

- **DIRECT VARIATION:** Linear function with a y-intercept of 0. In a direct variation, both of the quantities are either increasing or both are decreasing.
- There are two methods for solving a direct variation problem:
  - 1) Equation of Variation:  $y = kx$  where k is called the **constant of variation**
  - 2) ~~Proportion:  $\frac{y_1}{x_1} = \frac{y_2}{x_2}$~~

#1: The distance that a body near Earth's surface will fall from rest varies directly as the **square** of the number of seconds it has been falling. If a boulder falls from a cliff a distance of 122.5 m in 5 seconds, approximately how far will it fall in 8 seconds?

Method 1

$$y = kx$$

$$d = ks^2$$

$$122.5 = k(5)^2$$

$$\frac{122.5}{5^2} = \frac{k(5)^2}{5^2}$$

$$k = 4.9$$

Method 2

$$d = (4.9)(8)^2$$

$$d = 313.6m$$

- **JOINT VARIATION:** more than two quantities in a **direct variation** relationship
  - Equation of Variation:  $y = kxz$  where k is called the **constant of variaton**
- #2: If y varies jointly as x and z, and  $y = \frac{1}{2}$  when  $x = 27$  and  $z = \frac{-2}{3}$ , find y when  $x = 9$  and  $z = 18$ .
- $$y = kxz$$
- $$\frac{1}{2} = k(27)\left(\frac{-2}{3}\right)$$
- $$k = \frac{-1}{36}$$
- $$y = \left(\frac{-1}{36}\right)(9)(18)$$
- $$y = -\frac{9}{2} \text{ or } -4.5$$

- **INVERSE VARIATION:** Rational function with vertical and horizontal asymptotes. In an inverse variation, one of the quantities is increasing while the second quantity is decreasing.
  - Equation of Variation:  $y = \frac{k}{x}$  where k is called the **constant of variation**
- #3: The time of a trip varies inversely as the speed of the car. If a car being driven at 55 mph takes 2 hours to get from Wake Forest to Greensboro, how fast is the car traveling if the trip takes 2.5 hours?
- $$y = \frac{k}{x}$$
- $$h = \frac{k}{m}$$
- $$2 = \frac{k}{55}$$
- $$110 = k$$
- $$\frac{2.5}{x} = \frac{110}{55}$$
- $$2.5x = 110$$
- $$x = \frac{110}{2.5}$$
- $$x = 44$$

44 mph

➤ **COMPOUND VARIATION:** Both Inverse and Direct Variation in the same problem

➤ Equation of Variation:  $y = \frac{kx}{z}$  where k is called the **constant of variation**

#4: The volume of gas varies directly with Kelvin temperature and inversely with pressure. If a certain gas has a volume of 342 cubic meters at a temperature of 300 Kelvin degrees under a pressure of 200 KPa (kilopascals), what will be the volume of the same gas at a temperature of 320 Kelvin degrees under a pressure of 400 kPa?

$$y = \frac{kx}{z}$$

$$V = \frac{kT}{P}$$

$$\frac{342}{1} = \frac{300K}{200}$$

$$68400 = 300K$$

$$K = 228$$

$$V = \frac{(228)(320)}{400}$$

$$182.4 \text{ m}^3$$

➤ State whether each equation represents a direct, inverse, joint or compound variation. Then state the constant of variation.

1. $y = \frac{9}{x}$	2. $z = 5xy$	3. $y = \frac{8x}{z}$	4. $y = 2x$	5. $xy = 12$
6. $z = \frac{xy}{15}$	7. $y = \frac{3}{4}xz$	8. $y = \frac{1}{3}x$	9. $z = \frac{x}{12y}$	10. $y = \frac{x}{5}$

➤ Write a function for each variation relationship:

11.  $W$  varies directly as the square of  $d$ .  $W = kd^2$

12.  $V$  varies inversely as  $J$ .  $V = \frac{k}{J}$

13.  $V$  varies inversely as  $p$  and directly as  $T$ .  $V = \frac{kT}{p}$

14.  $F$  varies jointly as  $A$  and the square of  $v$ .  $F = kAv^2$

15.  $L$  varies directly as the fourth power of  $d$  and inversely as the square root of  $h$ .

$$L = \frac{kd^4}{\sqrt{h}}$$

Math 2 – Honors  
 Unit 4 - Radical & Rational Functions  
 Lesson 7 → Types of Variations HOMEWORK

Name \_\_\_\_\_  
 Date \_\_\_\_\_ Pd \_\_\_\_\_

Write an equation for each statement and then solve:

<p>1. If <math>y</math> varies directly as <math>x</math> and <math>y = 15</math> when <math>x = 3</math>, find <math>y</math> when <math>x = 12</math>.</p>	<p>2. If <math>y</math> varies directly as <math>x</math> and <math>x = 36</math> when <math>y = 4</math>, find <math>x</math> when <math>y = 24</math>.</p>	<p>3. If <math>y</math> varies directly as <math>x^2</math> and <math>y = 12</math> when <math>x = 4</math>, find <math>y</math> when <math>x = 6</math>.</p>
<p>4. If <math>y</math> varies inversely as <math>x</math> and <math>y = 2</math> when <math>x = 8</math>, find <math>x</math> when <math>y = 14</math>.</p>	<p>5. If <math>y</math> varies inversely as <math>x</math> and <math>x = 7</math> when <math>y = 21</math>, find <math>y</math> when <math>x = 42</math>.</p>	<p>6. If <math>y</math> varies inversely as <math>x^3</math> and <math>y = 6</math> when <math>x = \frac{-3}{4}</math>, find <math>y</math> when <math>x = 3</math>.</p>
<p>7. Suppose <math>y</math> varies jointly with <math>x</math> and <math>z</math>. If <math>y = 20</math> when <math>x = 2</math> and <math>z = 5</math>, find <math>y</math> when <math>x = 14</math> and <math>z = 8</math>.</p>	<p>8. Suppose <math>z</math> varies jointly with <math>x</math> and <math>y</math>. If <math>x = 3</math> and <math>y = 2</math> when <math>z = 12</math>, find <math>z</math> when <math>x = 4</math> and <math>y = 5</math>.</p>	<p>9. Suppose <math>m</math> varies jointly as <math>n</math> and <math>p</math>. If <math>n = 4</math> and <math>p = 5</math> when <math>m = 60</math>, find <math>m</math> when <math>n = 12</math> and <math>p = 2</math>.</p>
<p>10. Suppose that <math>y</math> varies directly as <math>x</math> and inversely as <math>z</math>. If <math>y = 5</math> when <math>x = 3</math> and <math>z = 4</math>, find <math>y</math> when <math>x = 6</math> and <math>z = 8</math>.</p>	<p>11. Suppose <math>y</math> varies directly as <math>\sqrt{x}</math> and inversely as <math>z</math>. If <math>y = 10</math> when <math>x = 9</math> and <math>z = 12</math>, find <math>y</math> when <math>x = 16</math> and <math>z = 10</math>.</p>	<p>12. Suppose <math>x</math> varies directly as <math>y^3</math> and inversely as <math>\sqrt{z}</math>. If <math>x = 7</math> when <math>y = 2</math> and <math>z = 4</math>, find <math>x</math> when <math>y = 3</math> and <math>z = 9</math>.</p>

# Direct, joint, inverse, compound

Determine the type of variation and then write an equation for each statement. Then solve.

13. The number (B) of bolts a machine can make *varies directly* as the time (T) it operates. If the machine can make 6578 bolts in 2 hours, how many bolts can it make in 5 hours? *Direct*
- $B = k h$   $\frac{6578}{2} = \frac{2k}{2}$   $k = 3289$   $b = 5(3289)$  **16,445**
14. The number of cooks needed to prepare lunch *varies inversely* with the time. If it takes 9 cooks four hours to prepare a school lunch, how long would it take 8 cooks to prepare the lunch?
15. The current (I) in an electrical conductor *varies inversely* as the resistance (r) of the conductor. If the current is 2 amperes when the resistance is 960 ohms, what is the current when the resistance is 480 ohms?
16. Cheers *varied jointly* as the number of fans and the **square** of the jubilation factor. If there were 100 cheers when the number of fans was 100 and the jubilation factor was 4, how many cheers were there when there were only 10 fans whose jubilation factor was 20?
17. The volume of a cone *varied jointly* as the height of the cone and the area of the base. If a cone has a volume of  $140 \text{ cm}^3$  when the height is 15 cm and the area of the base is  $28 \text{ cm}^2$ , find the volume of a cone with a height of 7 cm and a base area of  $12 \text{ cm}^2$ .
18. The number of girls *varies directly* as the number of boys and *inversely* as the number of teachers. When there were 50 girls, there were 10 boys and 20 teachers. How many boys were there when there were 10 girls and 100 teachers?
19. A pitcher's earned run average (ERA) *varies directly* as the number of earned runs allowed and *inversely* as the number of innings pitched. Joe Price had an ERA of 2.55 when he gave up 85 earned runs in 300 innings. What would be his ERA if he gave up 120 earned runs in 600 innings?
20. The maximum load that a cylindrical column with a circular cross section can hold *varies directly* as the fourth power of the diameter and *inversely* as the square of the height. A 9 meter column with a 2 meter diameter will support 64 metric tons. How many metric tons can be supported by a column 9 meters high and 3 meters in diameter?