Tehani has been studying the figure below. She knows that quadrilateral ADEG is a rectangle and that $ED$ bisects $BC$. She is wondering if with that information she can prove $\triangle BGE \cong \triangle EDC$.

She starts to organize her thinking by writing what she knows and the reasons she knows it.

I know $ED$ bisects $BC$ because I was given that information

I know that $BE \cong EC$ by definition of bisect.

I know that $GE$ must be parallel to $AD$ because the opposite sides in a rectangle are parallel.

I know that $GA || ED$ because they are opposite sides in a rectangle.

I know that $AD$ is contained in $AC$ so $AC$ is also parallel to $GE$.

I know that $GA$ is contained in $BA$ so $GA$ is also parallel to $BA$.

I know that $BC$ has the same slope everywhere because it is a line.

I know the angle that $BE$ makes with $GE$ must be the same as the angle that $EC$ makes with $AC$ since those 2 segments are parallel. So $\angle BGE \cong \angle ECD$. I think I can use that same argument for $\angle GBE \cong \angle DEC$.

I know that I now have an angle, a side, and an angle congruent to a corresponding angle, side, and angle. So $\triangle BGE \cong \triangle EDC$ by ASA.

14. Use Tehani’s “I know” statements and her reasons to write a two-column proof that proves $\triangle BGE \cong \triangle EDC$. Begin your proof with the “givens” and what you are trying to prove.

**Given:** quadrilateral ADEG is a rectangle, $ED$ bisects $AC$

**Prove:** $\triangle BGE \cong \triangle EDC$

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. quadrilateral ADEG is a rectangle</td>
<td>given</td>
</tr>
<tr>
<td>2. $ED$ bisects $AC$</td>
<td>given</td>
</tr>
<tr>
<td>3. $BE \cong EC$</td>
<td>Def of bisector</td>
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<tr>
<td>4. $AC \parallel GE$ and $GA \parallel BE$</td>
<td>Rectangles are parallelograms</td>
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<tr>
<td>5. $\angle BGE \cong \angle ECD$</td>
<td>Corresponding Angles</td>
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<td>6. $\angle GBE \cong \angle DEC$</td>
<td>ASA $\cong$</td>
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<tr>
<td>7. $\angle BGE \cong \angle EDC$</td>
<td></td>
</tr>
</tbody>
</table>

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5.4 Parallelism Preserved and Protected

A Solidify Understanding Task

In a previous task, *How Do You Know That*, you were asked to explain how you knew that this figure, which was formed by rotating a triangle about the midpoint of one of its sides, was a parallelogram.

You may have found it difficult to explain how you knew that sides of the original triangle and its rotated image were parallel to each other except to say, “It just has to be so.” There are always some statements we have to accept as true in order to convince ourselves that other things are true. We try to keep this list of statements as small as possible, and as intuitively obvious as possible. For example, in our work with transformations we have agreed that distance and angle measures are preserved by rigid motion transformations since our experience with these transformations suggest that sliding, flipping and turning figures do not distort the images in any way. Likewise, parallelism within a figure is preserved by rigid motion transformations: for example, if we reflect a parallelogram the image is still a parallelogram—the opposite sides of the new quadrilateral are still parallel.

Mathematicians call statements that we accept as true without proof postulates. Statements that are supported by justification and proof are called theorems.

Knowing that lines or line segments in a diagram are parallel is often a good place from which to start a chain of reasoning. Almost all descriptions of geometry include a parallel postulate among the list of statements that are accepted as true. In this task we develop some parallel postulates for rigid motion transformations.

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Translations
Under what conditions are the corresponding line segments in an image and its pre-image parallel after a translation? That is, which word best completes this statement?

After a translation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose “sometimes”, be very clear in your explanation about how to tell when the corresponding line segments before and after the translation are parallel and when they are not.

Rotations
Under what conditions are the corresponding line segments in an image and its pre-image parallel after a rotation? That is, which word best completes this statement?

After a rotation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose “sometimes”, be very clear in your explanation about how to tell when the corresponding line segments before and after the rotation are parallel and when they are not.
Reflections

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a reflection? That is, which word best completes this statement?

After a reflection, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose “sometimes” be very clear in your explanation about how to tell when the corresponding line segments before and after the reflection are parallel and when they are not.
READY
Topic: Special Quadrilateral
Identify each quadrilateral as a trapezoid, parallelogram, rectangle, rhombus, square, or none of these. List ALL that apply.

1. [Diagram of a trapezoid labeled as a parallelogram and a rhombus]
2. [Diagram of a rectangle labeled as a parallelogram]
3. [Diagram of a square labeled as a parallelogram]
4. [Diagram of a trapezoid labeled as a parallelogram]
5. [Diagram of a parallelogram labeled as a rhombus]
6. [Diagram of a nonagon labeled as a kite]

SET
Topic: Identifying parallel segments and lines produced from transformations
7. Verify the parallel postulates below by naming the line segments in the pre-image and its image that are still parallel. Use correct mathematical notation.

a. After a translation, corresponding line segments in an image and its pre-image are always parallel or lie along the same line.

b. After a rotation of 180°, corresponding line segments in a pre-image and its image are parallel or lie on the same line.

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c. After a reflection, line segments in the pre-image that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.

GO
Topic: Identifying congruence patterns in triangles

For each pair of triangles write a congruence statement and justify your statement by identifying the congruence pattern you used. Then justify that the triangles are congruent by connecting corresponding vertices of the pre-image and image with line segments. How should those line segments look?

8. SSS
\( \triangle CDB \cong \triangle EFG \)

9. SAS
\( \triangle GEF \cong \triangle JHL \)

10. ASA
\( \triangle FGH \cong \triangle MJK \)

11. SAS
\( \triangle FGE \cong \triangle JHL \)
1) Proof
2) Classifying Qvads
3) Matching Terms
4) $x/150^\circ = 50^\circ$