5.3 It's All In Your Head

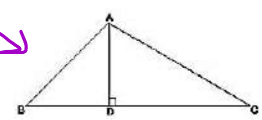
A Solidify Understanding Task

In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew. Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

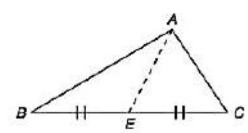
Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

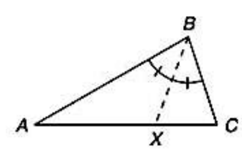
- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



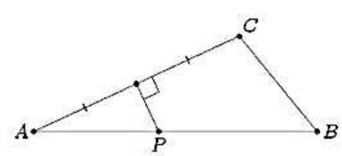
- In a triangle, a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



- In a triangle, an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- In a triangle, a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



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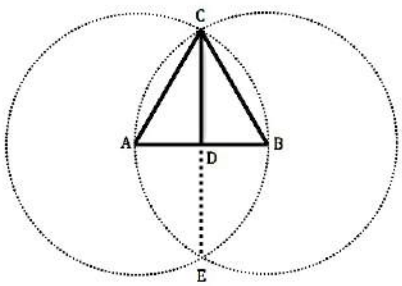
Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.



Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.

On the following page you will find a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas. You will use the flow diagram to identify the statements each of the students—Travis, Tehani, Carlos and Clarita—might use to make their case. To get ready to use the flow diagram, answer the following questions about what each student needs to know about the line of reflection to support their claim.

1. To support his claim that the line of reflection is a median of the equilateral triangle, Travis will need to show that:

   $\overline{AD} \cong \overline{BD}$

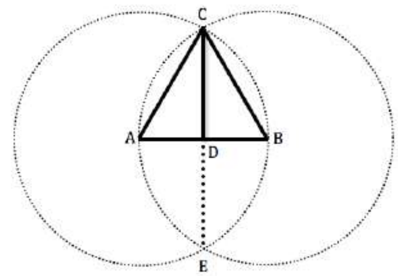
2. To support her claim that the line of reflection is an angle bisector of the equilateral triangle, Tehani will need to show that:

   $\angle ACD \cong \angle BCD$

3. To support her claim that the line of reflection is an altitude of the equilateral triangle, Clarita will need to show that: $\overline{CD} \perp \overline{AB}$

4. To support his claim that the line of reflection is a perpendicular bisector of a side of the equilateral triangle, Carlos will need to show that:

   $CD \perp AB$, $\overline{DA} \cong \overline{DB}$



Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.

7. Use four different colors to identify the statements each of the students—Travis, Tehani, Clarita and Carlos might use to make their case.

Given: ΔABC is equilateral AND 11 $\overleftrightarrow{CE}$ is a line of reflection

4

$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

1 1

$\angle CDA$ and $\angle CDB$ are right angles 5 $\overline{CD} \perp \overline{AB}$

D is the midpoint of $\overline{AB}$

6

$\overline{AD} \cong \overline{DB}$

17

$\overline{CD} \cong \overline{CD}$

14

$\angle ACD \cong \angle BCD$ $\Delta ACD \cong \Delta BCD$

16

7 8 9 10

therefore, $\overline{CD}$ is an altitude

therefore, $\overline{CD}$ is a median

therefore, $\overline{CD}$ is an angle bisector

therefore, $\overline{CD}$ is a perpendicular bisector

8. Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

1. Definition of reflection
2. Definition of translation
3. Definition of rotation
4. Definition of an equilateral triangle
5. Definition of perpendicular
6. Definition of midpoint
7. Definition of altitude
8. Definition of median
9. Definition of angle bisector
10. Definition of perpendicular bisector
11. Equilateral triangles can be folded onto themselves about a line of reflection
12. Equilateral triangles can be rotated 60° onto themselves
13. SSS triangle congruence criteria
14. SAS triangle congruence criteria
15. ASA triangle congruence criteria
16. Corresponding parts of congruent triangles are congruent
17. Reflexive Property

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two-column proof, as follows.

Statements	**Reasons**
ΔABC is equilateral	Given
$\overleftrightarrow{CE}$ is a line of reflection	Equilateral triangles can be folded onto themselves about a line of reflection
D is the midpoint of $\overline{AB}$	Definition of reflection
$\overline{CD}$ is a median	Definition of median

9. Write each of Clarita's, Tehani's, and Carlos' arguments in two-column proof format.

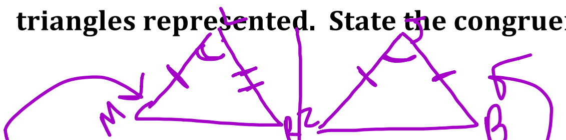
READY, SET, GO! Name Period Date

READY

Topic: Congruence statements and corresponding parts

Remember that when you write a congruence statement such as $\Delta ABC \cong \Delta FGH$, the corresponding parts of the two triangles must be the parts that are congruent.
For instance, $\angle A \cong \angle F$, $\overline{AB} \cong \overline{FG}$, $\angle B \cong \angle G$, $\overline{BC} \cong \overline{GH}$. Also, recall that the congruence patterns for triangles, ASA. SAS, and SSS, are what we can use to justify triangle congruence.

The segments and angles in each problem below are corresponding parts of 2 congruent triangles. Make a sketch of the two triangles. Then write a congruence statement for each pair of triangles represented. State the congruence pattern that justifies your statement.



	Congruence statement	Congruence pattern
1. $\overline{ML} \cong \overline{ZJ}, \overline{LR} \cong \overline{JB}, \angle L \cong \angle J$	a. $\Delta MLR \cong \Delta ZJB$	b. SAS
2. $\overline{WB} \cong \overline{QR}, \overline{BP} \cong \overline{RS}, \overline{WP} \cong \overline{QS}$	a.	b.
3. $\overline{CY} \cong \overline{RP}, \overline{EY} \cong \overline{BP}, \angle Y \cong \angle P$	a.	b.
4. $\overline{BC} \cong \overline{JK}, \overline{BA} \cong \overline{JM}, \angle B \cong \angle J$	a.	b.
5. $\overline{DF} \cong \overline{XZ}, \overline{FY} \cong \overline{ZW}, \angle F \cong \angle Z$	a.	b.
6. $\overline{WX} \cong \overline{AB}, \overline{XZ} \cong \overline{BC}, \overline{WZ} \cong \overline{AC}$	a.	b.

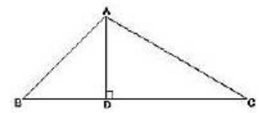
SET

Topic: Special triangle segments and proof.

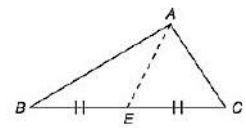
Recall the following definitions:

In a triangle:

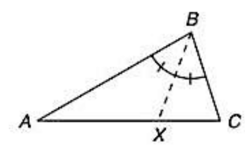
- an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



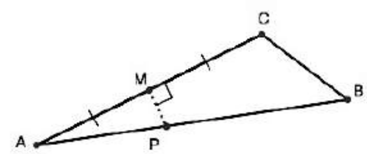
- a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



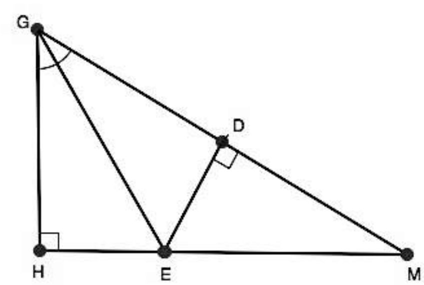
- an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Be sure to use the correct notation for a segment in the following problems.



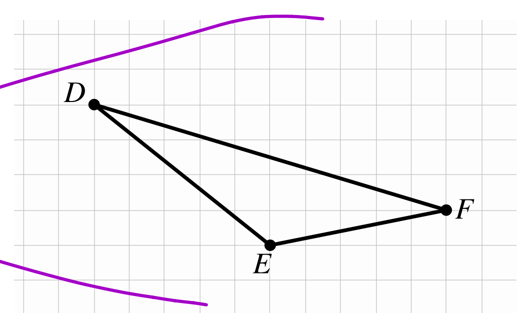
7. Name a segment in ΔGHM that is an altitude.

8. Name a segment in ΔGHM that is an angle bisector.

9. Name a segment in ΔGHM that is NOT an altitude.

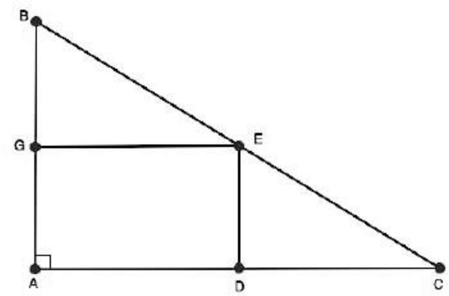
10. Create a perpendicular bisector by marking two segments congruent in ΔGHM. Name the segment that is now the perpendicular bisector.

Use ΔDEF in problems 11 – 13.



11. Construct the altitude from vertex D to $\overleftrightarrow{EF}$.
12. Construct the median from D to $\overline{EF}$.
13. Construct the perpendicular bisector of $\overline{EF}$.

Tehani has been studying the figure below. She knows that quadrilateral ADEG is a rectangle and that $\overline{ED}$ bisects $\overline{BC}$. She is wondering if with that information she can prove $\Delta BGE \cong \Delta EDC$.
She starts to organize her thinking by writing what she knows and the reasons she knows it.



I know $\overline{ED}$ bisects $\overline{BC}$ because I was given that information

I know that $\overline{BE} \cong \overline{EC}$ by definition of bisect.

I know that $\overline{GE}$ must be parallel to $\overline{AD}$ because the opposite sides in a rectangle are parallel.

I know that $\overline{GA} \parallel \overline{ED}$ because they are opposite sides in a rectangle.

I know that $\overline{AD}$ is contained in $\overline{AC}$ so $\overline{AC}$ is also parallel to $\overline{GE}$.

I know that $\overline{GA}$ is contained in $\overline{BA}$ so $\overline{GA}$ is also parallel to $\overline{BA}$

I know that $\overleftrightarrow{BC}$ has the same slope everywhere because it is a line.

I know the angle that $\overline{BE}$ makes with $\overline{GE}$ must be the same as the angle that $\overline{EC}$ makes with $\overline{AC}$ since those 2 segments are parallel. So $\angle BEG \cong \angle ECD$. I think I can use that same argument for $\angle GBE \cong \angle DEC$.

I know that I now have an angle, a side, and an angle congruent to a corresponding angle, side, and angle. So $\Delta BGE \cong \Delta EDC$ by ASA.

14. Use Tehani's "I know" statements and her reasons to write a two-column proof that proves $\Delta BGE \cong \Delta EDC$. Begin your proof with the "givens" and what you are trying to prove.

Given: quadrilateral ADEG is a rectangle, $\overline{ED}$ bisects $\overline{AC}$
Prove: $\Delta BGE \cong \Delta EDC$

STATEMENTS	REASONS
1. quadrilateral ADEG is a rectangle	given
2. $\overline{ED}$ bisects $\overline{AC}$	given