		Math 2 Honors
Day	**Date**	**Unit 4 Topics**
1	3/16/18	L1: Rational Exponents
2	3/19	L2: Square Root & Cube Roots Graphs
3	3/20	L3: Square Root & Cube Root Equations
4	3/21	L4: Square Root Applications
5	3/22	QUIZ on Lessons 1-4
6	3/23	L5: Graphs of Rational Functions
7	3/26	L6: Rational Equations
8	3/27	L7: Types of Variations
9	3/28	Review for Test
10	3/29	UNIT 4 TEST

QUIZ DATE: ____________________

TEST DATE: ____________________

Math 2 – Honors

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Unit 4 – Radical & Rational Functions

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Lesson 1 → Rational Exponents

❖ Rational or fractional exponents can be rewritten in radical form:

Converting from rational exponent to radical form:

$$x^{a/b} = \sqrt[b]{x^a}$$

The **numerator** of the exponent becomes the **exponent** of the radicand.

The **denominator** of the exponent becomes the **index** of the radicand.

➢ EXAMPLES:

1. $9^{1/2} =$	2. $64^{1/3} =$
3. $x^{2/3} =$	4. $16^{-1/2} =$ *** **Negative** exponents become fractions
5. $4x^{1/7} =$	6. $(3x)^{3/4} =$

➢ You Try: Write each expression in **simplest** radical form:

1. $2^{1/2}$	2. $3^{1/2}$	3. $9^{-1/2}$	4. $25^{1/2}$	5. $7^{1/3}$
6. $x^{4/7}$	7. $15^{-1/4}$	8. $x^{1/2}$	9. $y^{-1/2}$	10. $4x^{2/3}$
11. $3x^{-1/2}$	12. $(9a)^{1/2}$	13. $(16x^5)^{-1/2}$	14. $27^{5/3}$	15. $(5x)^{1/6}$

❖ Radicals can be rewritten in rational exponent form:

Converting from radical to rational exponent form:

$$\sqrt[b]{x^a} = x^{a/b}$$

The **exponent** of the radicand becomes the **numerator** of the fraction.

The **index** of the radicand becomes the **denominator** of the fraction.

➢ **EXAMPLES:**

1. $\sqrt{5} =$	2. $\sqrt[3]{7^2} =$
3. $\sqrt[4]{x} =$	4. $\frac{1}{\sqrt[3]{x^2}} =$
5. $5\sqrt[3]{x} =$	6. $\sqrt[5]{3x^2} =$

➢ You Try: Write each expression in **exponential** form:

16. $\sqrt{7}$	17. $\sqrt{6}$	18. $\sqrt[4]{8}$	19. $\sqrt[5]{18}$	20. $\sqrt[3]{x^2}$
21. $\sqrt[3]{(2x^2)}$	22. $\frac{1}{\sqrt[3]{5}}$	23. $2\sqrt[4]{15}$	24. $\sqrt{(3x)^7}$	25. $(\sqrt[3]{3v})^2$

Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 1 → Rational Exponents HOMEWORK

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➢ Rewrite each expression in radical form and then simplify completely:

1. $100^{1/2}$	2. $125^{1/3}$	3. $(17x)^{1/2}$	4. $64^{1/3}$	5. $16^{1/4}$
6. $16^{3/4}$	7. $\left(8^{1/2}\right)^2$	8. $\left(8^{1/3}\right)^3$	9. $(16x^4)^{1/4}$	10. $125^{-1/3}$

➢ Rewrite each expression in exponential form and then simplify completely:

11. $\sqrt{81}$	12. $\sqrt[3]{125}$	13. $\sqrt[4]{20x^3}$	14. $\sqrt[3]{-64}$	15. $\sqrt[3]{8}$
16. $\left(\sqrt[3]{8x}\right)^3$	17. $\left(\sqrt{98}\right)^2$	18. $\left(\sqrt[3]{98}\right)^3$	19. $\left(\sqrt[4]{98}\right)^4$	20. $\left(\frac{1}{\sqrt{x}}\right)^{-4}$

➢ Evaluate each of the following expressions. Give exact answers.

21. $27^{2/3}$	22. $1^{3.5}$	23. $\left(\frac{1}{32}\right)^{1/5}$	24. $(-27)^{-2/3}$	25. $4^{2.5}$
26. $\left(\frac{1}{16}\right)^{3/4}$	27. $216^{1/3}$	28. $16^{-1/4}$	29. $25^{3/2}$	30. $(x^6)^{1/2}$
31. $(9x^2)^{1/2}$	32. $\left(4x^{1/2}\right)^{1/2}$	33. $((8x^3)^2)^{1/3}$	34. $(9x^{-5}y^2)^{-1/2}$	35. $((-4x^3y^{-2})^3)^{1/2}$

What Happens When the King of Beasts Runs in Front of a Train?

Match each expression with its equivalent expression. Write the corresponding letter in the box at the bottom of the page.

1. $7^{1/2}$		**N**	$10^{3/2}$
2. $\sqrt[3]{2}$		**N**	216
3. $a^{6/5}$		**I**	3
4. $(\sqrt[3]{3a})^4$		**I**	$\sqrt{7}$
5. $(6v)^{1.5}$		**T**	$3n^2$
6. $\sqrt[6]{2}$		**F**	$10^{1/6}$
7. $4^{4/3}$		**S**	$(\sqrt[5]{a})^6$
8. $(\sqrt{10})^3$		**O**	$5^{5/4}$
9. $(x^6)^{1/2}$		**H**	$\frac{1}{\sqrt{m}}$
10. $\sqrt{6p}$		**O**	$(6p)^{1/2}$
11. $\sqrt[6]{10}$		**L**	$(\sqrt{10n})^3$
12. $(9n^4)^{1/2}$		**T**	$(3a)^{4/3}$
13. $m^{-1/2}$		**E**	$2^{5/3}$
14. $(\sqrt[3]{2})^5$		**E**	$2^{1/6}$
15. $(10n)^{3/2}$		**D**	x^3
16. $9^{1/2}$		**H**	$(\sqrt{6v})^3$
17. $(\sqrt[4]{5})^5$		**E**	$(\sqrt[3]{4})^4$
18. $36^{1.5}$		**T**	$2^{1/3}$

1	2	3		4	5	6		7	8	9		10	11		12	13	14		15	16	17	18

Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 2 → Square Root & Cube Root Graphs

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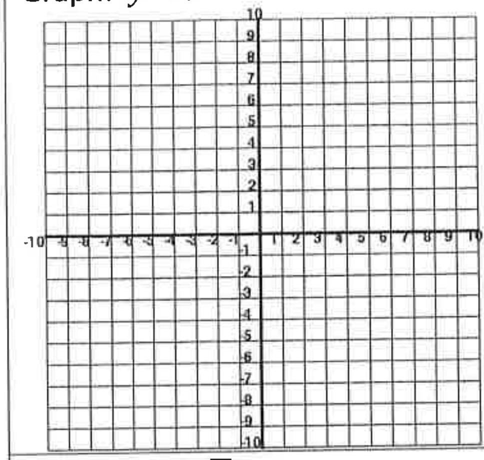
- Graphs of the **Parent Functions:**

Graph: $y = x^2$

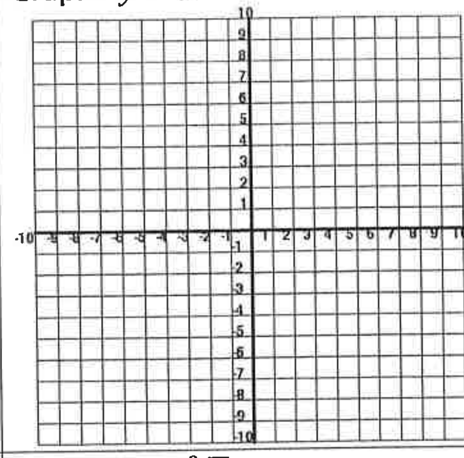


Vertex:

Domain:

Range:

Graph: $y = x^3$

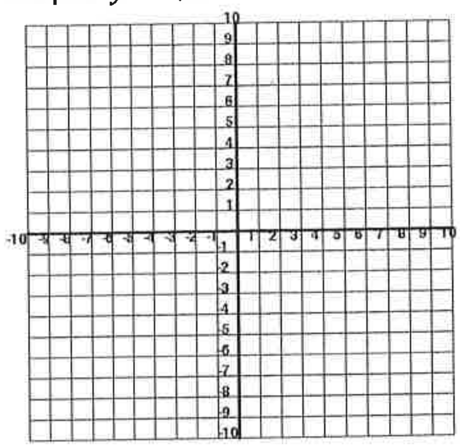


Vertex:

Domain:

Range:

Graph: $y = \sqrt{x}$

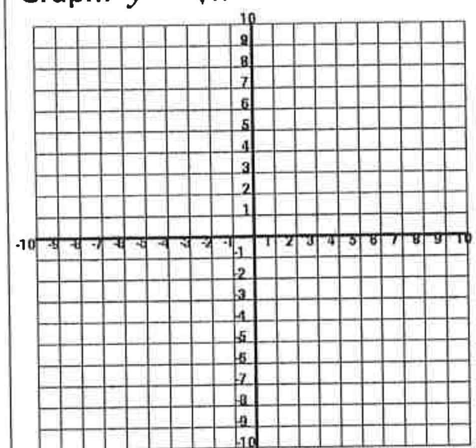


Vertex:

Domain:

Range:

Graph: $y = \sqrt[3]{x}$



Vertex:

Domain:

Range:

- **Recall Transformation Rules:**

$$y = a(x - h) + k$$

If a is negative, then the graph is a reflection across the x-axis

Vertical Stretch $|a| > 1$ (makes it narrower)

Vertical Compression $0 < |a| < 1$ (makes it wider)

Horizontal Translation (opposite of h)

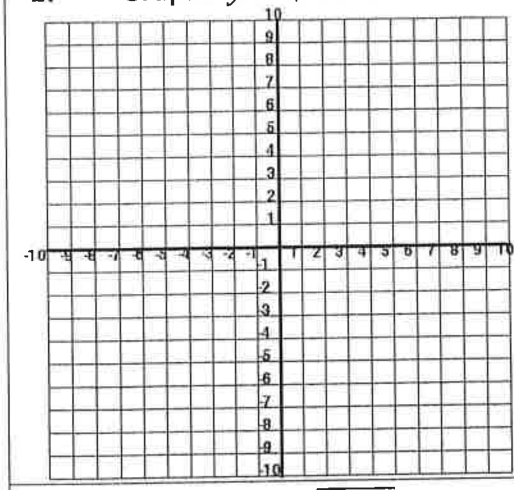
Vertical Translation

Quadratic Function	Shift Left or Right	Shift Up or Down
$y = (x-3)^2 + 6$		
$y = (x+1)^2$		
$y = x^2 - 4$		
Square Root Function	Shift Left or Right	Shift Up or Down
$y = \sqrt{x-2} + 5$		
$y = \sqrt{x} - 1$		
$y = \sqrt{x+3}$		

Cubic Function	Shift Left or Right	Shift Up or Down
$y = (x+2)^3 - 5$		
$y = x^3 + 7$		
$y = (x-8)^3$		
Cube Root Function	Shift Left or Right	Shift Up or Down
$y = \sqrt[3]{x} - 9$		
$y = \sqrt[3]{x+2} + 4$		
$y = \sqrt[3]{x-8}$		

➢ Graph using Transformation Rules:

1. **Graph:** $y = \sqrt{x+4}$

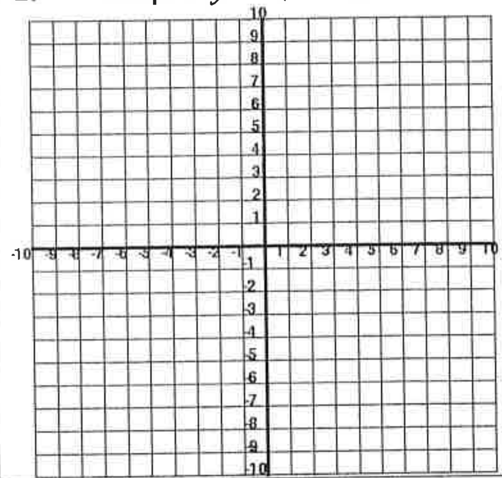


Vertex:

Domain:

Range:

2. **Graph:** $y = \sqrt[3]{x} + 2$

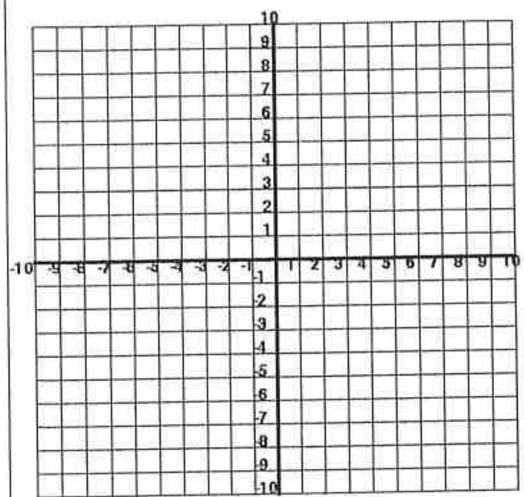


Vertex:

Domain:

Range:

3. **Graph:** $y = \sqrt{x+3} - 6$

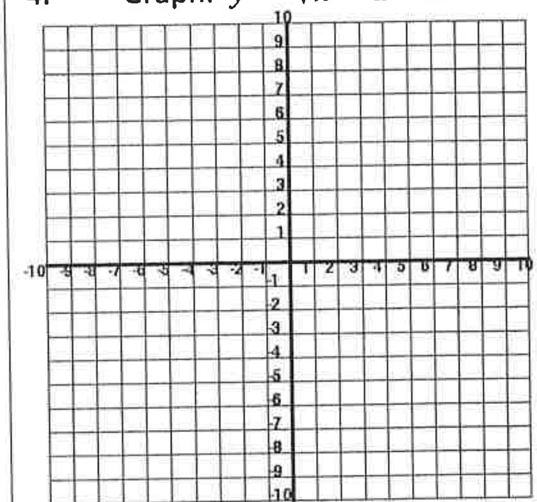


Vertex:

Domain:

Range:

4. **Graph:** $y = \sqrt[3]{x-1} - 3$

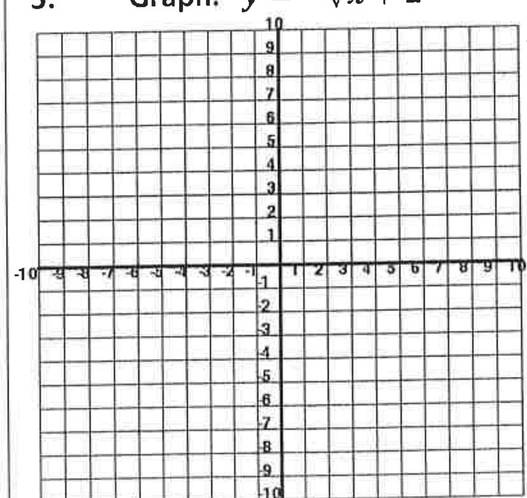


Vertex:

Domain:

Range:

5. **Graph:** $y = -\sqrt{x} + 2$

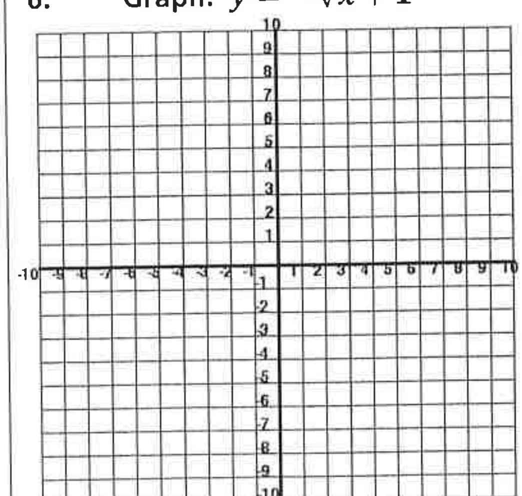


Vertex:

Domain:

Range:

6. **Graph:** $y = -\sqrt[3]{x+1}$



Vertex:

Domain:

Range:

7. Write the equation of a square root function that has been translated right ten units and up six units.

8. Write the equation of a cube root function that has been translated left three units and down two units.

9. Write the equation of a square root function that has been translated right four units and reflected across the $x - axis$.

10. Write the equation of a square root function with a domain of $x \leq -1$ and a range of $y \geq 2$.

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Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 2 → Square Root & Cube Root Graphs HOMEWORK

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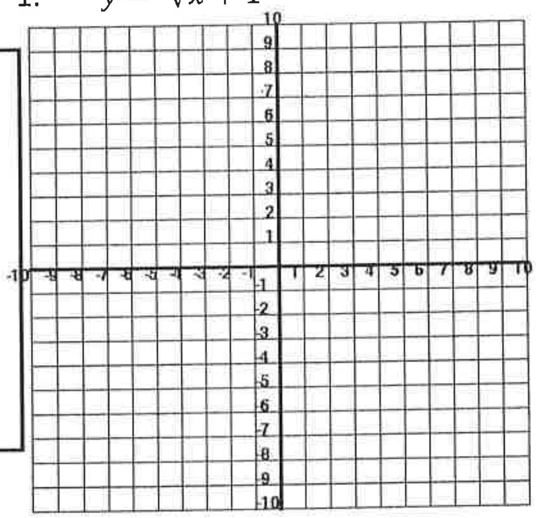
➢ Complete the table:

Function	Vertex	Horizontal Translation Left or Right	Vertical Translation Up or Down	Vertical Stretch or Compression	Reflection over x-axis	Domain	Range
$y = -\sqrt{x+4} - 1$							
$y = (x-3)^{1/2} + 2$							
$y = 2 - 3\sqrt{x+1}$							
$y = \sqrt[3]{x} + 4$							
$y = (x+4)^{1/3} - 5$							
$y = -4\sqrt[3]{x+3}$							
$y = \frac{1}{2}\sqrt{x+3} - 4$							

➢ Sketch each graph:

1. $y = \sqrt{x} + 1$

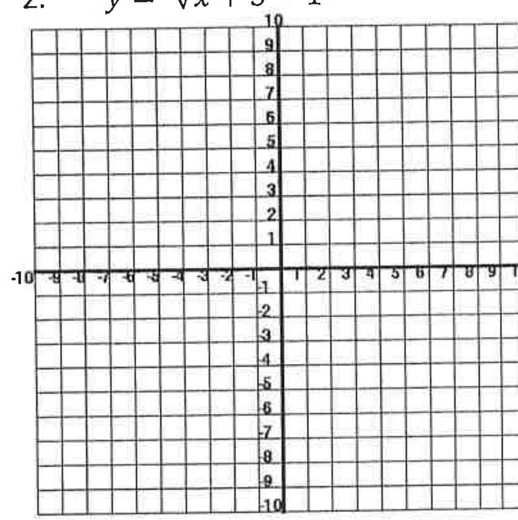


Vertex:

Domain:

Range:

2. $y = \sqrt{x+3} - 1$

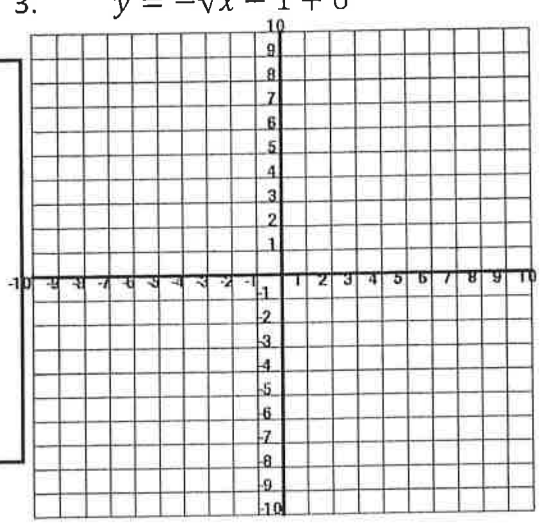


Vertex:

Domain:

Range:

3. $y = -\sqrt{x-1} + 6$

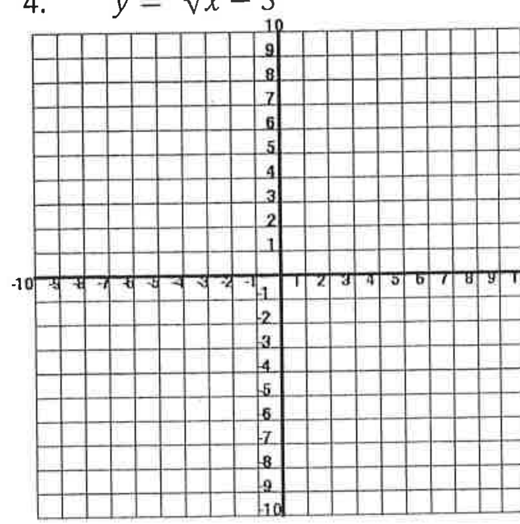


Vertex:

Domain:

Range:

4. $y = \sqrt[3]{x} - 3$



Vertex:

Domain:

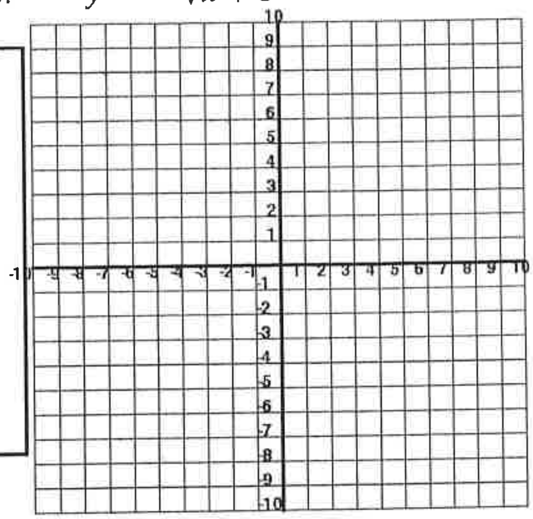
Range:

5. $y = -\sqrt[3]{x+3}$

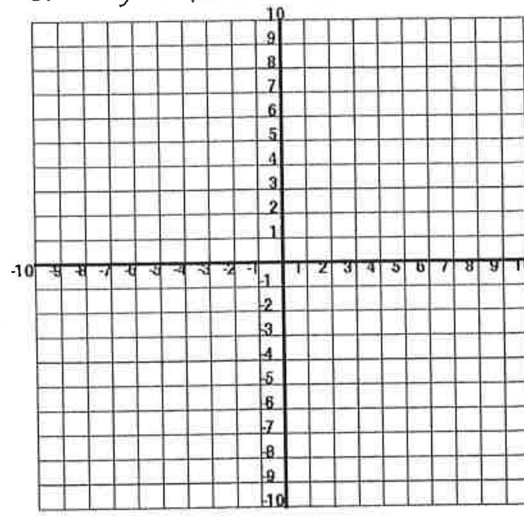
Vertex:

Domain:

Range:



6. $y = \sqrt[3]{x+2} - 5$



Vertex:

Domain:

Range:

➢ Write the equation of the function:

7. Write the equation of a **cubed** function that has been translated left four units and up six units.

8. Write the equation of a **cube root** function that has been translated left seven units and down one unit.

9. Write the equation of a **cube root** function that has been translated left four units and up six units and reflected across the $x - axis$.

10. Write the equation of a **square root** function that has been translated right three units and down two units.

11. Write the equation of a **square root** function that has been translated left two units and reflected across the $x - axis$.

12. Write the equation of a **square root** function that has been translated up two units and reflected across the $y - axis$.

13. Write the equation of a **square root** function with a domain of $x \geq 3$and a range of $y \leq 4$.

14. Write the equation of a **square root** function that passes through the points $(0,0)$ *and* $(1,7)$.

15. Write the equation of a **square root** function that passes through the points $(0,0)$ *and* $(1,-3)$.

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Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 3 → Square Root & Cube Root Equations

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There are four steps to solving a radical equation:
1) Isolate the radical.
2) Raise both sides to the power of the root.
3) Solve for x.
4) Check for extraneous solution(s).

What is an **EXTRANEOUS** solution? A solution to the final equation but not to the original equation. Extraneous solutions can occur when solving a **square root equation** but not when solving linear or quadratic equations.

- Examples:

1. $\sqrt{x} = 8$	2. $\sqrt{x+7} = 8$	3. $2\sqrt{x+6} = 14$
4. $-4\sqrt{x} + 11 = 3$	5. $(x-2)^{1/2} - 2 = 2$	6. $10 - 3\sqrt[3]{2x+5} = -11$
7. $\sqrt{10x^2 - 49} = 3x$	8. $\sqrt{2x-6} = \sqrt{5x-15}$	9. $(6x-5)^{1/3} = (3x+2)^{1/3}$

10. $\sqrt{3x+7} = x+1$

11. $(15-7x)^{1/2} = x-1$

12. $\sqrt{x+2} = 4 - \sqrt{x}$

13. $\sqrt{x} + 3 = \sqrt{x+4}$

14. $\sqrt{x+8} = \sqrt{x} + \sqrt{3}$

15. $\sqrt{x+3} = \sqrt{x+1} + 1$

Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 3 → Square Root & Cube Root Equations HOMEWORK

Name________________________
Date____________________ Pd______

1. $\sqrt{x-1}=3$	2. $2=\sqrt{\frac{x}{2}}$
3. $\sqrt{-8-2x}=0$	4. $(x+4)^{\frac{1}{2}}=7$
5. $\sqrt[3]{x-3}=5$	6. $\sqrt{2x-6}=\sqrt{3x-14}$
7. $\sqrt{8x}=x$	8. $\sqrt[3]{9-x}=\sqrt[3]{1-9x}$
9. $\sqrt{3-2x}=\sqrt{1-3x}$	10. $x=(20-x)^{\frac{1}{2}}$

11. $x = \sqrt{-10 + 7x}$

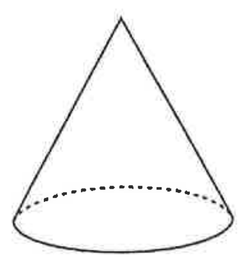
12. $\sqrt{2x - 7} = x - 3$

13. $x - 3 = \sqrt{37 - 3x}$

14. $\sqrt{3x - 11} = x - 5$

15. $\sqrt{3x - 2} = \sqrt{10 - x} + 2$

16. The surface area of a cone is found with the formula: $S = \pi r\sqrt{r^2 + h^2}$. Find h for the cone below of S = 1884 cm² and $r = 6$ cm. Use $\pi = 3.14$

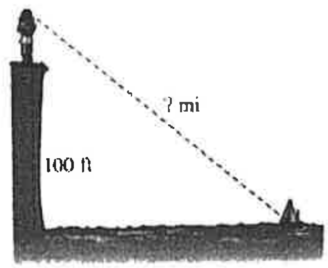


Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 4 → Square Root & Cube Root Applications

Name________________________
Date______________________ Pd_____

1. Did you ever stand on a beach and wonder how far out into the ocean you could see? Or have you wondered how close a ship has to be to spot land? In either case, the function $d(h) = \sqrt{2h}$ can be used to estimate the distance to the horizon (in miles) from a given height (in feet).

   a. Cordelia stood on a cliff gazing out at the ocean. Her eyes were $100\ ft$ above the ocean. She saw a ship on the horizon. Approximately how far was she from that ship?



   b. From a plane flying at $35{,}000\ ft$, how far away is the horizon?

   c. Given a distance, d, to the horizon, what altitude would allow you to see that far? Rewrite the formula and solve for h.

2. A weight suspended on the end of a string is a *pendulum*. The most common example of a pendulum (this side of Edgar Allen Poe) is the kind found in many clocks. The regular back-and-forth motion of the pendulum is *periodic*, and one such cycle of motion is called a *period*. The time, in seconds, that it takes for one period is given by the radical equation, $t = 2\pi\sqrt{\frac{l}{g}}$ in which g is the force of gravity $(10\ m/s^2)$ and l is the length of the pendulum $(meters)$.

   a. Find the period (to the nearest hundredth of a second) if the pendulum is $0.9\ m$ long.

   b. Find the period if the pendulum is $0.049\ m$ long.

   c. Solve the equation for length l.

   d. How long would the pendulum be if the period were exactly $1\ second$?

3. The current (in ampere) a machine need to reach a certain power can be modeled by the equation, $I = 0.2\sqrt{P}$, where P is the power of machine in watts. If the current is $10\ amperes$, what is the power of machine in watts?

4. When a car comes to a sudden stop, you can determine the skidding distance (in feet) for a given speed (in miles per hour) using the formula, $s(x) = 2\sqrt{5x}$, in which s is skidding distance and x is speed. Calculate the skidding distance for the following speeds (round to nearest tenth of a foot).

a. $55\ mph$

b. $65\ mph$

c. $75\ mph$

d. $40\ mph$

e. Given the skidding distance s, what formula would allow you to calculate the speed , x, in mph?

f. Use the formula obtained in (e) to determine the speed of a car in miles per hour if the skid marks were $35\ ft$ long.

Write an equation and then solve each of the following applications.

5. The sum of an integer and its square root is 12. Find the integer.

6. The difference between an integer and its square root is 12. What is the integer?

7. The sum of an integer and twice its square root is 24. What is the integer?

8. The sum of an integer and 3 times its square root is 40. Find the integer.

Math 2 – Honors
Unit 4 – Radical & Rational Functions
QUIZ Review Practice

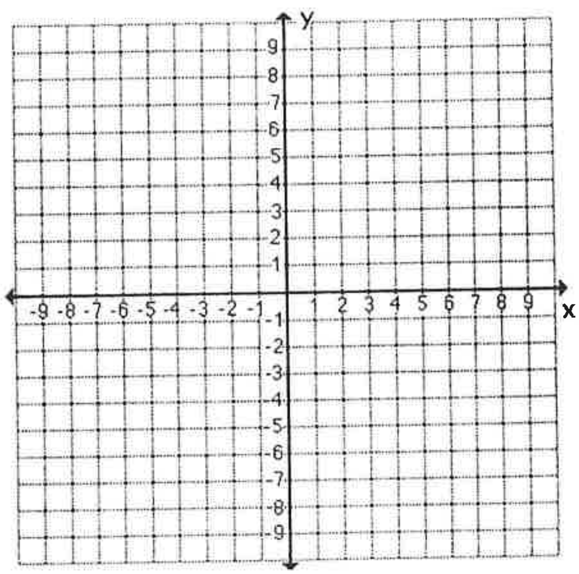
Name______________________________
Date______________________ Pd______

➤ Change each expression into its equivalent form and simplify if possible:

1. $\sqrt[3]{x^2}$	2. $64^{-1/3}$	3. $7x^{-1/5}$	4. $\sqrt[4]{5x^3}$

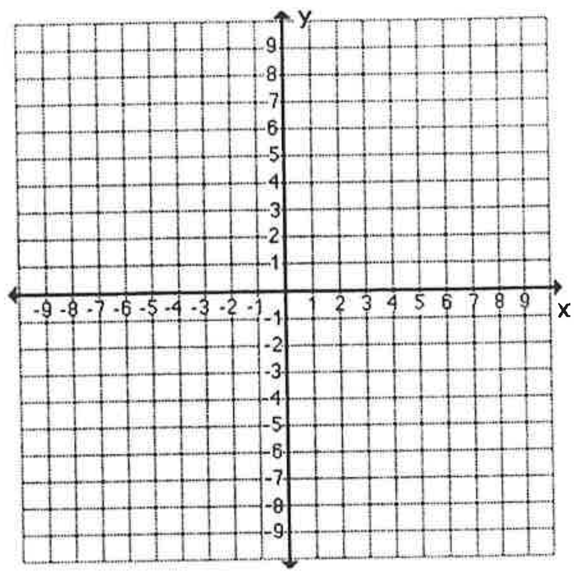
➤ Graph each function. Then state the Domain and the Range.

5. $y = \sqrt{x-3} + 2$



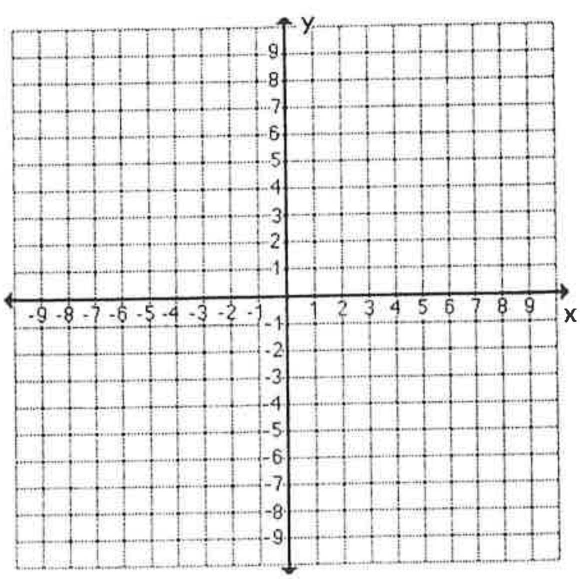
D: ____________ R: ____________

6. $y = -\sqrt{x} + 4$



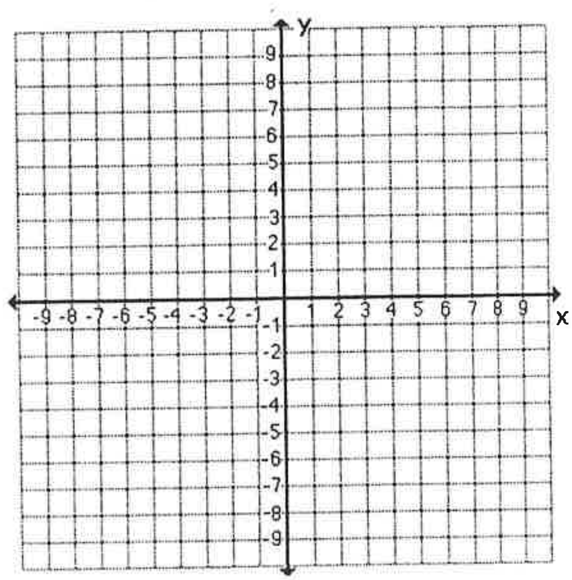
D: ____________ R: ____________

7. $y = \sqrt[3]{x-1}$



D: ____________ R: ____________

8. $y = -\sqrt[3]{x} - 2$



D: ____________ R: ____________

➤ Write an equation that describes the following function:

9. The parent function $y = x^3$ is translated 2 units to the left and one unit down.

10. The parent function $y = \sqrt{x}$ is translated 3 units to the right and reflected over the $x-axis$.

11. The parent function $y = \sqrt[3]{x}$ is compressed vertically by a factor of $\frac{1}{2}$ and then translated 2 units up.

➤ Solve each radical equation. Be sure to check for extraneous solutions.

12. $5\sqrt{x+7} = 25$	13. $\sqrt{x+8} + 4 = x$	14. $\sqrt[3]{3x-1} = \sqrt[3]{2x+4}$

15. Doctors can approximate the Body Surface Area of an adult (in $square\ meters$) using an index called BSA where H is height in $centimeters$ and W is weight in $kilograms$: $BSA = \sqrt{\frac{H*W}{3600}}$

Find the BSA of an adult who is 170 cm tall and weighs 68 kg.

Math 2 Honors
After Quiz Review of Rational Exponents

Name: ____________________
Date: ____________

Write each expression in radical form.

1. $3^{\frac{1}{5}}$

2. $7^{\frac{1}{3}}$

3. $a^{\frac{3}{5}}$

4. $\left(x^2\right)^{\frac{3}{5}}$

Write each expression using rational exponents.

5. $\sqrt{5}$

6. $\sqrt[3]{7}$

7. $\sqrt[5]{12y^2}$

8. $\sqrt[3]{3x^2y}$

Simplify each expression using the properties of exponents.

9. $36^{\frac{1}{2}}$

10. $(-27)^{\frac{2}{3}}$

11. $16^{-\frac{4}{3}}$

12. $\left(5^{\frac{1}{2}}\right)\left(5^{\frac{1}{2}}\right)$

13. $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{1}{3}}\right)\left(x^{\frac{1}{4}}\right)$

14. $8^{\frac{2}{3}}$

15. $64^{\frac{2}{3}}$

16. $(-8)^{\frac{2}{3}}$

17. $(-32)^{\frac{6}{5}}$

18. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

19. $\frac{4^{\frac{1}{3}}}{4}$

20. $\frac{16^{\frac{1}{2}}}{9^{\frac{1}{2}}}$

Math 2 – Honors
Unit 4 –Radical & Rational Functions
Lesson 5 → Graphs of Rational Functions

Name______________________________
Date___________________________Pd______

- A rational function is a function that can be written as the ratio of two polynomials where the denominator does not equal zero.

- $f(x) = \frac{p(x)}{q(x)}$ where $q(x) \neq 0$

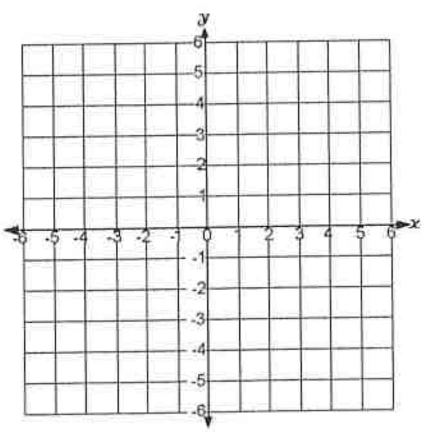
❖ **Steps to graph a rational function:** $y = \frac{n}{x-h} + k$

1) Determine the location of the asymptotes based on the transformations:
   A) Vertical asymptotes are placed based on the **horizontal translation:** $x = h$
   B) Horizontal asymptotes are placed based on the **vertical translation:** $y = k$
2) **Vertical Stretch or Compression:** ***n*** tells us how far the branches have been stretched from the asymptotes. We can use it to help us find out corner points to start our branches.

   Distance from asymptotes = $\sqrt{n}$
3) Look at the table on the calculator for other points and then sketch the two branches.

❖ Graph each of the following examples:

1. $y = \frac{1}{x}$



Equation of VA:

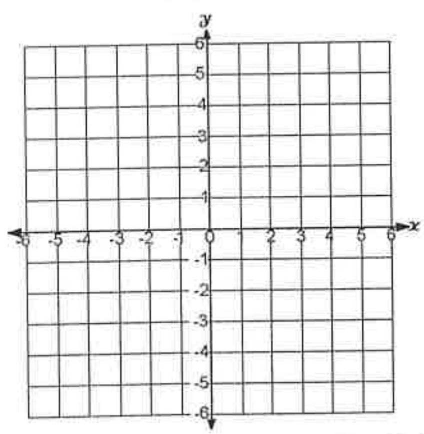
Equation of HA:

Describe translations:

Domain:

Range:

2. $y = \frac{1}{x-2} + 1$



Equation of VA:

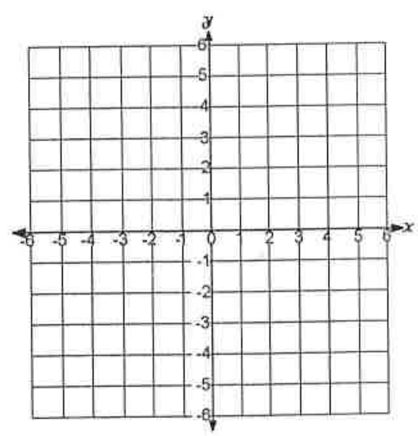
Equation of HA:

Describe translations:

Domain:

Range:

3. $y = -\frac{4}{x+1}$



Equation of VA:

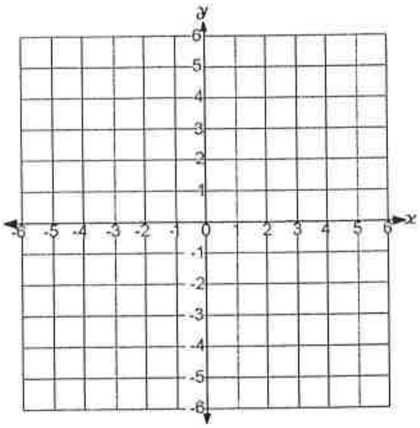
Equation of HA:

Describe translations:

Domain:

Range:

4. $y = \frac{1}{x} - 4$



Equation of VA:

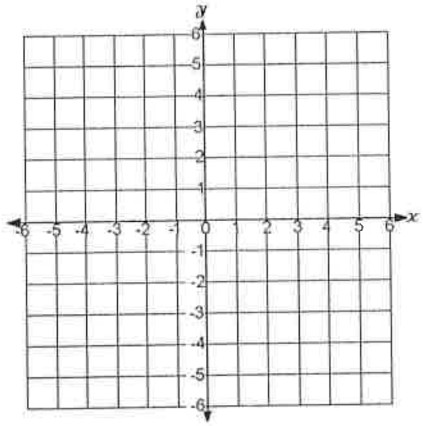
Equation of HA:

Describe translations:

Domain:

Range:

5. $xy = 9$



Equation of VA:

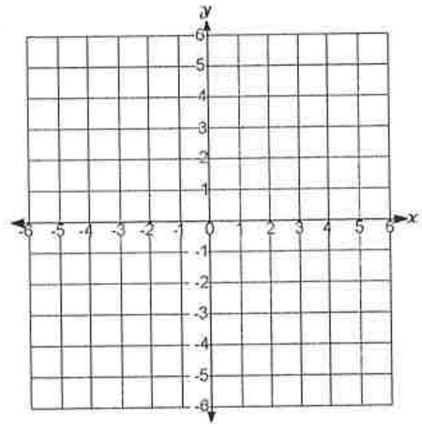
Equation of HA:

Describe translations:

Domain:

Range:

6. $y = \frac{3}{x-2} - 3$



Equation of VA:

Equation of HA:

Describe translations:

Domain:

Range:

7. Describe each graph as compared to the parent graph $y = \frac{1}{x}$.

$y = \frac{-2}{x-7} + 5$ The graph of this ________________ function has been translated ________ seven units and translated _____ units ________. It has been vertically stretched by a factor of ______ and ______________ across the x-axis. The graph is increasing from _________ to _________. The function has a domain of ______________ and a range of ______________.	$y = \frac{7}{x+2} - 4$ The graph of this ________________ function has been translated ________ two units and translated _____ units _________. It has been vertically stretched by a factor of _______. The graph is ______________ from left to right. The function has a domain of ______________ and a range of ______________.

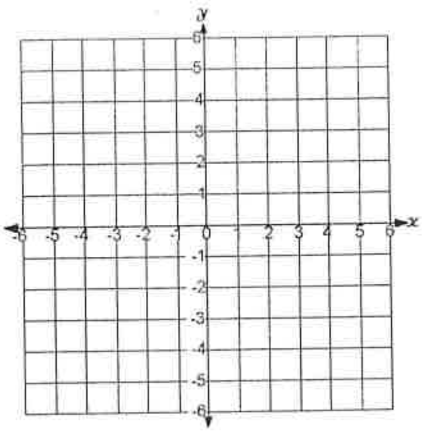
8. Write the equation of a rational function $y = \frac{1}{x}$ with following transformations:

A. Right 4 and Down 5	B. Left 3 and Up 2 and Reflected across $x - axis$.
C. Left 6 and Vertically Stretched by a factor of 4.	D. Right 5 and graph will be in II & IV quadrants

Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 5 → Graphing Rational Functions HOMEWORK

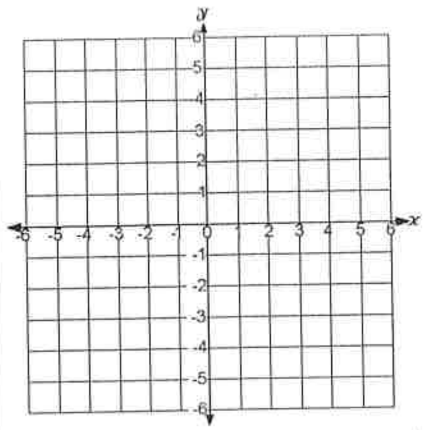
Name________________________________
Date_________________________ Pd______

1. $y = \frac{1}{x} + 3$



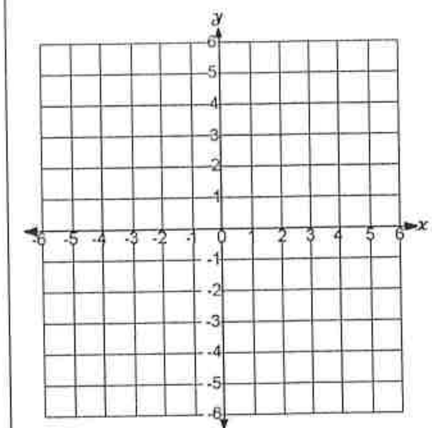
D: ____________ **R:** ____________

2. $y = \frac{1}{x-3}$



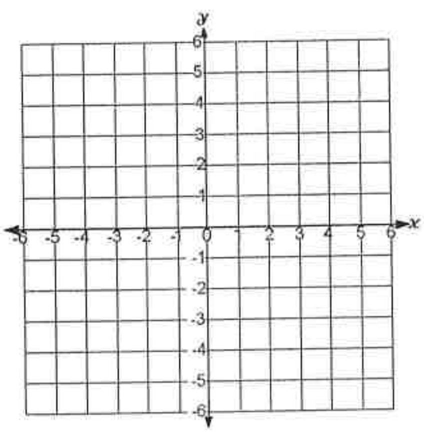
D: ____________ **R:** ____________

3. $y = \frac{1}{x+2} - 1$



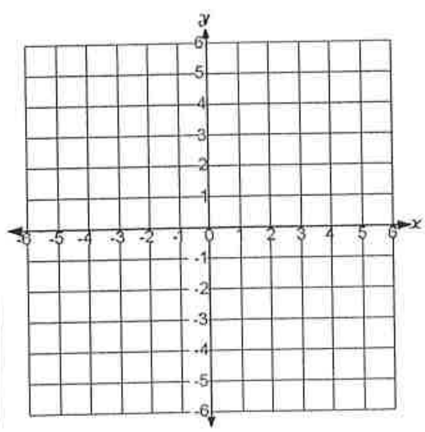
D: ____________ **R:** ____________

4. $y = \frac{2}{x}$



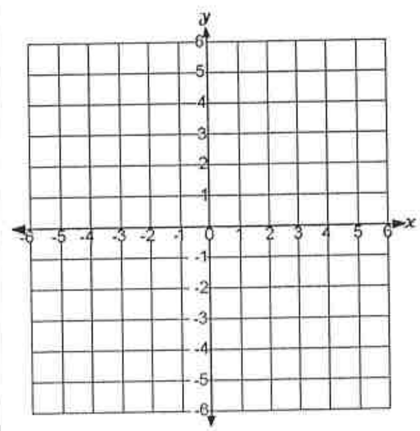
D: ____________ **R:** ____________

5. $y = \frac{3}{x+1}$



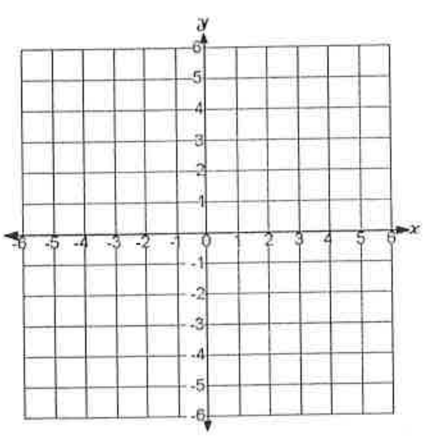
D: ____________ **R:** ____________

6. $y = \frac{4}{x-4} + 2$



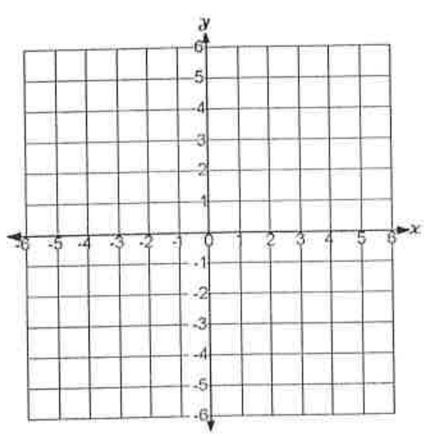
D: ____________ **R:** ____________

7. $y = -\frac{1}{x}$



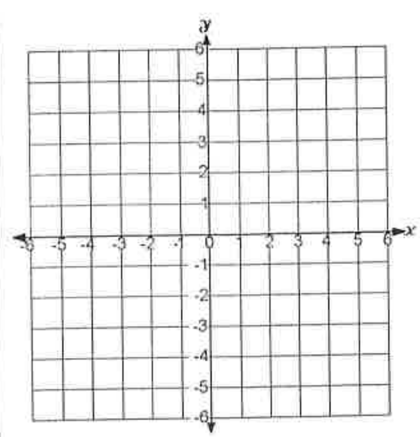
D: ____________ **R:** ____________

8. $y = -\frac{3}{x-2} + 1$



D: ____________ **R:** ____________

9. $y = -\frac{2}{x+1} - 2$



D: ____________ **R:** ____________

10. Consider the equation: $y = \frac{9}{x+1} - 2$

A) For what value is the function undefined (makes denominator = 0)? ____________________

B) What is the equation of the vertical asymptote? ____________________

C) What is the equation of the horizontal asymptote? ____________________

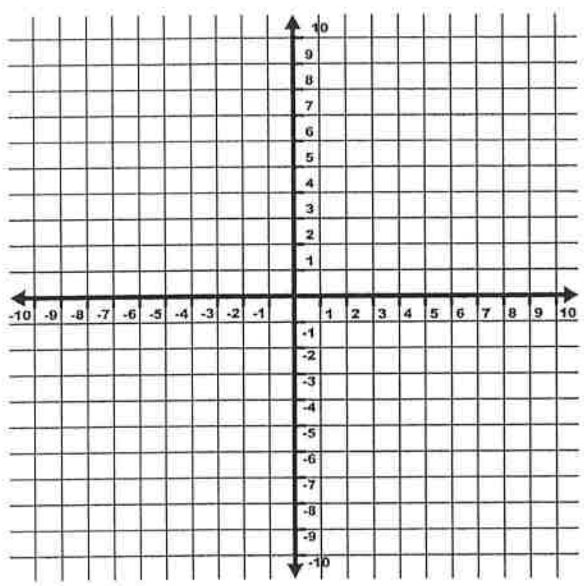
D) What is the domain of the function? ____________________

E) What is the range of the function? ____________________

F) What is the distance of the turning point from the intersection of the asymptotes? __________

G) In which quadrant is the center point located? ____________________

H) Graph the equation:



11. Write the equation of a rational function that has a domain of $x \neq 10$ and a range of $y \neq 5$ and passes through the point $(8, 4)$.

12. Write the equation of a rational function that has a domain of $x \neq 5$ and a range of $y \neq -7$ and passes through the point $(8, -6)$.

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Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 6 → Rational Equations

Name______________________________
Date________________________ Pd______

- **Recall:** A rational function is a function that can be written as the ratio of two polynomials where the denominator does not equal zero: $f(x) = \frac{p(x)}{q(x)}$ where $q(x) \neq 0$

- When solving rational equations with variables in the denominator, you must check the solution to be sure the denominator will not equal zero. The solution will be eliminated if the denominator is zero.

Examples: Solve for x.

1. $\frac{6}{x} = \frac{3}{7}$	2. $\frac{4}{x-7} = \frac{6}{x}$
3. $\frac{-5}{x+4} = \frac{1}{x+4}$	4. $\frac{4}{x+5} = \frac{x}{6}$

5. $\frac{x-4}{4}+\frac{x}{3}=6$	6. $\frac{3}{2x}-\frac{2x}{x+1}=-2$
7. $\frac{6}{x}=\frac{1}{4}+\frac{9}{x-1}$	8. $\frac{2x}{x-1}+\frac{x-5}{(x-1)(x+1)}=1$

Math 2 – Honors
Unit 4 – Radical & Rational Functions
Lesson 6 → Rational Equations HOMEWORK

Name________________________________
Date_________________________Pd______

- Solve for x:

1. $\frac{3}{x} = \frac{2}{x+4}$	2. $\frac{x+1}{2x+5} = \frac{2}{x}$
3. $\frac{3}{x+2} + 5 = \frac{4}{x+2}$	4. $\frac{6}{x-3} = \frac{x}{18}$
5. $\frac{5x}{x+2} + \frac{2}{x} = 5$	6. $\frac{2x-3}{7} - \frac{x}{2} = \frac{x+3}{14}$

7. $\frac{4x}{3x-2}+\frac{2x}{3x+2}=2$

8. $\frac{5}{5-x}-\frac{x^2}{5-x}=-2$

9. $\frac{2x-5}{x-2}-2=\frac{3}{x+2}$

10. $\frac{4}{(x-2)(x-6)}=\frac{x}{x-2}+\frac{1}{x-6}$

Math 2 – Honors
Unit 4 - Radical & Rational Functions
Lesson 7 → Types of Variations

Name________________________
Date____________________**Pd**_____

- **DIRECT VARIATION:** Linear function with a y-intercept of 0. In a direct variation, both of the quantities are either increasing or both are decreasing.
- There are two methods for solving a direct variation problem:
  1) Equation of Variation: $y = kx$ where k is called the **constant of variation**
  2) Proportion: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

#1: The distance that a body near Earth's surface will fall from rest varies directly as the **square** of the number of seconds it has been falling. If a boulder falls from a cliff a distance of 122.5 m in 5 $seconds$, approximately how far will it fall in 8 $seconds$?

Method 1	Method 2

- **JOINT VARIATION**: more than two quantities in a **direct variation** relationship
- Equation of Variation: $y = kxz$ where k is called the **constant of variaton**

#2: If y $varies\ jointly\ as\ x\ and\ z$, and $y = \frac{1}{2}$ when $x = 27$ and $z = \frac{-2}{3}$, find y when $x = 9$ and $z = 18$.

- **INVERSE VARIATION:** Rational function with vertical and horizontal asymptotes. In an inverse variation, one of the quantities is increasing while the second quantity is decreasing.
- Equation of Variation: $y = \frac{k}{x}$ where k is called the **constant of variation**

#3: The time of a trip varies inversely as the speed of the car. If a car being driven at 55 mph takes 2 $hours$ to get from Wake Forest to Greensboro, how fast is the car traveling if the trip takes 2.5 $hours$?

- **COMPOUND VARIATION:** Both Inverse and Direct Variation in the same problem
- Equation of Variation: $y = \frac{kx}{z}$ where k is called the **constant of variation**

#4: The volume of gas varies directly with Kelvin temperature and inversely with pressure. If a certain gas has a volume of 342 *cubic meters* at a temperature of 300 *Kelvin degrees* under a pressure of 200 *KPa (kilopascals)*, what will be the volume of the same gas at a temperature of 320 *Kelvin degrees* under a pressure of 400 *kPA*?

- **State whether each equation represents a direct, inverse, joint or compound variation. Then state the constant of variation.**

1. $y = \frac{9}{x}$	2. $z = 5xy$	3. $y = \frac{8x}{z}$	4. $y = 2x$	5. $xy = 12$
6. $z = \frac{xy}{15}$	7. $y = \frac{3}{4}xz$	8. $y = \frac{1}{3}x$	9. $z = \frac{x}{12y}$	10. $y = \frac{x}{5}$

- **Write a function for each variation relationship:**

11. W varies directly as the square of d.

12. V varies inversely as J.

13. V varies inversely as p and directly as T.

14. F varies jointly as A and the square of v.

15. L varies directly as the fourth power of d and inversely as the square root of h.

Math 2 – Honors
Unit 4 - Radical & Rational Functions
Lesson 7 → Types of Variations HOMEWORK

Name______________________________
Date________________________**Pd**_____

Write an equation for each statement and then solve:

1. If y *varies directly as* x and $y = 15$ when $x = 3$, find y when $x = 12$.
2. If y *varies directly as* x and $x = 36$ when $y = 4$, find x when $y = 24$.
3. If y *varies directly as* x^2 and $y = 12$ when $x = 4$, find y when $x = 6$.
4. If y *varies inversely as* x and $y = 2$ when $x = 8$, find x when $y = 14$.
5. *If* y *varies inversely as* x and $x = 7$ when $y = 21$, find y when $x = 42$.
6. If y *varies inversely as* x^3 and $y = 6$ when $x = \frac{-3}{4}$, find y when $x = 3$.
7. Suppose y *varies jointly with* x *and* z. If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 14$ and $z = 8$.
8. Suppose z *varies jointly with* x *and* y. If $x = 3$ and $y = 2$ when $z = 12$, find z when $x = 4$ and $y = 5$.
9. Suppose m *varies jointly* as n *and* p. If $n = 4$ and $p = 5$ when $m = 60$, find m when $n = 12$ and $p = 2$.
10. Suppose that y *varies directly as* x *and inversely as* z. If $y = 5$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.
11. Suppose y *varies directly as* $\sqrt{x}$ and *inversely as* z. If $y = 10$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 10$.
12. Suppose x *varies directly as* y^3 *and inversely as* $\sqrt{z}$. If $x = 7$ when $y = 2$ and $z = 4$, find x when $y = 3$ and $z = 9$.

Determine the type of variation and then write an equation for each statement. Then solve.

13. The number (B) of bolts a machine can make *varies directly* as the time (T) it operates. If the machine can make 6578 *bolts* in 2 *hours*, how many bolts can it make in 5 *hours*?

14. The number of cooks needed to prepare lunch *varies inversely* with the time. If it takes 9 *cooks four hours* to prepare a school lunch, how long would it take 8 *cooks* to prepare the lunch?

15. The current (I) in an electrical conductor *varies inversely* as the resistance (r) of the conductor. If the current is 2 *amperes* when the resistance in 960 *ohms*, what is the current when the resistance is 480 *ohms*?

16. Cheers *varied jointly* as the number of fans and the **square** of the jubilation factor. If there were 100 *cheers* when the number of fans was 100 and the jubilation factor was 4, how many cheers were there when there were only 10 *fans* whose jubilation factor was 20?

17. The volume of a cone *varied jointly* as the height of the cone and the area of the base. If a cone has a volume of 140 cm^3 when the height is 15 cm and the area of the base is 28 cm^2, find the volume of a cone with a height of 7 cm and a base area of 12 cm^2.

18. The number of girls *varies directly* as the number of boys and *inversely* as the number of teachers. When there were 50 *girls*, there were 10 *boys* and 20 *teachers*. How many boys were there when there were 10 *girls* and 100 *teachers*?

19. A pitcher's earned run average (ERA) *varies directly* as the number of earned runs allowed and *inversely* as the number of innings pitched. Joe Price had an ERA of 2.55 when he gave up 85 *earned runs in* 300 *innings*. What would be his ERA if he gave up 120 *earned runs in* 600 *innings*?

20. The maximum load that a cylindrical column with a circular cross section can hold *varies directly* as the fourth power of the diameter and *inversely* as the square of the height. A 9 *meter column* with a 2 *meter diameter* will support 64 *metric tons*. How many metric tons can be supported by a column 9 *meters high* and 3 *meters* in *diameter*?

JOINT AND COMBINED VARIATION

Translate each statement into a formula. Use k as the constant of variation.

1. V varies jointly as B and h. ____________
2. t varies directly as W and inversely as n. ____________
3. P varies directly as the square of V and inversely as R. ____________
4. h varies directly as W and inversely as the square of r. ____________
5. E varies jointly as m and the square of v. ____________
6. I varies jointly as A and H and inversely as T. ____________
7. The mass, m, of a cement block varies jointly as the length, ℓ, width, w, and thickness, t, of the block. ____________
8. The volume, V, of a gas varies directly as the temperature, T, and inversely as the pressure, P. ____________
9. The collision impact, I, of an automobile varies jointly as the mass, m, and the square of the speed, s. ____________
10. The intensity of a sound, i, varies directly as the amplitude, A, of the sound source, and inversely as the square of the distance, d, from the source. ____________
11. The safe load, s, for a beam, varies jointly as the breadth, b, and the square of the depth, d, and inversely as the length, ℓ, between supports. ____________
12. The gravitational force, g, between two objects varies jointly as the mass of the first, m_1, and the mass of the second, m_2, and inversely as the square of the distance, d, between them. ____________

Math 2 - Honors
Unit 4 Radical & Rational Functions
Test Review

Name ______________________
Date: ________________ PD: ______

➢ Write each expression in **simplest radical** form

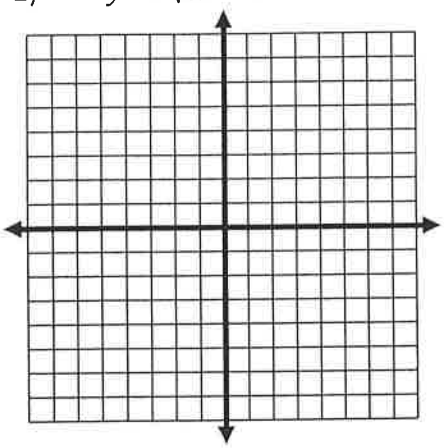
1. $7^{1/2}$	2. $x^{-2/3}$	3. $5y^{2/3}$	4. $(7x)^{1/4}$	5. $36^{-1/2}$

➢ Write each expression in **exponential** form:

6. $\sqrt{5}$	7. $\sqrt[4]{2x}$	8. $\sqrt[3]{x^2}$	9. $3\sqrt[5]{x^3}$	10. $\frac{1}{\sqrt{11}}$

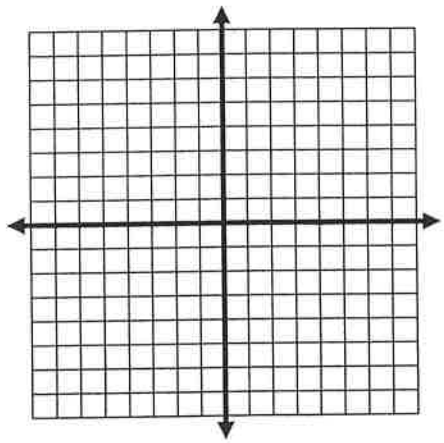
➢ **Graph each function. Then state the Domain & Range.**

1) $y = \sqrt{x} - 1$



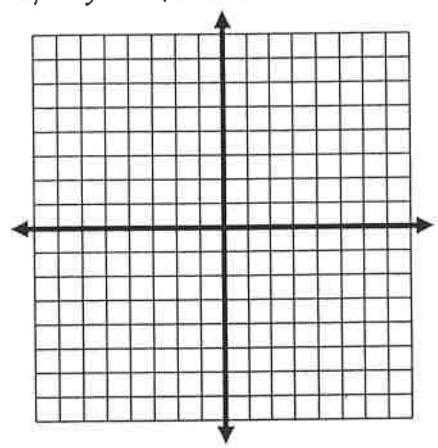
D: __________ R: __________

2) $f(x) = -\sqrt{x+2} - 4$



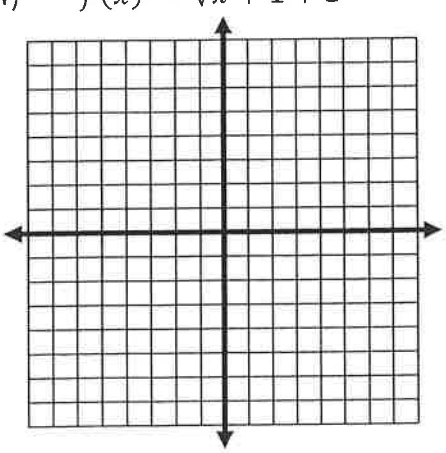
D: __________ R: __________

3) $y = \sqrt[3]{x} + 2$



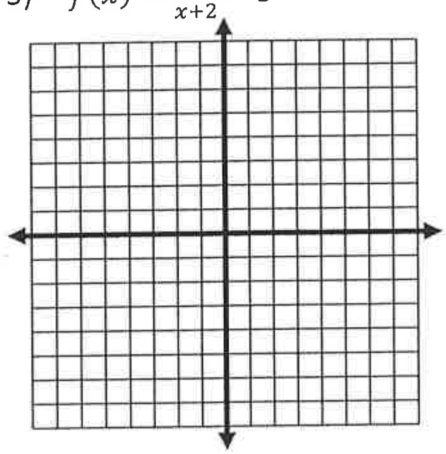
D: __________ R: __________

4) $f(x) = \sqrt[3]{x+1} + 2$



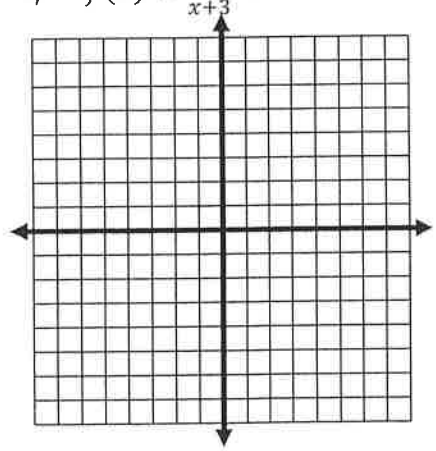
D: __________ R: __________

5) $f(x) = \frac{4}{x+2} - 3$



D: __________ R: __________

6) $f(x) = \frac{-1}{x+3} + 1$



D: __________ R: __________

7) Write the equation of a square root function that has been translated four units left and five units down and reflected across the $x - axis$.

8) Write the equation of a rational function that has a domain of $x \neq 2$ and a range of $y \neq -4$ with a vertical stretch of 9.

Solve each equation. Be sure to check for extraneous solutions!!

9) $\sqrt{x+10} - 7 = -5$	10) $\sqrt{-3x+40} = x$	11) $\sqrt{x+14} = x - 16$
12) $\frac{-2}{x+4} = \frac{4}{x+3}$	13) $\frac{x+4}{x-2} = \frac{x-5}{x-8}$	14) $\frac{5}{6x} + \frac{1}{x} = 4$
15) $\frac{2}{(x-1)(x+1)} - 1 = \frac{1}{x-1}$	16) Your distance from lightning *varies directly* with the time it takes you to hear thunder. If you hear thunder 10 *seconds* after you see lightning, you are about 2 *miles* from the lightning. About how many seconds would it take for thunder to travel a distance of 4 *miles*?	17) The drama club is planning a bus trip to New York City. The cost per person *varies inversely* as the number of people going on the trip. It will cost \$30 per person if 44 *people* go on the trip. How much will it cost per person if 60 *people* go on the trip?
18) For a given interest rate, simple interest *varies jointly* as principal and time. If \$2000 left in an account for 4 *years* earns interest of \$320, how much interest would be earned in if you deposit \$5000 for 7 *years*?	19) The volume of gas *varies directly* as the temperature and *inversely* as the pressure. If the volume is 230 *cubic centimeters* when the temperature is 300ºK and the pressure is 20 *pounds* per square centimeter, what is the volume when the temperature is 270ºK and the pressure is 30 *pounds* per square centimeter?	20) A. In a thunderstorm, the wind velocity in *meters per second* can be described by the function, $v(p) = 5.7\sqrt{998 - p}$ where p is the air pressure in millibars. What is the wind velocity if the air pressure is 437 *millibars*? B. What is the air pressure of a thunderstorm in which the wind velocity is 49.3 *meters per second*?