

- There are four steps to solving a radical equation:
- 1) Isolate the radical.
 - 2) Raise both sides to the power of the root.
 - 3) Solve for x.
 - 4) Check for extraneous solution(s).

What is an **EXTRANEIOUS** solution? A solution to the final equation but not to the original equation. Extraneous solutions can occur when solving a **square root equation** but not when solving linear or quadratic equations.

➤ Examples:

<p>1. $(\sqrt{x}) = 8$</p> <p>$x = 64$ ✓</p>	<p>2. $(\sqrt{x+7}) = 8$</p> <p>$x+7 = 64$ $-7 \quad -7$</p> <p>$x = 57$ ✓</p>	<p>3. $2\sqrt{x+6} = 14$</p> <p>$(\sqrt{x+6})^2 = (7)^2$</p> <p>$x+6 = 49$</p> <p>$x = 43$ ✓</p>
<p>4. $-4\sqrt{x} + 11 = 3$</p> <p>$-4x + 11 = 3$ ($\sqrt{x} = 2$)</p> <p>$-11 \quad -11$ $x = 4$ ✓</p> <p>$-4\sqrt{x} = -8$ $\frac{-4}{-4} \quad \frac{-4}{-4}$</p>	<p>5. $(x-2)^{1/2} - 2 = 2$</p> <p>$+2 \quad +2$</p> <p>$(x-2)^{1/2} = 4$</p> <p>$(\sqrt{x-2})^2 = (4)^2$</p> <p>$x-2 = 16$ $x = 18$ $+2 \quad +2$ ✓</p>	<p>6. $10 - 3\sqrt[3]{2x+5} = -11$</p> <p>$-10 \quad -10$</p> <p>$-3\sqrt[3]{2x+5} = -21$ $\frac{-3}{-3} \quad \frac{-21}{-3}$</p> <p>$(\sqrt[3]{2x+5})^3 = (7)^3$</p> <p>$2x+5 = 343$ $-5 \quad -5$</p> <p>$2x = 338$ $x = 169$ ✓</p>
<p>7. $(\sqrt{10x^2 - 49}) = (3x)^2$ ($3x \times 3x$)</p> <p>$10x^2 - 49 = 9x^2$ $-9x^2 \quad -9x^2$</p> <p>$x^2 - 49 = 0$ $x^2 = 49$ $x = \pm 7$</p> <p>$(x-7)(x+7) = 0$</p> <p>$x = 7$ $x = -7$ ✓</p>	<p>8. $(\sqrt{2x-6}) = (\sqrt{5x-15})$</p> <p>$2x-6 = 5x-15$ $-2x \quad -2x$</p> <p>$-6 = 3x-15$ $+15 \quad +15$</p> <p>$9 = 3x$ $\frac{9}{3} = \frac{3x}{3}$</p> <p>$x = 3$ ✓</p>	<p>9. $(6x-5)^{1/3} = (3x+2)^{1/3}$</p> <p>$(6x-5) = (3x+2)$</p> <p>$-3x+5 \quad -3x+5$</p> <p>$3x = 7$ $7/3$</p> <p>$x = \frac{7}{3}$ ✓</p>

10. $(\sqrt{3x+7})^2 = (x+1)^2$ $(x+1)(x+1)$

$3x+7 = x^2 + 2x + 1$

$-3x - 7$ $-3x - 7$

$0 = x^2 - x - 6$ $0 = (x-3)(x+2)$

$x=3$ $x=-2$

11. $(15-7x)^{1/2} = (x-1)^2$

$15-7x = x^2 - 2x + 1$

$-15 + 7x$ $+ 7x - 15$

$0 = x^2 + 5x - 14$

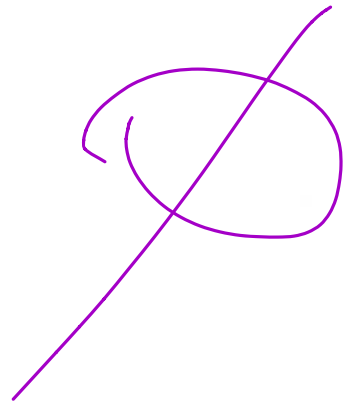
$0 = (x+7)(x-2)$

$x = -7$ $x = 2$

12. $\sqrt{x+2} = 4 - \sqrt{x}$

$\frac{49}{16}$

13. $\sqrt{x+3} = \sqrt{x+4}$



14. $\sqrt{x+8} = \sqrt{x} + \sqrt{3}$

$\frac{25}{12}$

15. $\sqrt{x+3} = \sqrt{x+1} + 1$

12. $(\sqrt{x+2})^2 = (4-\sqrt{x})^2$ * write twice and foil

$$\begin{aligned}
 x+2 &= (4-\sqrt{x})(4-\sqrt{x}) \\
 x+2 &= 16 - 4\sqrt{x} - 4\sqrt{x} + x \\
 x+2 &= 16 - 8\sqrt{x} + x \\
 -x & \qquad \qquad \qquad -x
 \end{aligned}$$

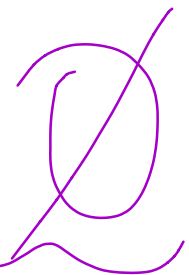
$$\begin{aligned}
 2 &= 16 - 8\sqrt{x} \\
 -16 & \quad -16 \\
 -14 &= -8\sqrt{x} \\
 -8 & \quad -8 \\
 \left(\frac{7}{4}\right)^2 &= (\sqrt{x})^2 \\
 \frac{49}{16} &= x
 \end{aligned}$$

13. $(\sqrt{x+3})^2 = (\sqrt{x+4})^2$

$$\begin{aligned}
 (\sqrt{x+3})(\sqrt{x+3}) &= x+4 \\
 x+3\sqrt{x}+3\sqrt{x}+9 & \\
 x+6\sqrt{x}+9 &= x+4 \\
 -x & \qquad \qquad \qquad -x \\
 6\sqrt{x}+9 &= 4 \\
 -9 & \quad -9
 \end{aligned}$$

$$\begin{aligned}
 \frac{6\sqrt{x}}{6} &= \frac{-5}{6} \\
 (\sqrt{x})^2 &= \left(\frac{-5}{6}\right)^2
 \end{aligned}$$

$$x = \frac{25}{36}$$



14. $(\sqrt{x+8})^2 = (\sqrt{x+3})^2$

$$\begin{aligned}
 x+8 &= (\sqrt{x+3})(\sqrt{x+3}) \\
 x+8 &= x + \sqrt{3x} + \sqrt{3x} + 3 \\
 x+8 &= x + 2\sqrt{3x} + 3 \\
 -x & \quad -x
 \end{aligned}$$

$$\begin{aligned}
 8 &= 2\sqrt{3x} + 3 \\
 -3 & \quad -3 \\
 5 &= 2\sqrt{3x} \\
 \frac{5}{2} &= \frac{2\sqrt{3x}}{2} \\
 \left(\frac{5}{2}\right)^2 &= \frac{4 \cdot 3x}{4} \\
 \frac{25}{4} &= 3x \\
 \frac{25}{12} & \checkmark
 \end{aligned}$$

$$\begin{aligned}
 3+3+3 & \quad \sqrt{x} + \sqrt{x} \\
 3 \cdot 3 & \quad + \sqrt{x} \\
 x+x+x & \\
 3x &
 \end{aligned}$$

$$15. (\sqrt{x+3})^2 = (\sqrt{x+1} + 1)^2$$

$$x+3 = (\sqrt{x+1} + 1)(\sqrt{x+1} + 1)$$

$$x+3 = x+1 + \sqrt{x+1} + \sqrt{x+1} + 1$$

$$x+3 = x+1 + 2\sqrt{x+1} + 1$$

$$x+3 = x+2 + 2\sqrt{x+1}$$

$$-x \quad -x$$

$$3 = 2 + 2\sqrt{x+1}$$

$$-2 \quad -2$$

$$\frac{1}{2} = \frac{2\sqrt{x+1}}{2}$$

$$\left(\frac{1}{2}\right)^2 = (\sqrt{x+1})^2$$

$$\frac{1}{4} = x+1$$

$$-1$$

$$-3/4 \text{ or } -.75 = x$$

Direct, joint, inverse, compound

Determine the type of variation and then write an equation for each statement. Then solve.

13. The number (B) of bolts a machine can make varies directly as the time (T) it operates. If the machine can make 6578 bolts in 2 hours, how many bolts can it make in 5 hours? **Direct**

$$B = kh \quad \frac{6578}{2} = \frac{2k}{2} \quad k = 3289 \quad b = 5(3289) \quad \boxed{16,445}$$

14. The number of cooks needed to prepare lunch varies inversely with the time. If it takes 9 cooks four hours to prepare a school lunch, how long would it take 8 cooks to prepare the lunch?

$$k = 36 \quad 4.5$$

15. The current (I) in an electrical conductor varies inversely as the resistance (r) of the conductor. If the current is 2 amperes when the resistance is 960 ohms, what is the current when the resistance is 480 ohms?

$$k = 1920 \quad 4$$

16. Cheers varied jointly as the number of fans and the **square** of the jubilation factor. If there were 100 cheers when the number of fans was 100 and the jubilation factor was 4, how many cheers were there when there were only 10 fans whose jubilation factor was 20?

$$C = kff^2$$

$$C = (.0625)(10)(20)^2 \quad k = .0625$$

17. The volume of a cone varied jointly as the height of the cone and the area of the base. If a cone has a volume of 140 cm³ when the height is 15 cm and the area of the base is 28 cm², find the volume of a cone with a height of 7 cm and a base area of 12 cm².

$$k = \frac{1}{3} \quad 28$$

18. The number of girls varies directly as the number of boys and inversely as the number of teachers. When there were 50 girls, there were 10 boys and 20 teachers. How many boys were there when there were 10 girls and 100 teachers?

$$k = 100 \quad 10$$

19. A pitcher's earned run average (ERA) varies directly as the number of earned runs allowed and inversely as the number of innings pitched. Joe Price had an ERA of 2.55 when he gave up 85 earned runs in 300 innings. What would be his ERA if he gave up 120 earned runs in 600 innings?

$$k = 9 \quad 1.8$$

20. The maximum load that a cylindrical column with a circular cross section can hold varies directly as the fourth power of the diameter and inversely as the square of the height. A 9 meter column with a 2 meter diameter will support 64 metric tons. How many metric tons can be supported by a column 9 meters high and 3 meters in diameter?

$$W = \frac{kd^4}{h^2} \quad 64 = \frac{k(2)^4}{9^2} \quad 64 = \frac{16k}{81} \quad \frac{5184}{16} = \frac{16k}{20} \quad k = 324$$