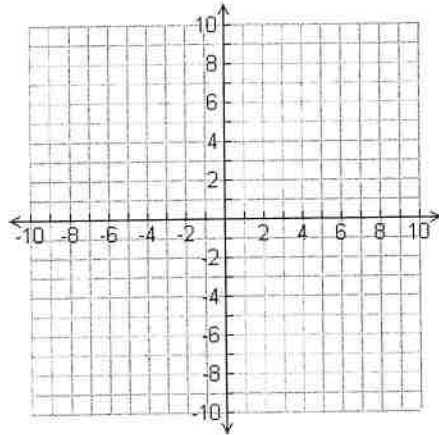


➤ Graph each system of inequalities. Be sure to shade the solution.

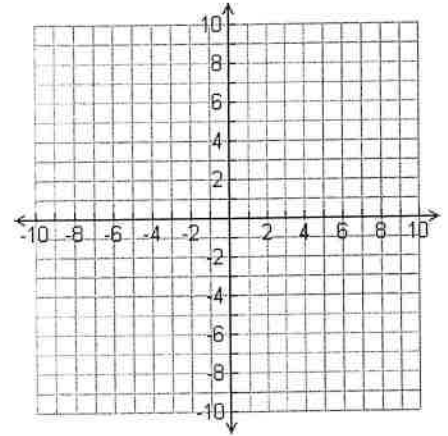
7) $y \geq x^2 + 4x + 3$
 $y \leq 2x + 6$



8) $y < -x^2 + 2x + 4$
 $y > -x + 4$



9) $y \geq x^2 - 6x + 8$
 $y \geq -x(x - 4)$



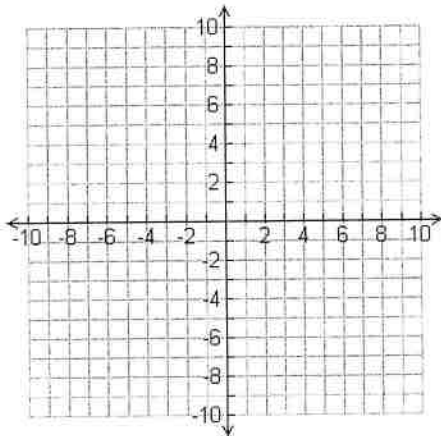
➤ How can we use graphing to solve an inequality in one-variable?

- Solve each of the inequalities. Write your solution as an inequality and graph on a number line.

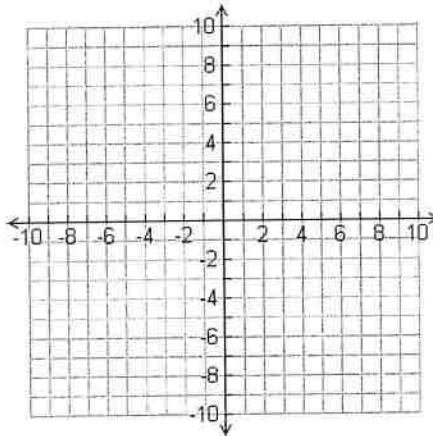
| | | |
|------------------------|----------------------|----------------------|
| $x^2 - x - 6 \leq 0$ | $x^2 - x - 6 \geq 0$ | $x^2 + 2x > 0$ |
| $x^2 + 2x - 24 \leq 0$ | $3x^2 - 5x > 8$ | $x^2 + 2x > 2x + 36$ |

➤ Graph each quadratic inequality. Be sure to shade the solution.

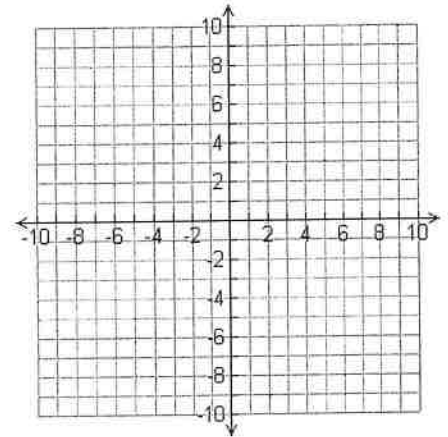
1) $y \geq x^2 - 1$



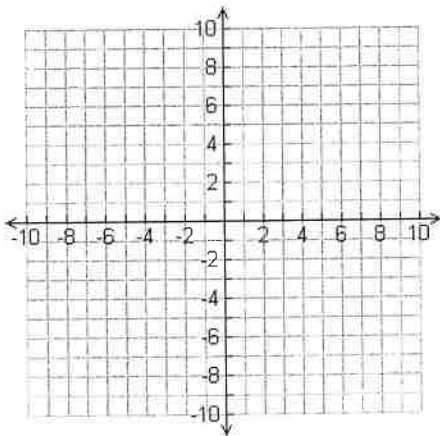
2) $y < x^2 - 4x - 4$



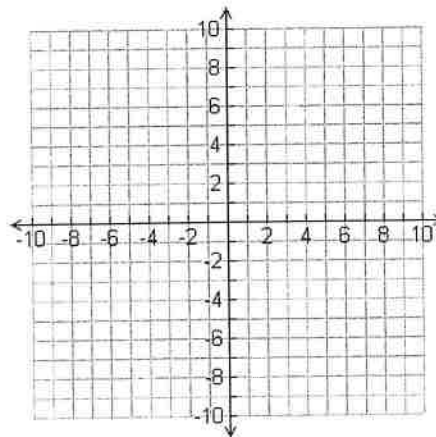
3) $y \leq -x^2 + 2x - 3$



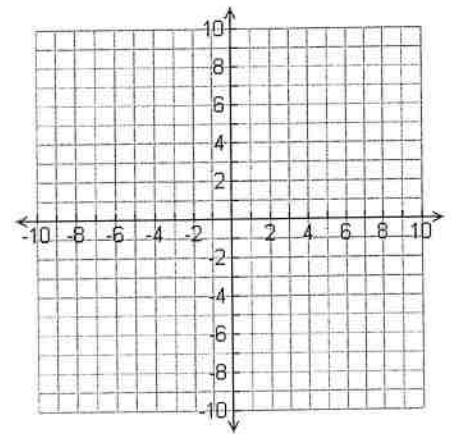
4) $y > -x^2 + 4x + 5$



5) $y \leq 4x^2 - 1$

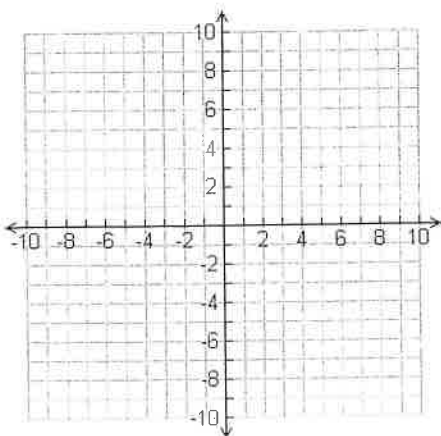


6) $y \leq x^2 + 6x + 8$



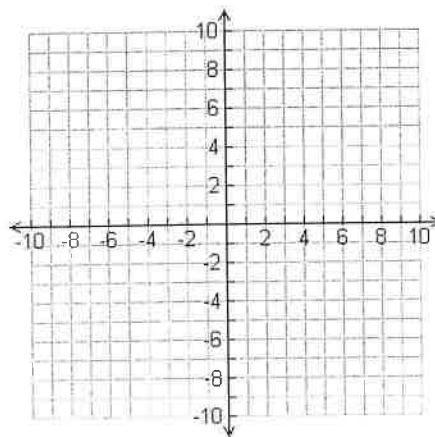
7) $y \geq x^2 - 3$

$y \leq 2x$



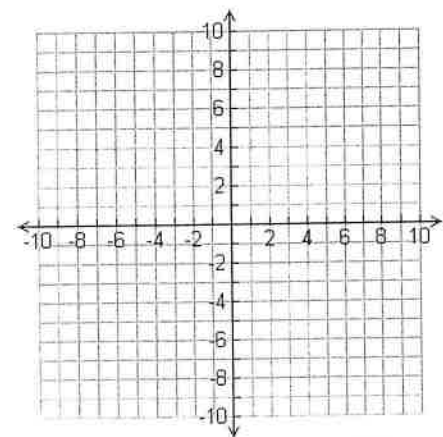
8) $y > x^2 - 5x + 4$

$y > -x + 1$

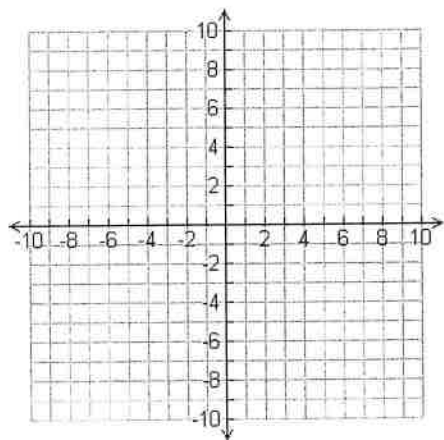


9) $y \leq -x^2 + 4x$

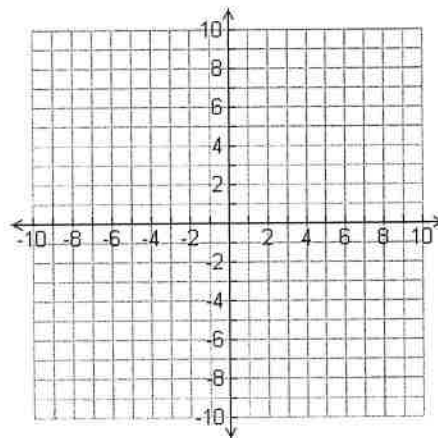
$y \geq 3x + 2$



$$10) \begin{aligned} y &\geq x^2 - 4 \\ y &\leq -x^2 - x + 2 \end{aligned}$$



$$11) \begin{aligned} y &> x^2 + 2x + 1 \\ y &> x^2 - 4x + 4 \end{aligned}$$



- Solve each of the inequalities. Write your solution as an inequality and graph on a number line.

| | | |
|-----------------------------|----------------------------|--------------------------|
| 12. $(x + 3)(x - 4) \leq 0$ | 13. $x^2 - 9x + 14 \geq 0$ | 14. $x^2 - 7x > 0$ |
| 15. $5x^2 - 180 \leq 0$ | 16. $x^2 - 12x + 32 > -3$ | 17. $x^2 + 14x \leq -49$ |

➤ Simplify each of the following radicals.

| | | | | |
|------------------|---------------------------|-------------------|--------------------|-----------------------------|
| 1. $\sqrt{-24}$ | 2. $\pm\sqrt{252}$ | 3. $-3\sqrt{-48}$ | 4. $\sqrt{50}$ | 5. $\pm\sqrt{63}$ |
| 6. $2\sqrt{147}$ | 7. $\frac{3}{4}\sqrt{64}$ | 8. $5\sqrt{-17}$ | 9. $\pm\sqrt{162}$ | 10. $-\sqrt{\frac{25}{81}}$ |

➤ Solve by **Completing the Square**.

Solve by **Quadratic Formula**.

| | |
|-------------------------|----------------------|
| 11. $4x^2 - 4x + 3 = 0$ | 12. $2x^2 + 6x = -3$ |
|-------------------------|----------------------|

➤ Solve each quadratic equation by the best method: **Factoring, Completing the Square or the Quadratic Formula**

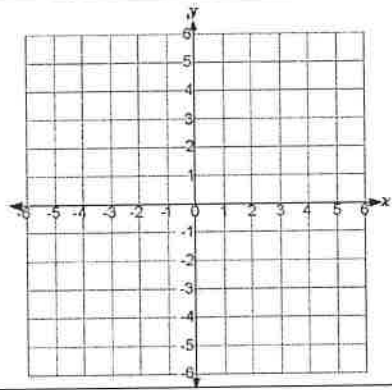
| | |
|--------------------------|-------------------------|
| 13. $9x^2 - 6x - 11 = 0$ | 16. $7x^2 - 5x = 0$ |
| 14. $8x^2 + 5 = -6x$ | 17. $3x^2 - 6x + 3 = 0$ |
| 15. $x^2 + 5x = 6$ | 18. $4x^2 + 4x - 8 = 1$ |

➤ **Quadratic Systems – Solve by substitution.**

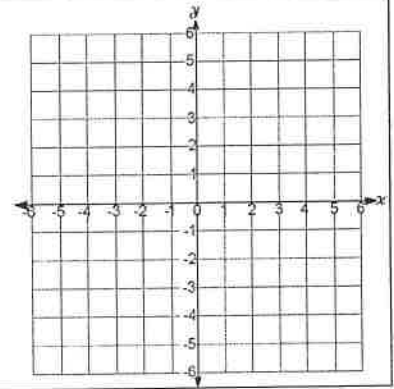
| | |
|-------------------------------|---|
| 19. $y = x^2 + 3$ $y = 4x$ | 20. $y = 3x^2 - 12x + 1$ $y = -2x - 7$ |
|-------------------------------|---|

➤ Graphing Quadratic Inequalities

21. $y \leq x^2 - 6x + 10$

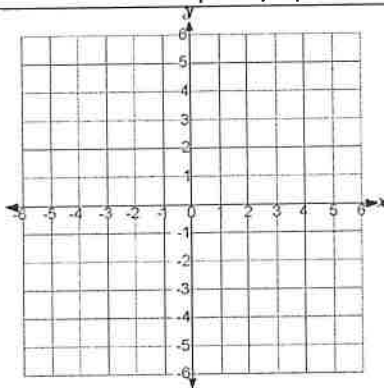


22. $y > -2(x - 1)^2 + 5$

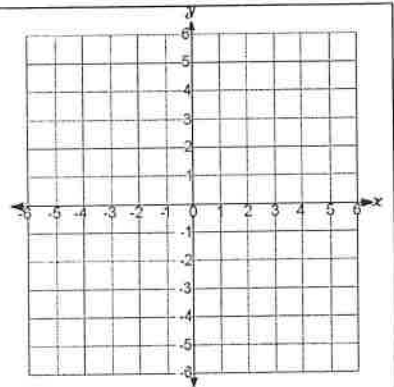


➤ Graphing Quadratic and Linear Inequality Systems

23. $y \geq x^2 + 8x + 14$
 $y \leq -2x - 4$



24. $y < -(x - 2)^2 + 4$
 $y > (x - 2)^2$



➤ Solve each Quadratic Inequality. Write your solution in interval notation.

25. $(x - 5)(x - 2) \leq 0$

26. $x^2 - 12x + 32 \geq -3$

27. $x^2 - 64 < 0$

➤ Application of Quadratic and Linear Inequalities

28. Each year the 'Rock the Vote' committee organizes a public rally. Based on previous years, the organizers decided that the income from ticket sales, $I(t)$, is related to ticket price (t) by the equation $I(t) = -50t^2 + 500t$. Cost, $C(t)$, of operating the public event is also related to ticket price (t) by the equation $C(t) = -50t + 500$.

A) What ticket price would generate the maximum income? Where is this shown on the graph?

B) For what ticket price would the operating cost be equal to the income from ticket sales?

C) Write and solve an inequality to show where the operating cost is greater than the income from ticket sales.

D) Write and solve an inequality to show where the income from ticket sales is greater than the operating cost.

