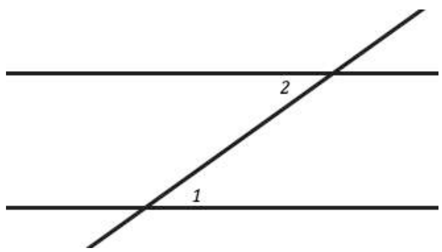
READY, SET, GO! Name Period Date

READY

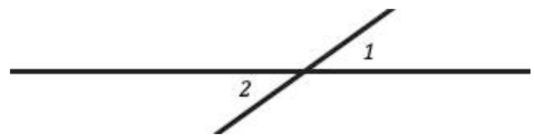
Topic: Basic angle relationships

Match the diagrams below with the best name or phrase that describes the angles.

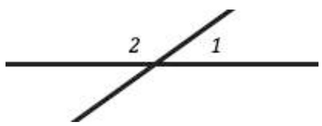
A 1.



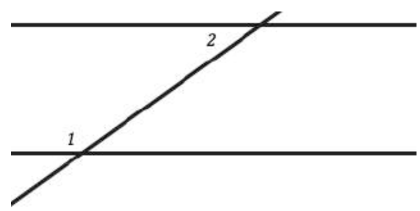
B 2.



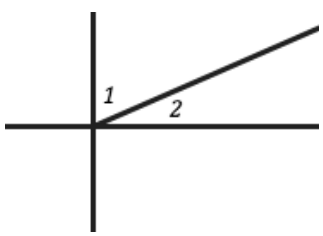
C 3.



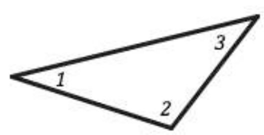
F 4.



E 5.



D 6.



a. Alternate Interior Angles b. Vertical Angles c. Complementary Angles

d. Triangle Sum Theorem e. Linear Pair f. Same Side Interior Angles

6. 3 Similar Triangles and Other Figures

A Solidify Understanding Task

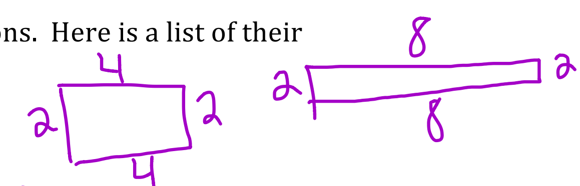
Two figures are said to be *congruent* if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In Mathematics I we found that we only needed three pieces of information to guarantee that two triangles were congruent: SSS, ASA or SAS.

What about AAA? Are two triangles congruent if all three pairs of corresponding angles are congruent? In this task we will consider what is true about such triangles.

Part 1

Definition of Similarity: Two figures are *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Mason and Mia are testing out conjectures about similar polygons. Here is a list of their conjectures.



Conjecture 1: *All rectangles are similar.* Not all

Conjecture 2: *All equilateral triangles are similar.* True

Conjecture 3: *All isosceles triangles are similar.* Not true

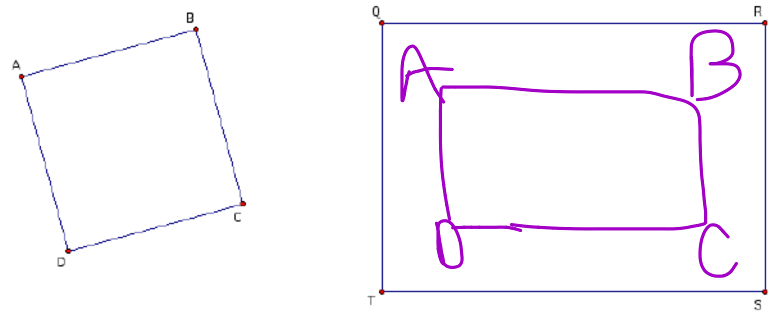
Conjecture 4: *All rhombuses are similar.* Not true

Conjecture 5: *All squares are similar.* True

1. Which of these conjectures do you think are true? Why?

Mason is explaining to Mia why he thinks conjecture 1 is true using the diagram given below.

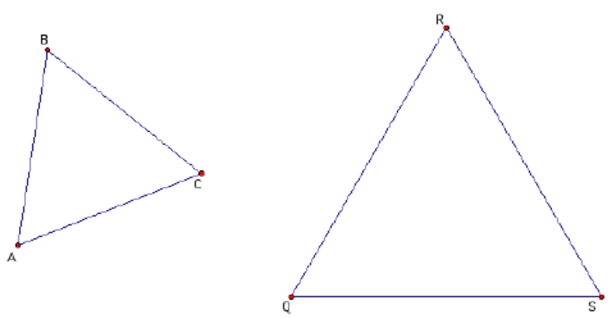
"All rectangles have four right angles. I can translate and rotate rectangle *ABCD* until vertex *A* coincides with vertex *Q* in rectangle *QRST*. Since $\angle A$ and $\angle Q$ are both right angles, side *AB* will lie on top of side *QR*, and side *AD* will lie on top of side *QT*. I can then dilate rectangle *ABCD* with point *A* as the center of dilation, until points *B*, *C*, and *D* coincide with points *R*, *S*, and *T*.



2. Does Mason's explanation convince you that rectangle *ABCD* is similar to rectangle *QRST* based on the definition of similarity given above? Does his explanation convince you that *all rectangles are similar*? Why or why not?

From B to R may not be same as scale factor as D to T.

Mia is explaining to Mason why she thinks conjecture 2 is true using the diagram given below.



"All equilateral triangles have three 60° angles. I can translate and rotate ΔABC until vertex *A* coincides with vertex *Q* in ΔQRS. Since $\angle A$ and $\angle Q$ are both 60° angles, side *AB* will lie on top of side *QR*, and side *AC* will lie on top of side *QS*. I can then dilate ΔABC with point *A* as the center of dilation, until points *B* and *C* coincide with points *R* and *S*."

3. Does Mia's explanation convince you that ΔABC is similar to ΔQRS based on the definition of similarity given above? Does her explanation convince you that *all equilateral triangles are similar*? Why or why not? The dilation matching B to R will be same as from C to S since sides of equilateral Δ's are $\cong$.

4. For each of the other three conjectures, write an argument like Mason's and Mia's to convince someone that the conjecture is true, or explain why you think it is not always true.

a. Conjecture 3: *All isosceles triangles are similar.*

b. Conjecture 4: *All rhombuses are similar.*

c. Conjecture 5: *All squares are similar.*

While the definition of similarity given at the beginning of the task works for all similar figures, an alternative definition of similarity can be given for polygons: **Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.**

5. How does this definition help you find the error in Mason's thinking about conjecture 1?

Since scale factors for the dilation that matches B to R and D to T may not be equal, the ratio $\overline{AB}/\overline{QR}$ is not necessarily equal to AD/QT, therefore, corresponding sides need not be proportional

6. How does this definition help confirm Mia's thinking about conjecture 2?

Since scale factors for the dilation that matches B to R and C to S are equal, the ratio $\overline{AB}/\overline{QR}$ is equal to AC/QS, therefore, corresponding sides need to be proportional

7. How might this definition help you think about the other three conjectures?

a. Conjecture 3: *All isosceles triangles are similar.*

b. Conjecture 4: *All rhombuses are similar.*

c. Conjecture 5: *All squares are similar.*

Part 2 (AAA, SAS and SSS Similarity)

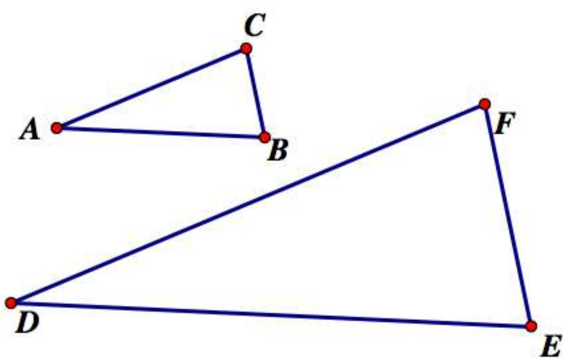
From our work above with rectangles it is obvious that knowing that all rectangles have four right angles (an example of AAAA for quadrilaterals) is not enough to claim that all rectangles are similar. What about triangles? In general, are two triangles similar if all three pairs of corresponding angles are congruent?

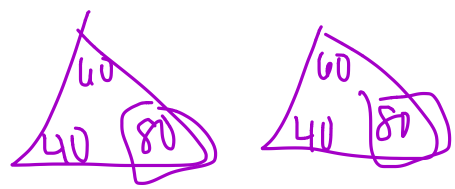
8. Decide if you think the following conjecture is true.

   Conjecture: *Two triangles are similar if their corresponding angles are congruent.*

   True

9. Explain why you think the conjecture—*two triangles are similar if their corresponding angles are congruent*—is true. Use the following diagram to support your reasoning, Remember to start by marking what you are given to be true (AAA) in the diagram.





Hint: *Begin by translating A to D.*

10. Mia thinks the following conjecture is true. She calls it "*AA Similarity for Triangles*." What do you think? Is it true? Why?

    Conjecture: *Two triangles are similar if they have two pair of corresponding congruent angles.*

11. Using the diagram given in problem 9, how might you modify your proof that $\Delta ABC \sim \Delta DEF$ if you are given the following information about the two triangles:

    a. $\angle A \cong \angle D,\ DE = k \cdot AB,\ DF = k \cdot AC$; that is, $\frac{DE}{AB} = \frac{DF}{AC}$

    b. $DE = k \cdot AB,\ DF = k \cdot AC$ and $EF = k \cdot BC$; that is, $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

READY, SET, GO! Name Period Date

READY

Topic: Solving proportions in multiple ways

Solve each proportion. Show your work and check your solution.

1. $\frac{3}{4} = \frac{x}{20}$

$\frac{60}{4} = \frac{4x}{4}$

$15 = x$

2. $\frac{x}{7} = \frac{18}{21}$

3. $\frac{3}{6} = \frac{8}{x}$

4. $\frac{9}{c} = \frac{6}{10}$

5. $\frac{3}{4} = \frac{b+3}{20}$

6. $\frac{7}{12} = \frac{a}{24}$

7. $\frac{a}{2} = \frac{13}{20}$

8. $\frac{3}{(b+2)} = \frac{6}{5}$ $\frac{6b+12}{15}$

$\frac{2b+4}{5}$

9. $\frac{\sqrt{3}}{2} = \frac{\sqrt{12}}{c}$

SET

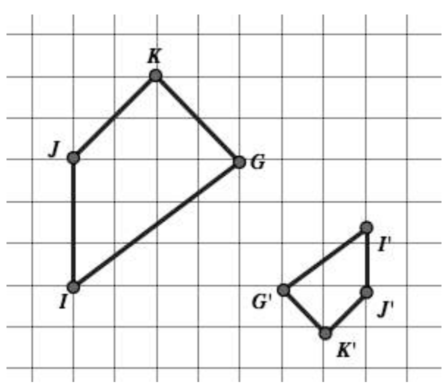
Topic: Proving Shapes are similar

Provide an argument to prove each conjecture, or provide a counterexample to disprove it.

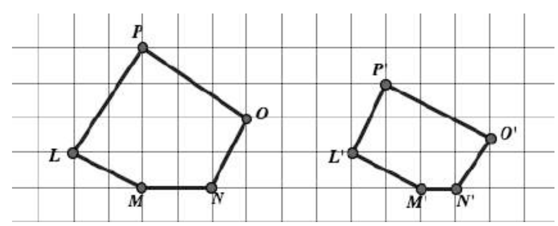
10. All right triangles are similar

11. All regular polygons are similar to other regular polygons with the same number of sides.

12. The polygons on the grid below are similar.

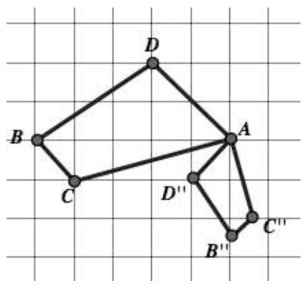


13. The polygons on the grid below are similar.

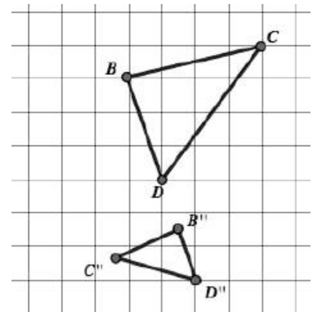


A sequence of transformations occurred to create the two similar polygons. Provide a specific set of steps that can be used to create the image from the pre-image.

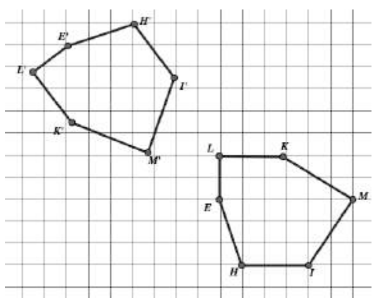
14.



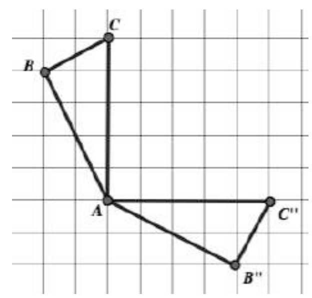
15.



16.



17.

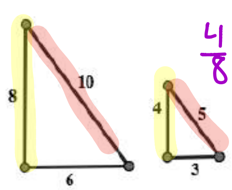


GO

Topic: Ratios in similar polygons

For each pair of similar polygons give three ratios that would be equivalent.

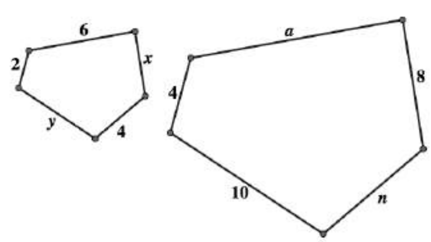
18.



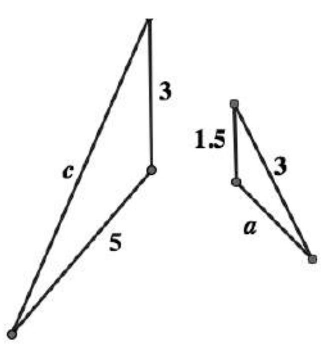
$\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$

$\frac{1}{2}$

19.



20.



21.