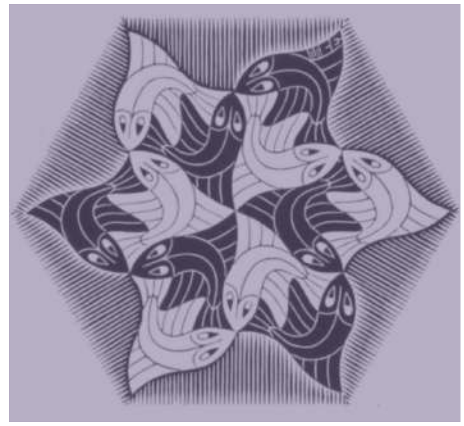
5.6 Justification and Proof

A Practice Understanding Task



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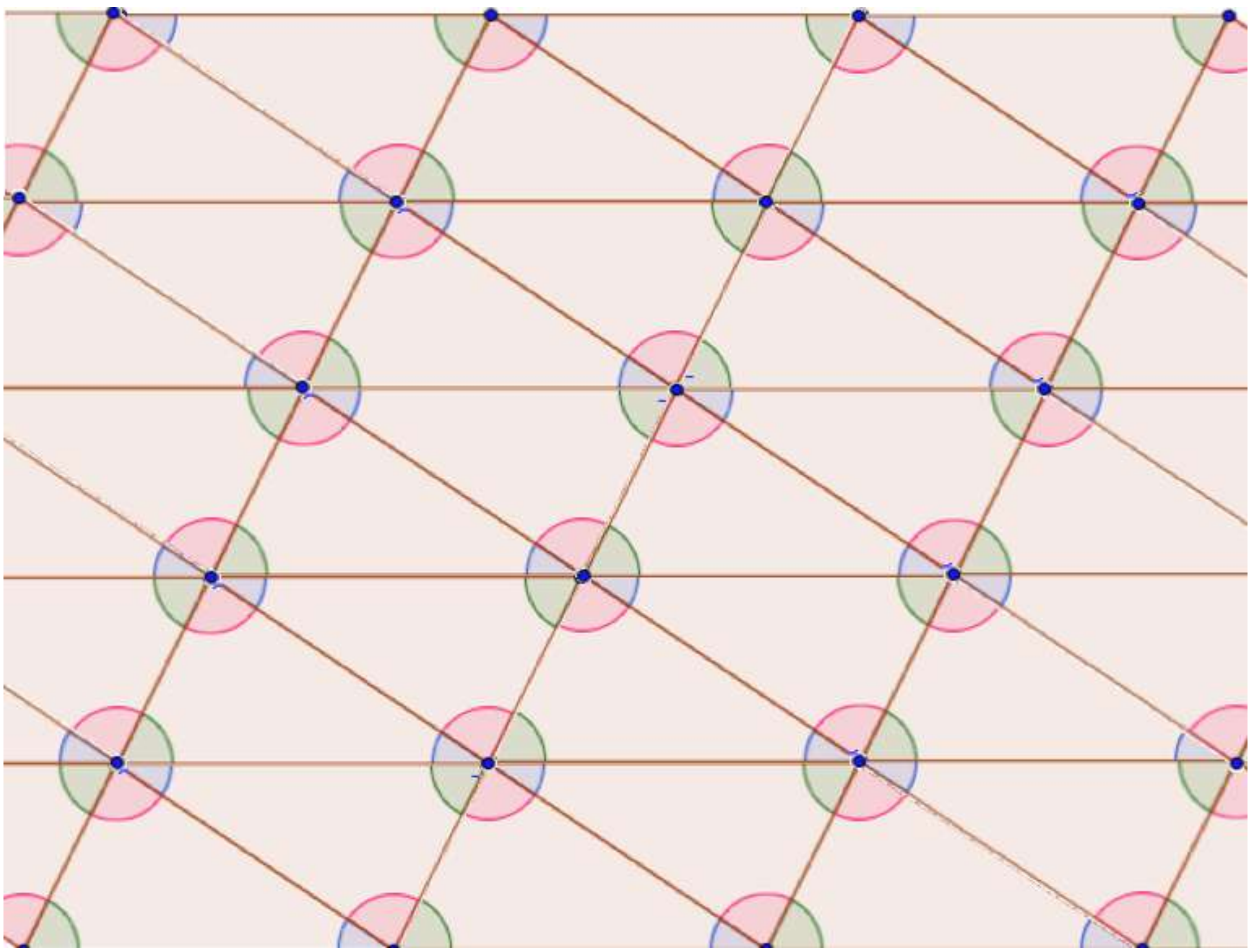
The diagram from *How Do You Know That?* has been extended by repeatedly rotating the image triangles around the midpoints of their sides to form a tessellation of the plane, as shown below.



Using this diagram, you have made some conjectures about lines, angles and triangles. In this task you will write proofs to convince yourself and others that these conjectures are always true.

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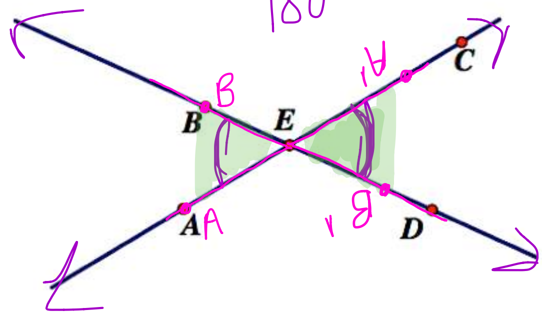
Reflect
180°

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram, $\angle AEB$ and $\angle CED$ form a pair of vertical angles.



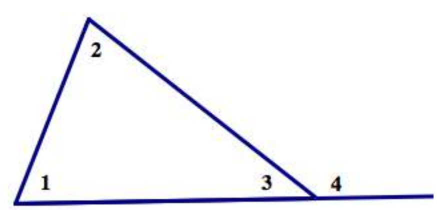
1. Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ intersect at E.
   Prove: $\angle AEB \cong \angle CED$

[Note: For each of the following proofs you may use any format you choose to write your proof: a flow proof diagram, a two-column proof, or a narrative paragraph.]

Statement	Reason
1) $\overleftrightarrow{AC}$ + $\overleftrightarrow{BD}$ intersect at E	Given
2) $\angle BEA$ maps onto $\angle DEC$	Rotate 180° about E.
3) $\angle BEA \cong \angle DEC$	$\angle$ measures are preserved through rotations

Exterior Angles of a Triangle

When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, $\angle 4$ is an exterior angle, and $\angle 1$ and $\angle 2$ are the two remote interior angles for this exterior angle.



2. Given: $\angle 4$ is an exterior angle of the triangle
   Prove: $m\angle 4 = m\angle 1 + m\angle 2$

Statement	Reason
1) $\angle 4$ is an exterior	Given
2) $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	Δ Sum theorem
3) $m\angle 3 + m\angle 4 = 180°$	Linear Pairs Supplementary
4) $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$	Transitive Prop of =
5) $m\angle 1 + m\angle 2 = m\angle 4$	Subtraction Prop of =

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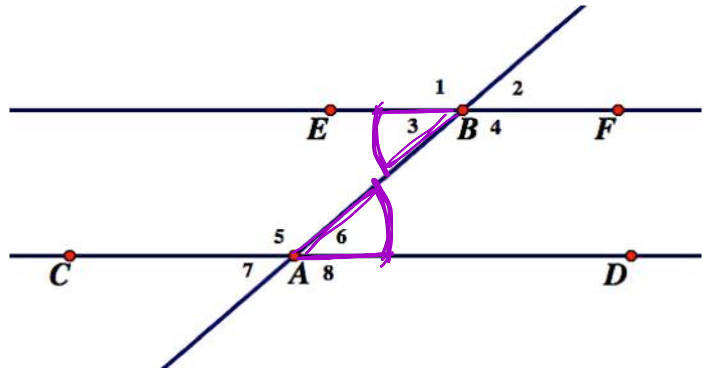
Parallel Lines Cut By a Transversal

When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram, $\angle 1$ and $\angle 5$ are called *corresponding angles*, $\angle 3$ and $\angle 6$ are called *alternate interior angles*, and $\angle 3$ and $\angle 5$ are called *same side interior angles*.



3. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$

   Prove: Corresponding angles $\angle 1$ and $\angle 5$ are congruent

Statement	Reason
1) $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$	Given
2) $\angle 1$ maps to $\angle 5$	Translate B to A
3) $\angle 1 \cong \angle 5$	Translations preserve $\angle$'s measure

4. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$

   Prove: Alternate interior angles $\angle 3$ and $\angle 6$ are congruent

S	R
$\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$	Given
$\angle 3$ maps to $\angle 7$	translate B to A
$\angle 3$ maps to $\angle 7$ which maps to $\angle 6$	Rotate $180°$ about A
$\angle 3 \cong \angle 6$	Composition of translate then rotate preserves angle measure

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