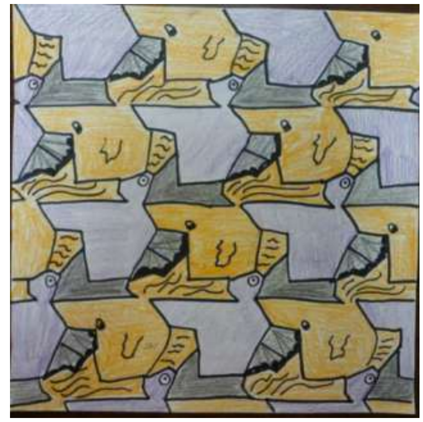
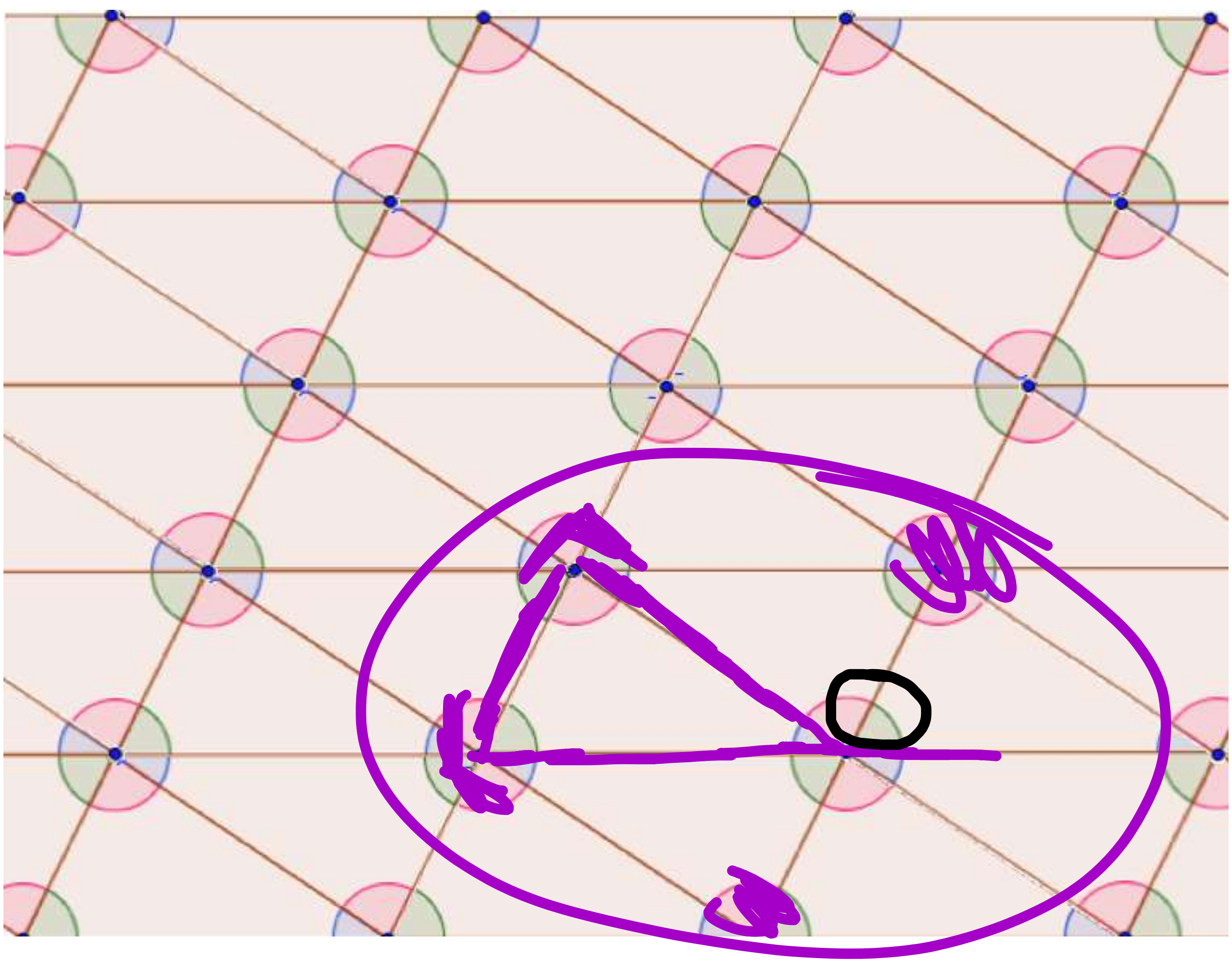
5.5 Claims and Conjectures

A Solidify Understanding Task



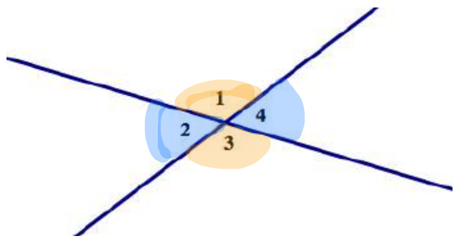
The diagram from *How Do You Know That?* has been extended by repeatedly rotating the image triangles around the midpoints of their sides to form a tessellation of the plane, as shown below.



Using this diagram, we will make some conjectures about lines, angles and triangles and then write proofs to convince ourselves that our conjectures are always true.

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram below, ∠ 1 and ∠ 3 form a pair of vertical angles, and ∠ 2 and ∠ 4 form another pair of vertical angles.



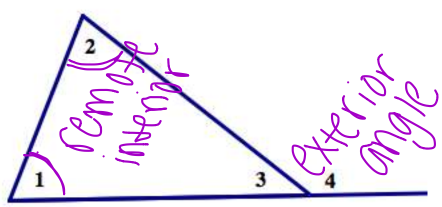
Examine the tessellation diagram above, looking for places where vertical angles occur. (You may have to ignore some line segments and angles in order to focus on pairs of vertical angles. This is a skill we have to develop when trying to see specific images in geometric diagrams.)

Based on several examples of vertical angles in the diagram, write a conjecture about vertical angles.

My conjecture: vertical ∠'s are ≅
"congruent"

Exterior Angles of a Triangle

When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, ∠ 4 is an exterior angle, and ∠ 1 and ∠ 2 are the two remote interior angles for this exterior angle



Examine the tessellation diagram above, looking for places where exterior angles of a triangle occur. (Again, you may have to ignore some line segments and angles in order to focus on triangles and their vertical angles.)

Based on several examples of exterior angles of triangles in the diagram, write a conjecture about exterior angles.

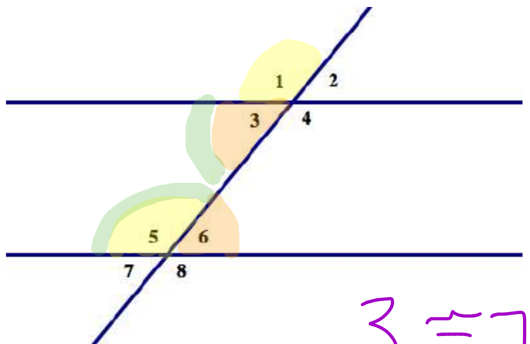
My conjecture: $\angle 4 \cong \angle 1 + \angle 2$

$* \; m\angle 4 = m\angle 1 + m\angle 2 \; *$

The sum of the remote interior ∠'s = the exterior angle

Parallel Lines Cut By a Transversal

When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram below, ∠ 1 and ∠ 5 are called *corresponding angles*, ∠ 3 and ∠ 6 are called *alternate interior angles*, and ∠ 3 and ∠ 5 are called *same side interior angles*.



Examine the tessellation diagram above, looking for places where parallel lines are crossed by a transversal line.

Based on several examples of parallel lines and transversals in the diagram, write some conjectures about corresponding angles, alternate interior angles and same side interior angles.

3 ≅ 7

My conjectures: ∠1 ≅ ∠5 b/c corresponding ∠'s

2 ≅ 6
4 ≅ 8

4 ≅ 5
∠3 ≅ ∠6 b/c alt int ∠'s

∠3 and ∠5 same side int ∠'s = 180°

Justifying Our Conjectures

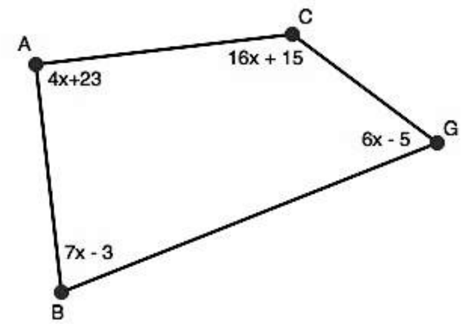
In the next task you will be asked to write a proof that will convince you and others that each of the conjectures you wrote above is always true. You will be able to use ideas about transformations, linear pairs, congruent triangle criteria, etc. to support your arguments. A good way to start is to write down everything you know about the diagram, and then identify which statements you might use to make your case. To get ready for the next task, revisit each of the conjectures you wrote about and record some ideas that seem helpful in proving that the conjecture is true.

READY, SET, GO! Name Period Date

READY

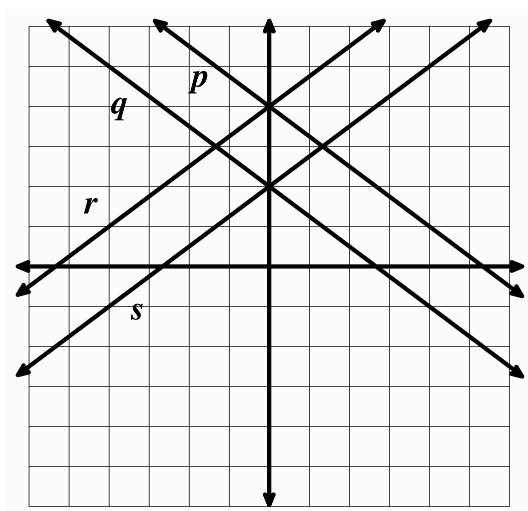
Topic: Properties of Quadrilaterals

1. Use what you know about triangles to write a paragraph proof that proves that the sum of the angles in a quadrilateral is 360º.



2. Find the measure of x in quadrilateral *ABGC*.

Match the equation with the correct line in the graph of lines *p*, *q*, *r*, and *s*.



3. $y = \frac{3}{4}x + 2$

4. $y = -\frac{3}{4}x + 2$

5. $y = \frac{3}{4}x + 4$

6. $y = -\frac{3}{4}x + 4$

7. Describe the shape made by the intersection 4 lines. List as many observations as you shape and its features.

SET

Topic: Parallel lines cut by transversal, vertical angles and exterior angle of a triangle

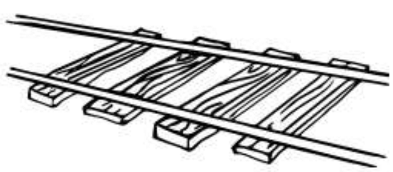
Label each picture as showing *parallel lines with a transversal, vertical angles*, or an *exterior angle of a triangle*. Highlight the geometric feature you identified. Can you find all 3 features in 1 picture? Where?

8.

9.

10.

11.

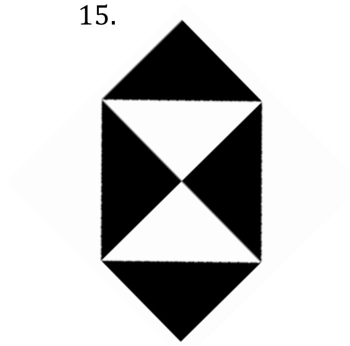


12.

13.

14.

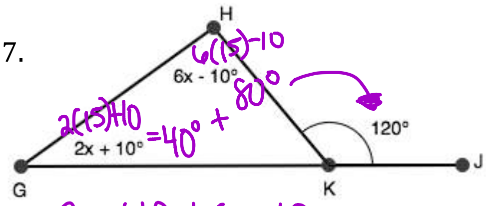
15.



16.

Find the value of the 2 remote interior angles in the figures below.

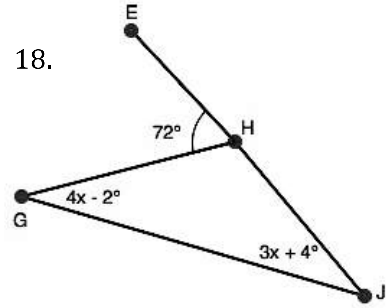
17.



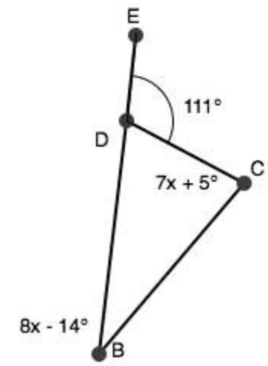
$2x+10+6x-10=120$

$\frac{8x}{8}=\frac{120}{8}$ $\quad x=15$

18.



19.



Indicate whether each pair of angles is *congruent* or *supplementary* by trusting how they look. Lines *p* and *q* are parallel.

20. $\angle 5$ *and* $\angle 8$

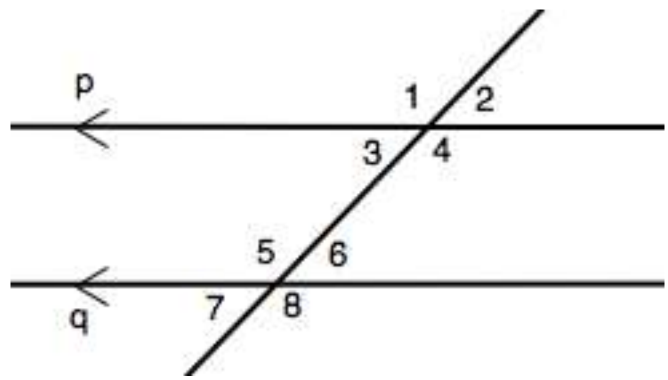
21. $\angle 2$ *and* $\angle 6$

22. $\angle 2$ *and* $\angle 8$

23. $\angle 4$ *and* $\angle 6$

24. $\angle 3$ *and* $\angle 5$

25. $\angle 1$ *and* $\angle 3$



GO

Topic: Complementary and supplementary angles

Find the complement and the supplement of the given angles. It is possible for the complement or supplement not to exist.

26. 37°

27. 59°

28. 89°

29. 111°

30. 3°

31. 90°