Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

WCPSS Math 2 Unit 7: MVP MODULE 5
Geometric Figures

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The Mathematics Vision Project

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MODULE 5 - TABLE OF CONTENTS

Geometric Figures

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5.1 How Do You Know That?

A Develop Understanding Task

You probably know that the sum of the interior angles of any triangle is 180°. (If you didn't know that, you do now!) But an important question to ask yourself is, "How do you know that?"

We know a lot of things because we *accept it on authority*—we believe what other people tell us; things such as the distance from the earth to the sun is 93,020,000 miles or that the population of the United States is growing about 1% each year. Other things are just defined to be so, such as the fact that there are 5,280 feet in a mile. Some things we accept as true based on experience or repeated experiments, such as the sun always rises in the east, or "I get grounded every time I stay out after midnight." In mathematics we have more formal ways of deciding if something is true.

Experiment #1

1. Cut out several triangles of different sizes and shapes. Tear off the three corners (angles) of the triangle and arrange the vertices so they meet at a single point, with the edges of the angles (rays) touching each other like pieces of a puzzle. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

2. Since you and your classmates have performed this experiment with several different triangles, does it guarantee that we will observe this same result for *all* triangles? Why or why not?

Experiment #2

Perhaps a different experiment will be more convincing. Cut out another triangle and trace it onto a piece of paper. It will be helpful to color-code each vertex angle of the original triangle with a different color. As new images of the triangle are produced during this experiment, color-code the corresponding angles with the same colors.

- Locate the midpoints of each side of your cut out triangle by folding the vertices that form the endpoints of each side onto each other.

- Rotate your triangle 180° about the midpoint of one of its sides. Trace the new triangle onto your paper and color-code the angles of this image triangle so that corresponding image/pre-image pairs of angles are the same color.

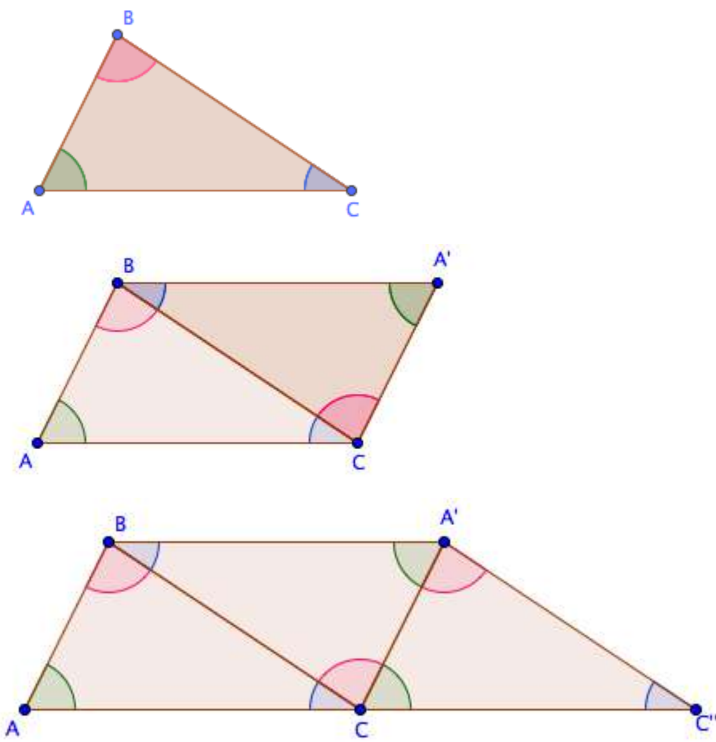
- Now rotate the new "image" triangle 180° about the midpoint of one of the other two sides. Trace the new triangle onto your paper and color-code the angles of this new image triangle so that corresponding image/pre-image pairs of angles are the same color.

3. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

4. Do you think you can rotate *all* triangles in the same way about the midpoints of its sides, and get the same results? Why or why not?

Examining the Diagram

Experiment #2 produced a sequence of triangles, as illustrated in the following diagram.



Here are some interesting questions we might ask about this diagram:

5. Will the second figure in the sequence always be a parallelogram? Why or why not?

6. Will the last figure in the sequence always be a trapezoid? Why or why not?

READY, SET, GO! Name Period Date

READY

Topic: Geometric Figures

One of the cool things about geometric figures is that our world is filled with them. For instance, my bathroom mirror is a perfect rectangle and the tiles on my floor are squares. Plus, the edges of these shapes are straight lines or line segments which are pieces of lines, since theoretically a line goes on forever.

1. Look around your world and make a list of the things you see that have a geometric shape. Here are some shapes to begin with. Think of all you can and be prepared to share your lists with the class.

Triangle **Trapezoid** **Parallelogram** **Cube** **Perpendicular lines**

SET

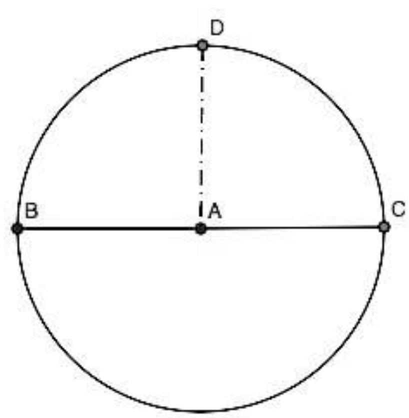
Topic: Linear Pairs

2. Fold a piece of paper, making a smooth crease. Open the paper and examine the shape that you made. Is it a line? Will it always be a line? Justify your thinking.

3. Look at a wall where it meets the ceiling. How would you describe the intersection of the wall and the ceiling?

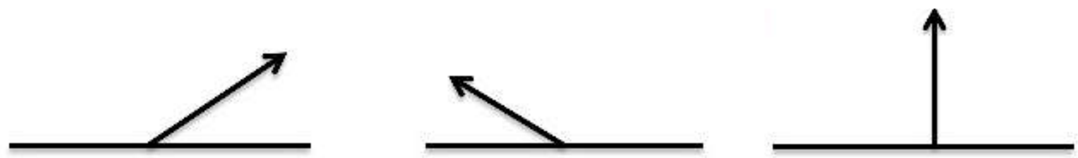
Imagine folding a circle exactly in half so that the fold passes through the center of the circle. This fold is called the diameter of the circle. It is a line segment with a length, but it is also a special kind of angle called a **straight angle**.

In order to "see" the angle, think of the center of the circle. That point is the vertex of the angle. Either side of the vertex is a radius of the circle. Whenever you draw 2 radii of the circle you make an angle. <u>When the two radii extend in exactly opposite directions and share a common endpoint (the center), they make a line or a</u> <u>**straight angle**</u>.



14. How many degrees do you think are in a *straight angle*?
 Use features of the diagram to justify your answer.

If two angles share a vertex and together they make a straight angle, then the two angles are called a **linear pair**. *(Below are 3 examples of* **linear pairs***.)*



Examples of linear pairs in real life:

http://www.flicker.com/photos/angle_dore/63650608

http://www.flicker.com/photos/truthlying/3845031/siz

5. Draw at least 2 diagrams of a real life linear pair.

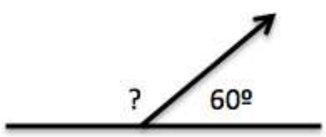
GO

Topic: Algebra of Linear Pairs

For 2 angles to be a **linear pair**, *they must share a vertex and a side, and the sum of their measures must equal 180°.*

Find the measure of the missing angle.

6.



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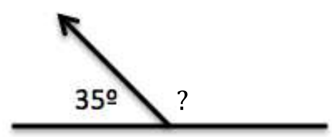

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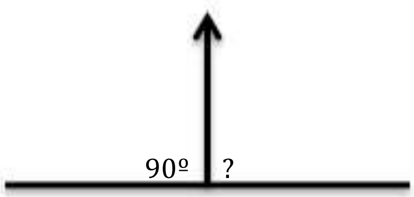

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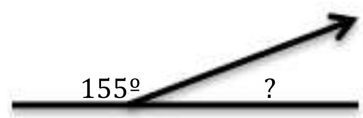

10. Linear pairs could be defined as being **supplementary angles** because they always add up to 180°. Are all supplementary angles linear pairs? Explain your answer.

Find the supplement of the given angle. Then draw the two angles as linear pairs. Label each angle with its measure.

11. m∠ABC = 72° B will be the vertex.

12. m∠GHK = 113° H will be the vertex.

13. m∠XYZ = 24° Y will be the vertex

14. m∠JMS = 168° M will be the vertex

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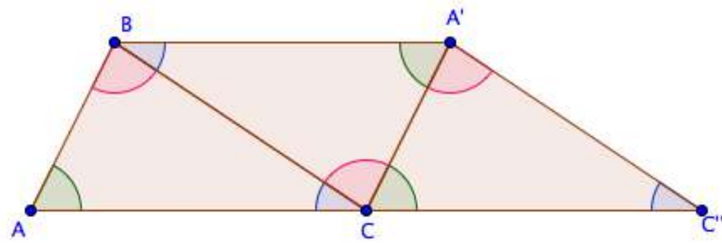
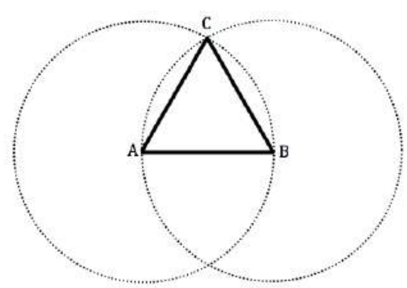
5.2 Do You See What I See?

A Develop Understanding Task

In the previous task, *How Do You Know That*, we saw how the following diagram could be constructed by rotating a triangle about the midpoint of two of its sides. The final diagram suggests that the sum of the three angles of a triangle is 180°. This diagram "tells a story" because you saw how it was constructed through a sequence of steps. You may even have carried out those steps yourself.



Sometimes we are asked to draw a conclusion from a diagram when we are given the last diagram in a sequence steps. We may have to mentally reconstruct the steps that got us to this last diagram, so we can believe in the claim the diagram wants us to see.



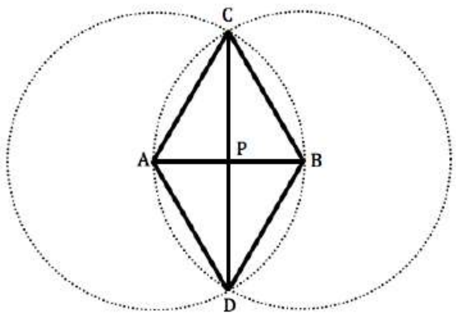
1. For example, what can you say about the triangle in this diagram?

2. What convinces you that you can make this claim? What assumptions, if any, are you making about the other figures in the diagram?

3. What is the sequence of steps that led to this final diagram?

4. What can you say about the triangles, quadrilateral, or diagonals of the quadrilateral that appear in the following diagram? List several conjectures that you believe are true.

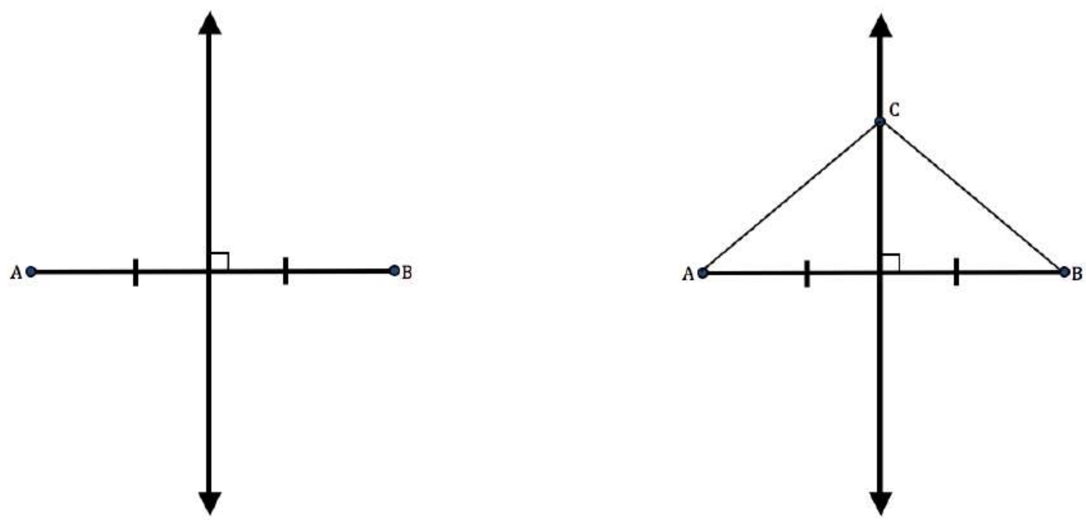
Given: $\odot A \cong \odot B$



5. Select one of your conjectures and write a paragraph convincing someone else that your conjecture is true. Think about the sequence of statements you need to make to tell your story in a way that someone else can follow the steps and construct the images you want them to see.

6. Now pick a second claim and write a paragraph convincing someone else that this claim is true. You can refer to your previous paragraph, if you think it supports the new story you are trying to tell.

7. Here is one more diagram. Describe the sequence of steps that you think were used to construct this diagram beginning with the figure on the left and ending with the figure on the right.



Travis and Tehani are doing their math homework together. One of the questions asks them to prove the following statement.

The points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment?

Travis and Tehani think the diagram above will be helpful to prove this statement, but they know they will need to say more than just describe how to create this diagram. Travis starts by describing the things they know, and Tehani tries to keep a written record by jotting notes down on a piece of paper.

8. In the table below, record in symbolic notation what Tehani may have written to keep track of Travis' statements. In the examples given, note how Tehani is introducing symbols for the lines and points in the diagram, so she can reference them again without using a lot of words.

Tehani's Notes	**Travis' Statements**
Draw $\overline{AB}$. Locate its midpoint M, and draw a perpendicular line ℓ through the midpoint	We need to start with a segment and its perpendicular bisector already drawn.
Pick any point C on line ℓ	We need to show that *any* point on the perpendicular bisector is equidistant from the two endpoints, so I can pick any arbitrary point on the perpendicular bisector. Let's call it C.
Prove:	We need to show that this point is the same distance from the two endpoints.
First prove:	If we knew the two triangles were congruent, we could say that the point on the perpendicular bisector is the same distance from each endpoint. So, what do we know about the two triangles that would let us say that they are congruent?
	We know that both triangles contain a right angle.
	And we know that the perpendicular bisector cuts segment AB into two congruent segments.
	Obviously, the segment from C to the midpoint of segment AB is a side of both triangles.
	So, the triangles are congruent by the SAS triangle congruence criteria.
	Since the triangles are congruent, segments AC and BC are congruent.
Any point C on line ℓ, the perpendicular bisector of $\overline{AB}$, is equidistant from the endpoints A and B.	And, that proves that point C is equidistant from the two endpoints!

9. Tehani thinks Travis is brilliant, but she would like the ideas to flow more smoothly from start to finish. Arrange Tehani's symbolic notes in a way that someone else could follow the argument and see the connections between ideas.

10. Would your justification be true regardless of where point C is chosen on the perpendicular bisector? Why?

READY, SET, GO! Name Period Date

READY

Topic: Symbols in Geometry

Throughout the study of mathematics, you have encountered many symbols that help you write mathematical sentences and phrases without using words. Symbols help the mathematician calculate efficiently and communicate concisely.

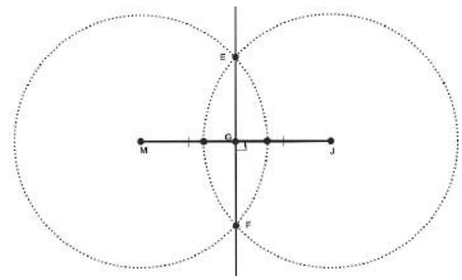
Below is a set of common mathematical symbols. Your job is to match them to their definitions. Are the symbols logical?

Symbol	Definitions
_____1. $=$	A. **Absolute value** – it is always equal to the positive value of the number inside the lines. It represents distance from zero.
_____2. $m\angle C$	B. **Congruent** – Figures that are the same size and shape are said to be congruent.
_____3. GH	C. **Parallel** – used between segments, lines, rays, or planes
_____4. ΔABC	D. **Line segment with endpoints G and H**. Line segments can be congruent to each other. You would not say they were equal.
_____5. $\perp$	E. **Ray GH** – The letter on the left indicates the endpoint of the ray.
_____6. $\angle ABC$	F. Used when comparing numbers of **equal value**.
_____7. $\overleftrightarrow{GH}$	G. **Plus or minus** – indicates 2 values, the positive value and the negative value
_____8. $\cong$	H. **Triangle ABC**
_____9. $\sim$	J. Indicates the **measure of an angle**. It would be set equal to a number.
_____10. $\overline{GH}$	K. **Perpendicular** - Lines, rays, segments, and planes can all be perpendicular
_____11. $\overrightarrow{GH}$	L. **Angle ABC** – The middle letter is always the vertex of the angle.
_____12. $\|\|$	M. **Similar** – Figures that have been dilated are similar.
_____13. $\pm$	N. **The length of GH**. It would equal a number.
_____14. $\|x\|$	P. Refers to the **infinite line GH**. Lines are not equal or congruent to other lines.

SET

Topic: Construction of midpoint, perpendicular bisector, and angle bisector, using "givens" to solve problems.

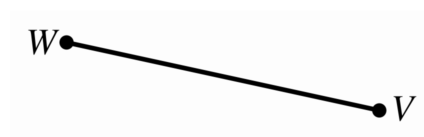
The figure on the right demonstrates the construction of a perpendicular bisector of a segment.



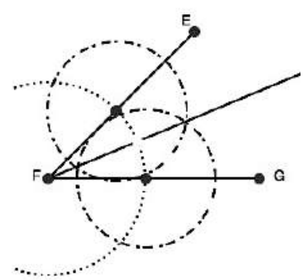
Use the diagram to guide you in constructing the perpendicular of the following line segments. Mark the right angle with the correct symbol for right angles. Indicate the segments are congruent by using slash marks.

15.

16.



The figure on the right demonstrates the construction of an angle bisector.
Use the diagram to guide you in constructing the angle bisector of the following angles. Mark your bisected angles as congruent.



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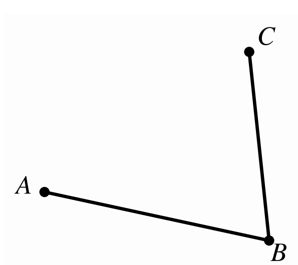

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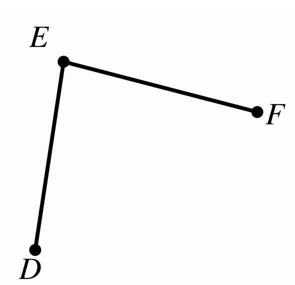

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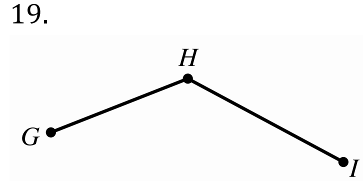
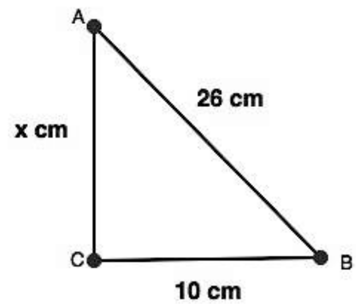
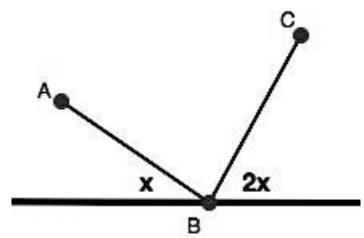


Examine the diagram and add any information that you are given. Think how you can use what you have been given and what you know to answer the question. Plan a strategy for finding the value of x. Follow your plan. Justify each step.

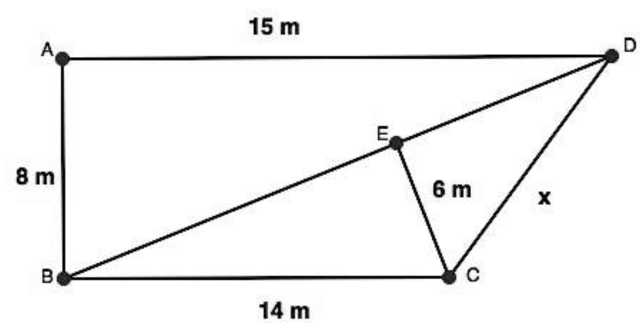
20. Given: $m\angle C = 90°$



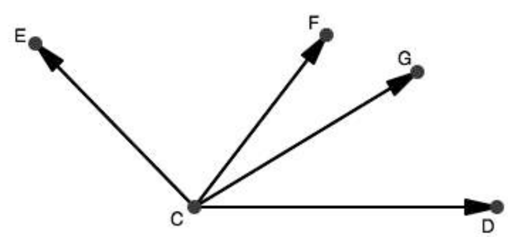
21. Given $m\angle ABC = 90°$



22. Given: $\Delta BEC, \Delta CED, and\ \Delta DAB$ are right triangles.



23. Given: $\overrightarrow{CF}$ bisects $\angle ECD, m\angle ECF = 2x + 10$, and $m\angle FCD = 3x - 18$. Find $m\angle FCE$.



Have you answered the question?
This problem asks you to do more than find the value of x.

GO

Topic: Translations, reflections, rotations

Perform the following transformations on the diagram below.

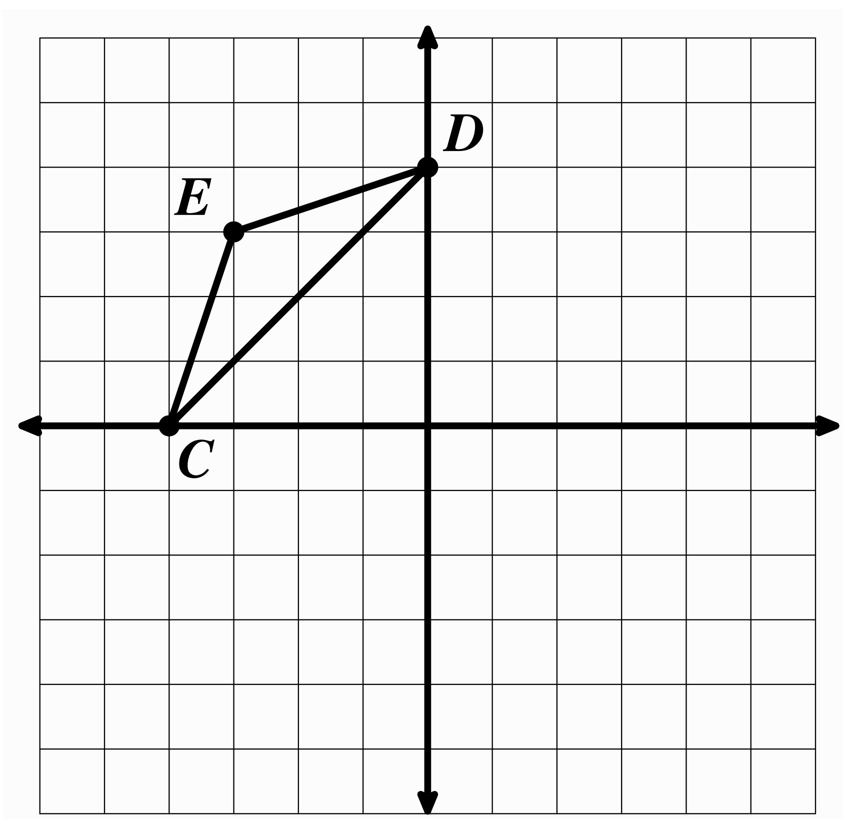
24. Label points C, E, D with the correct ordered pairs.

25. Translate ΔCED down 4 and right 6. Label the image as $\Delta C'E'D'$ and include the new ordered pairs.

26. Draw $\overline{CC'}$, $\overline{EE'}$, and $\overline{DD'}$. What is the slope of each of these line segments?

27. Reflect ΔCED across the x = 0 line. Label the image $\Delta C''E''D''$. Include the new ordered pairs.
Draw $\overline{CC''}$ *and* $\overline{EE''}$ Why didn't you need to draw $\overline{DD''}$?
What is the relationship between $\overline{CC''}$ and $\overline{EE''}$ to the $x = 0$ line?

28. Rotate ΔCED 180º about the point (-2, 0). Label the image $\Delta C'''E'''D'''$.
Include the new ordered pairs.



5.3 It's All In Your Head

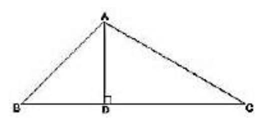
A Solidify Understanding Task

In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew. Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

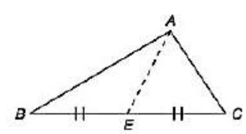
Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

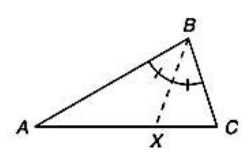
- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



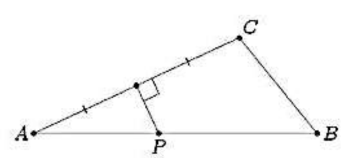
- In a triangle, a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



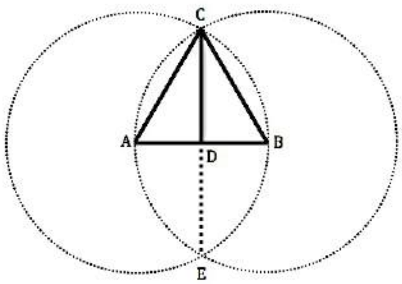
- In a triangle, an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- In a triangle, a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.



Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.

On the following page you will find a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas. You will use the flow diagram to identify the statements each of the students—Travis, Tehani, Carlos and Clarita—might use to make their case. To get ready to use the flow diagram, answer the following questions about what each student needs to know about the line of reflection to support their claim.

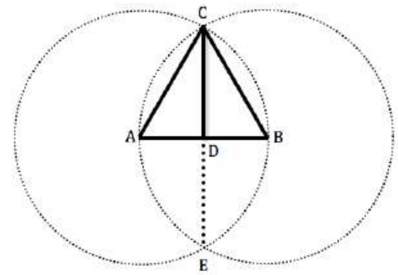
1. To support his claim that the line of reflection is a median of the equilateral triangle, Travis will need to show that:

2. To support her claim that the line of reflection is an angle bisector of the equilateral triangle, Tehani will need to show that:

3. To support her claim that the line of reflection is an altitude of the equilateral triangle, Clarita will need to show that:

4. To support his claim that the line of reflection is a perpendicular bisector of a side of the equilateral triangle, Carlos will need to show that:

Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.



7. Use four different colors to identify the statements each of the students—Travis, Tehani, Clarita and Carlos might use to make their case.

Given: ΔABC is equilateral AND $\overleftrightarrow{CE}$ is a line of reflection

$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

$\angle CDA$ and $\angle CDB$ are right angles ← $\overline{CD} \perp \overline{AB}$

D is the midpoint of $\overline{AB}$

$\overline{AD} \cong \overline{DB}$

$\overline{CD} \cong \overline{CD}$

$\angle ACD \cong \angle BCD$ ← $\Delta ACD \cong \Delta BCD$

therefore, $\overline{CD}$ is an altitude

therefore, $\overline{CD}$ is a median

therefore, $\overline{CD}$ is an angle bisector

therefore, $\overline{CD}$ is a perpendicular bisector

8. Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

1. Definition of reflection
2. Definition of translation
3. Definition of rotation
4. Definition of an equilateral triangle
5. Definition of perpendicular
6. Definition of midpoint
7. Definition of altitude
8. Definition of median
9. Definition of angle bisector
10. Definition of perpendicular bisector
11. Equilateral triangles can be folded onto themselves about a line of reflection
12. Equilateral triangles can be rotated 60° onto themselves
13. SSS triangle congruence criteria
14. SAS triangle congruence criteria
15. ASA triangle congruence criteria
16. Corresponding parts of congruent triangles are congruent
17. Reflexive Property

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two-column proof, as follows.

Statements	**Reasons**
ΔABC is equilateral	Given
$\overleftrightarrow{CE}$ is a line of reflection	Equilateral triangles can be folded onto themselves about a line of reflection
D is the midpoint of $\overline{AB}$	Definition of reflection
$\overline{CD}$ is a median	Definition of median

9. Write each of Clarita's, Tehani's, and Carlos' arguments in two-column proof format.

READY, SET, GO! Name Period Date

READY

Topic: Congruence statements and corresponding parts

Remember that when you write a congruence statement such as $\Delta ABC \cong \Delta FGH$, the corresponding parts of the two triangles must be the parts that are congruent.

For instance, $\angle A \cong \angle F, \overline{AB} \cong \overline{FG}, \angle B \cong \angle G, \overline{BC} \cong \overline{GH}$. Also, recall that the congruence patterns for triangles, ASA. SAS, and SSS, are what we can use to justify triangle congruence.

The segments and angles in each problem below are corresponding parts of 2 congruent triangles. Make a sketch of the two triangles. Then write a congruence statement for each pair of triangles represented. State the congruence pattern that justifies your statement.

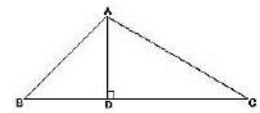
	Congruence statement	Congruence pattern
1. $\overline{ML} \cong \overline{ZJ}, \overline{LR} \cong \overline{JB}, \angle L \cong \angle J$	a.	b.
2. $\overline{WB} \cong \overline{QR}, \overline{BP} \cong \overline{RS}, \overline{WP} \cong \overline{QS}$	a.	b.
3. $\overline{CY} \cong \overline{RP}, \overline{EY} \cong \overline{BP}, \angle Y \cong \angle P$	a.	b.
4. $\overline{BC} \cong \overline{JK}, \overline{BA} \cong \overline{JM}, \angle B \cong \angle J$	a.	b.
5. $\overline{DF} \cong \overline{XZ}, \overline{FY} \cong \overline{ZW}, \angle F \cong \angle Z$	a.	b.
6. $\overline{WX} \cong \overline{AB}, \overline{XZ} \cong \overline{BC}, \overline{WZ} \cong \overline{AC}$	a.	b.

SET

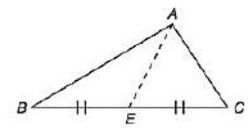
Topic: Special triangle segments and proof.
Recall the following definitions:

In a triangle:

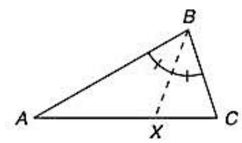
- an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



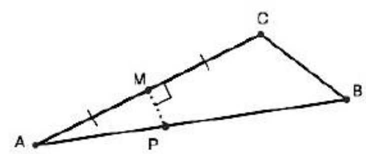
- a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



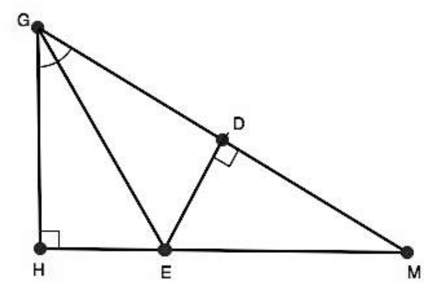
- an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Be sure to use the correct notation for a segment in the following problems.



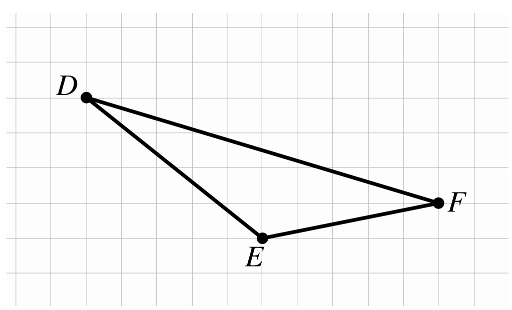
7. Name a segment in ΔGHM that is an altitude.

8. Name a segment in ΔGHM that is an angle bisector.

9. Name a segment in ΔGHM that is NOT an altitude.

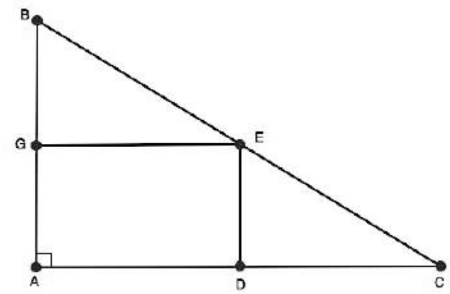
10. Create a perpendicular bisector by marking two segments congruent in ΔGHM . Name the segment that is now the perpendicular bisector.

Use ΔDEF in problems 11 – 13.



11. Construct the altitude from vertex D to $\overleftrightarrow{EF}$.
12. Construct the median from D to $\overline{EF}$.
13. Construct the perpendicular bisector of $\overline{EF}$.

Tehani has been studying the figure below. She knows that quadrilateral ADEG is a rectangle and that $\overline{ED}$ *bisects* $\overline{BC}$. *She is wondering if with that information she can prove* $\Delta BGE \cong \Delta EDC$.
She starts to organize her thinking by writing what she knows and the reasons she knows it.



I know $\overline{ED}$ bisects $\overline{BC}$ because I was given that information

I know that $\overline{BE} \cong \overline{EC}$ by definition of bisect.

I know that $\overline{GE}$ must be parallel to $\overline{AD}$ because the opposite sides in a rectangle are parallel.

I know that $\overline{GA} \parallel \overline{ED}$ because they are opposite sides in a rectangle.

I know that $\overline{AD}$ is contained in $\overline{AC}$ so $\overline{AC}$ is also parallel to $\overline{GE}$.

I know that $\overline{GA}$ is contained in $\overline{BA}$ so $\overline{GA}$ is also parallel to $\overline{BA}$

I know that $\overleftrightarrow{BC}$ has the same slope everywhere because it is a line.

I know the angle that $\overline{BE}$ makes with $\overline{GE}$ must be the same as the angle that $\overline{EC}$ makes with $\overline{AC}$ since those 2 segments are parallel. So $\angle BEG \cong \angle ECD$. I think I can use that same argument for $\angle GBE \cong \angle DEC$.

I know that I now have an angle, a side, and an angle congruent to a corresponding angle, side, and angle. So $\Delta BGE \cong \Delta EDC$ by ASA.

14. Use Tehani's "I know" statements and her reasons to write a two-column proof that proves $\Delta BGE \cong \Delta EDC$. Begin your proof with the "givens" and what you are trying to prove.

Given: quadrilateral ADEG is a rectangle, $\overline{ED}$ bisects $\overline{AC}$
Prove: $\Delta BGE \cong \Delta EDC$

STATEMENTS	REASONS
1. quadrilateral ADEG is a rectangle	given
2. $\overline{ED}$ bisects $\overline{AC}$	given

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GO

Topic: Transformations

Perform the following transformations on ΔABC. Use a straight edge to connect the corresponding points with a line segment. Answer the questions.

15. Reflect ΔABC over $\overleftrightarrow{LK}$. Label your new image $\Delta A'B'C'$.
16. What do you notice about the line segments $\overline{AA'}, \overline{BB'}, and\ \overline{CC'}$?

17. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'B'}, \overline{B'C'}, \overline{C'A'}$. What is the same and what is different about these segments?

18. Translate ΔABC down 8 units and right 10 units. Label your new image $\Delta A''B''C''$.

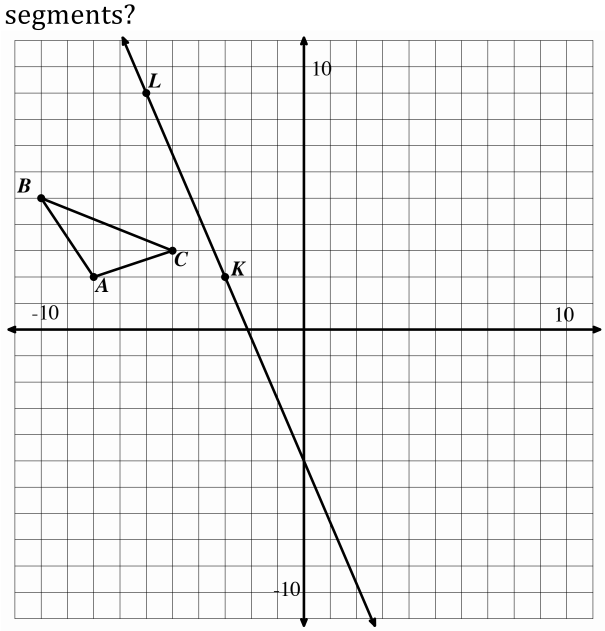
19. What do you notice about the line segments $\overline{AA''}, \overline{BB''}, and\ \overline{CC''}$?

20. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A''B''}, \overline{B''C''}, \overline{C''A''}$. What is the same and what is different about these segments?

21. Translate ΔABC down 10 units and reflect it over the Y–axis. Label your new image $\Delta A'''B'''C'''$.

22. What do you notice about the line segments $\overline{AA'''}, \overline{BB'''}, and\ \overline{CC'''}$?

23. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'''B'''}, \overline{B'''C'''}, \overline{C'''A'''}$. What is the same and what is different about these segments?



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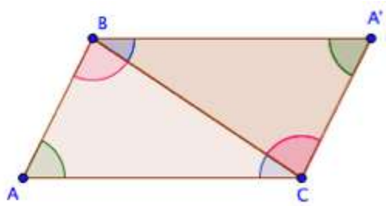
5.4 Parallelism Preserved and Protected

A Solidify Understanding Task

In a previous task, *How Do You Know That*, you were asked to explain how you knew that this figure, which was formed by rotating a triangle about the midpoint of one of its sides, was a parallelogram.



You may have found it difficult to explain how you knew that sides of the original triangle and its rotated image were parallel to each other except to say, "It just has to be so." There are always some statements we have to accept as true in order to convince ourselves that other things are true. We try to keep this list of statements as small as possible, and as intuitively obvious as possible. For example, in our work with transformations we have agreed that distance and angle measures are preserved by rigid motion transformations since our experience with these transformations suggest that sliding, flipping and turning figures do not distort the images in any way. Likewise, parallelism within a figure is preserved by rigid motion transformations: for example, if we reflect a parallelogram the image is still a parallelogram—the opposite sides of the new quadrilateral are still parallel.

Mathematicians call statements that we accept as true without proof *postulates.* Statements that are supported by justification and proof are called *theorems.*

Knowing that lines or line segments in a diagram are parallel is often a good place from which to start a chain of reasoning. Almost all descriptions of geometry include a *parallel postulate* among the list of statements that are accepted as true. In this task we develop some parallel postulates for rigid motion transformations.

Translations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a translation? That is, which word best completes this statement?

After a translation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how to tell when the corresponding line segments before and after the translation are parallel and when they are not.

Rotations

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a rotation? That is, which word best completes this statement?

After a rotation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how to tell when the corresponding line segments before and after the rotation are parallel and when they are not.

Reflections

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a reflection? That is, which word best completes this statement?

> *After a reflection, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

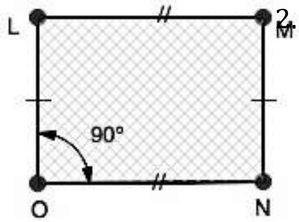
Give reasons for your answer. If you choose "sometimes" be very clear in your explanation about how to tell when the corresponding line segments before and after the reflection are parallel and when they are not.

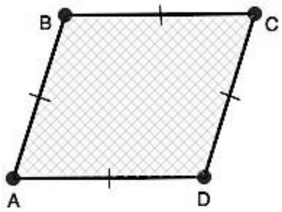
READY, SET, GO! Name Period Date

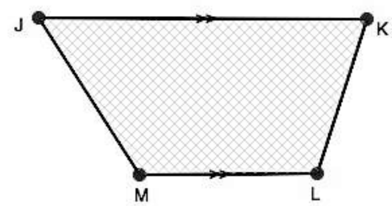
READY

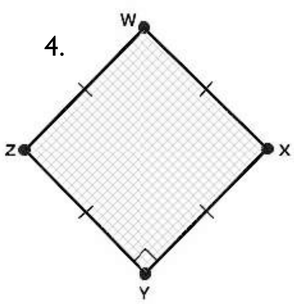
Topic: Special Quadrilateral

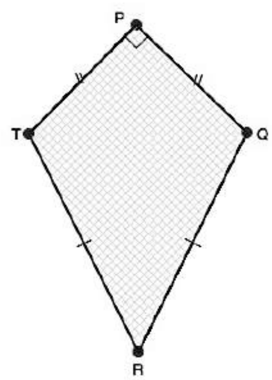
Identify each quadrilateral as a trapezoid, parallelogram, rectangle, rhombus, square, or none of these. List ALL that apply.

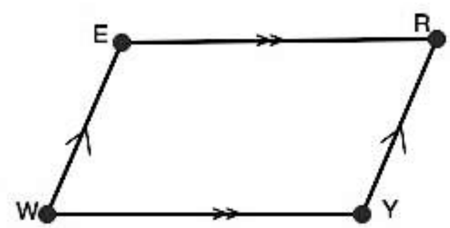
1.


2.




4.




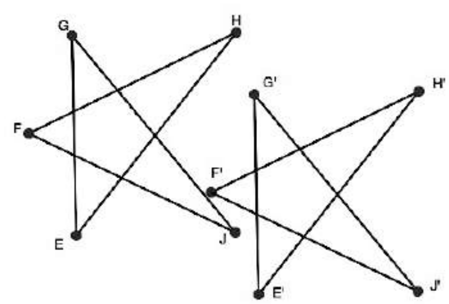


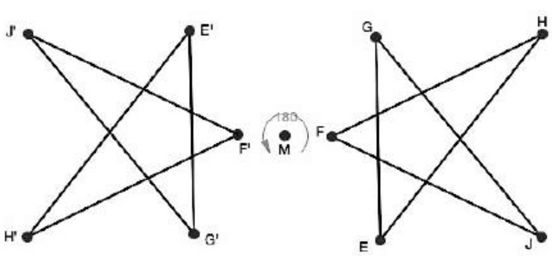
SET

Topic: Identifying parallel segments and lines produced from transformations

7. Verify the parallel postulates below by naming the line segments in the pre-image and its image that are still parallel. Use correct mathematical notation.

a. After a translation, corresponding line segments in an image and its pre-image are always parallel or lie along the same line.





b. After a rotation of 180°, corresponding line segments in a pre- image and its image are parallel or lie on the same line.

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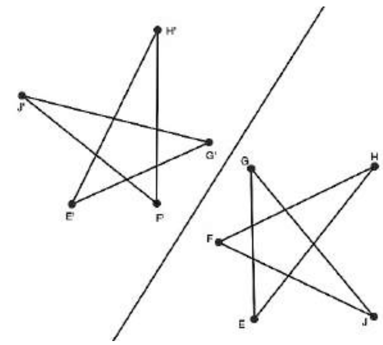
c. After a reflection, line segments in the pre-mage that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.

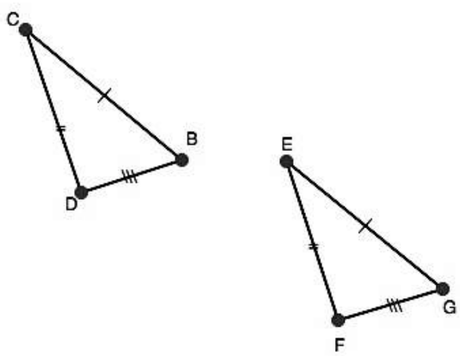


GO

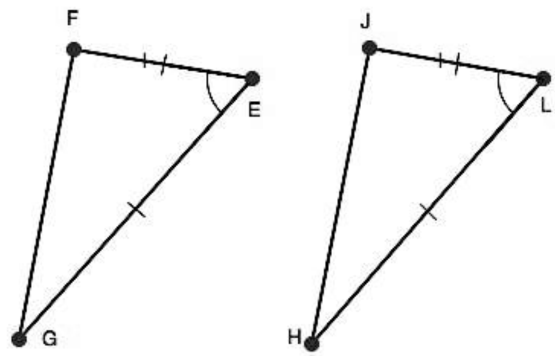
Topic: Identifying congruence patterns in triangles

For each pair of triangles write a congruence statement and justify your statement by identifying the congruence pattern you used. Then justify that the triangles are congruent by connecting corresponding vertices of the pre-image and image with line segments.
How should those line segments look?

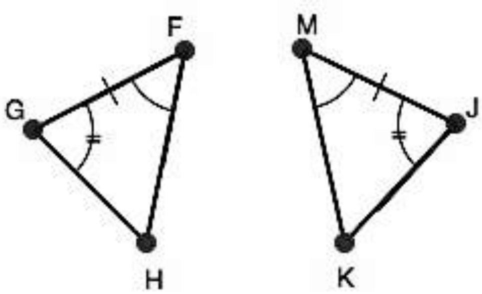
8.



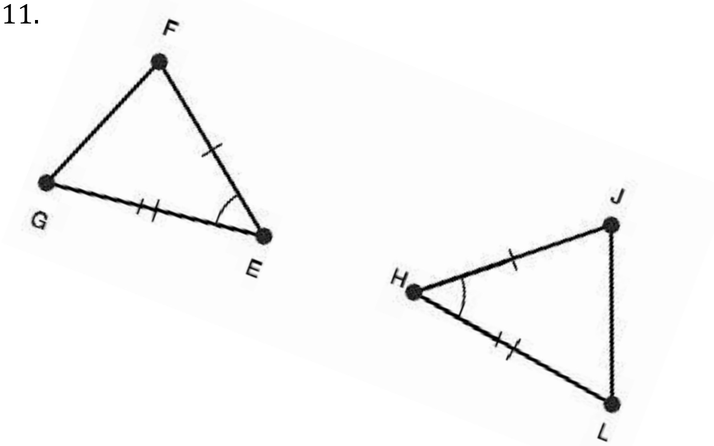
9.



10.



11.



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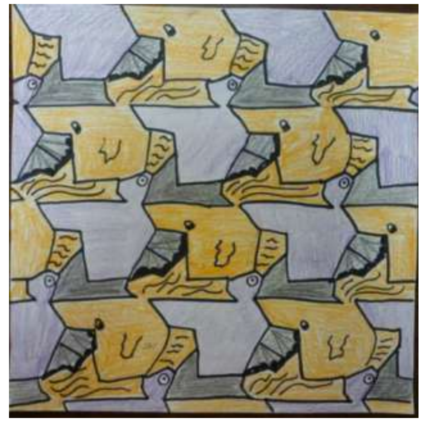
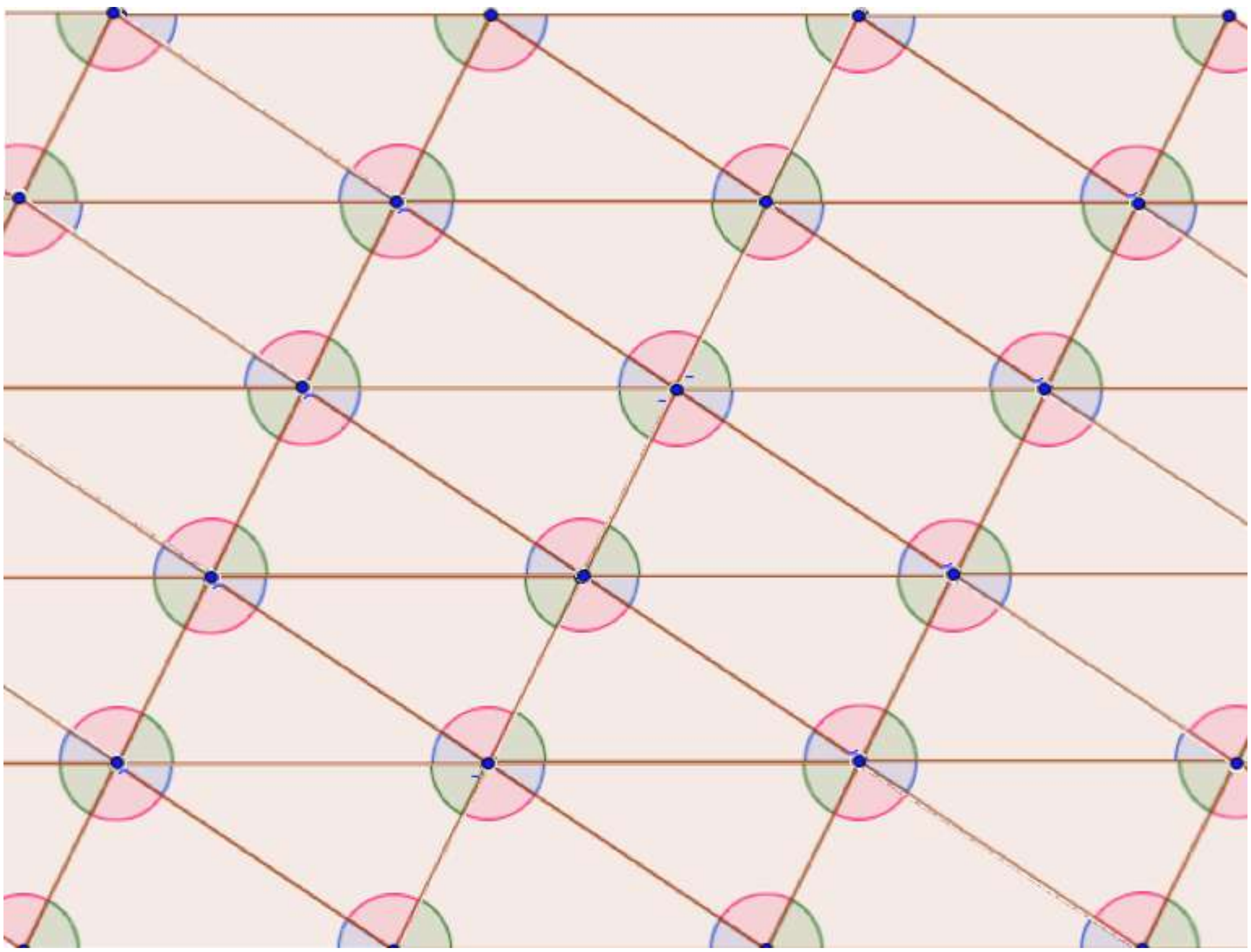
5.5 Claims and Conjectures

A Solidify Understanding Task



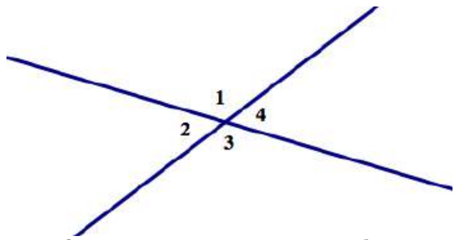
The diagram from *How Do You Know That?* has been extended by repeatedly rotating the image triangles around the midpoints of their sides to form a tessellation of the plane, as shown below.



Using this diagram, we will make some conjectures about lines, angles and triangles and then write proofs to convince ourselves that our conjectures are always true.

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram below, $\angle 1$ and $\angle 3$ form a pair of vertical angles, and $\angle 2$ and $\angle 4$ form another pair of vertical angles.



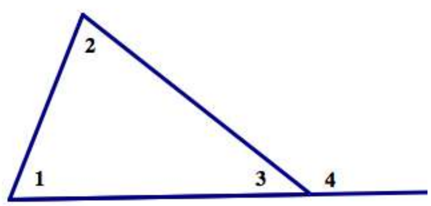
Examine the tessellation diagram above, looking for places where vertical angles occur. (You may have to ignore some line segments and angles in order to focus on pairs of vertical angles. This is a skill we have to develop when trying to see specific images in geometric diagrams.)

Based on several examples of vertical angles in the diagram, write a conjecture about vertical angles.

My conjecture:

Exterior Angles of a Triangle

When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, $\angle 4$ is an exterior angle, and $\angle 1$ and $\angle 2$ are the two remote interior angles for this exterior angle



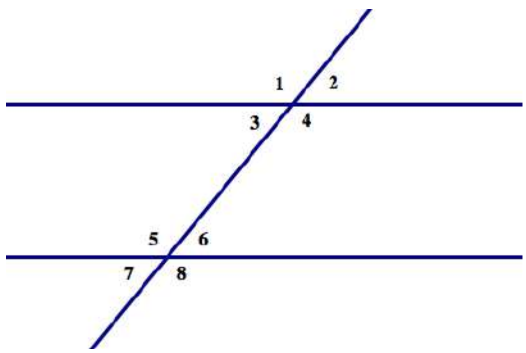
Examine the tessellation diagram above, looking for places where exterior angles of a triangle occur. (Again, you may have to ignore some line segments and angles in order to focus on triangles and their vertical angles.)

Based on several examples of exterior angles of triangles in the diagram, write a conjecture about exterior angles.

My conjecture:

Parallel Lines Cut By a Transversal

When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram below, $\angle 1$ and $\angle 5$ are called *corresponding angles*, $\angle 3$ and $\angle 6$ are called *alternate interior angles*, and $\angle 3$ and $\angle 5$ are called *same side interior angles*.



Examine the tessellation diagram above, looking for places where parallel lines are crossed by a transversal line.

Based on several examples of parallel lines and transversals in the diagram, write some conjectures about corresponding angles, alternate interior angles and same side interior angles.

My conjectures:

Justifying Our Conjectures

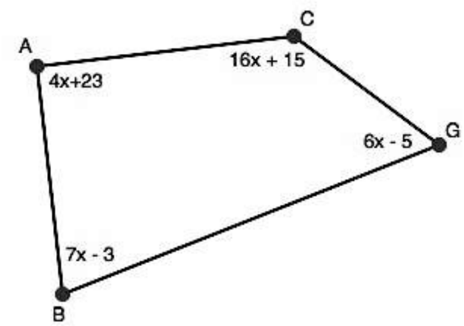
In the next task you will be asked to write a proof that will convince you and others that each of the conjectures you wrote above is always true. You will be able to use ideas about transformations, linear pairs, congruent triangle criteria, etc. to support your arguments. A good way to start is to write down everything you know about the diagram, and then identify which statements you might use to make your case. To get ready for the next task, revisit each of the conjectures you wrote about and record some ideas that seem helpful in proving that the conjecture is true.

READY, SET, GO! Name Period Date

READY

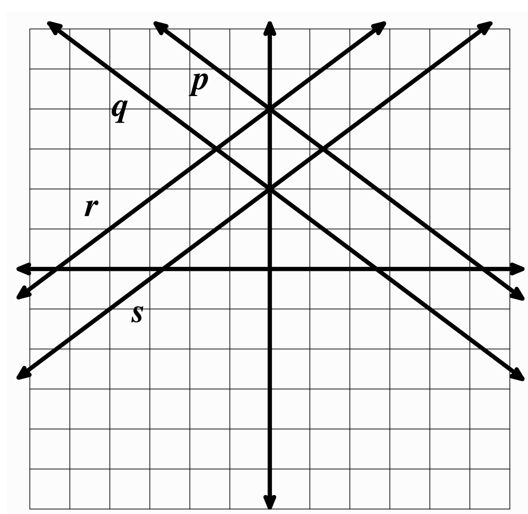
Topic: Properties of Quadrilaterals

1. Use what you know about triangles to write a paragraph proof that proves that the sum of the angles in a quadrilateral is 360º.



2. Find the measure of x in quadrilateral *ABGC*.

Match the equation with the correct line in the graph of lines *p*, *q*, *r*, and *s*.



3. $y = \frac{3}{4}x + 2$

4. $y = -\frac{3}{4}x + 2$

5. $y = \frac{3}{4}x + 4$

6. $y = -\frac{3}{4}x + 4$

7. Describe the shape made by the intersection 4 lines. List as many observations as you shape and its features.

SET

Topic: Parallel lines cut by transversal, vertical angles and exterior angle of a triangle

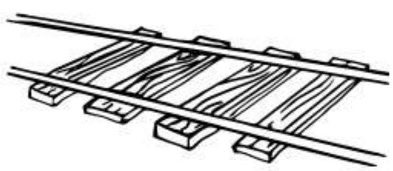
Label each picture as showing *parallel lines with a transversal, vertical angles*, or an *exterior angle of a triangle*. Highlight the geometric feature you identified. Can you find all 3 features in 1 picture? Where?

8.

9.

10.

11.

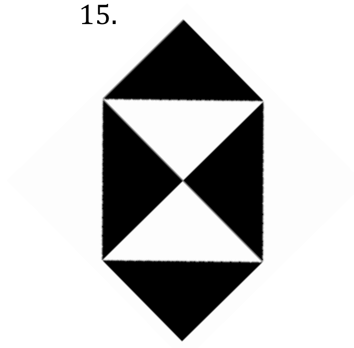


12.

13.

14.

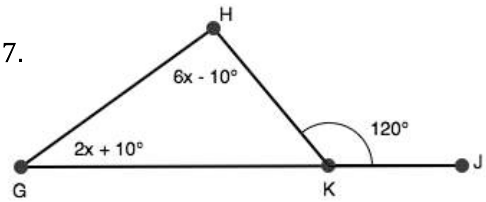
15.



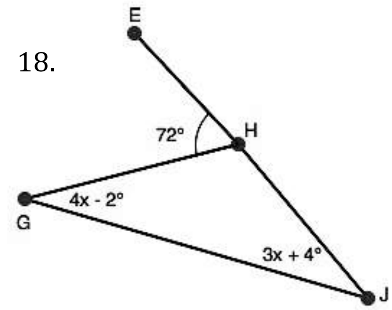
16.

Find the value of the 2 remote interior angles in the figures below.

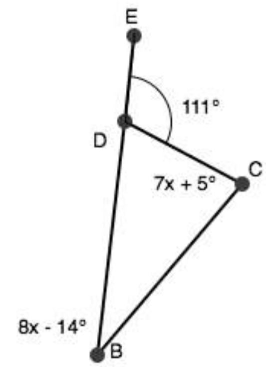
17.



18.



19.



Indicate whether each pair of angles is *congruent* or *supplementary* by trusting how they look. Lines *p* and *q* are parallel.

20. $\angle 5$ *and* $\angle 8$

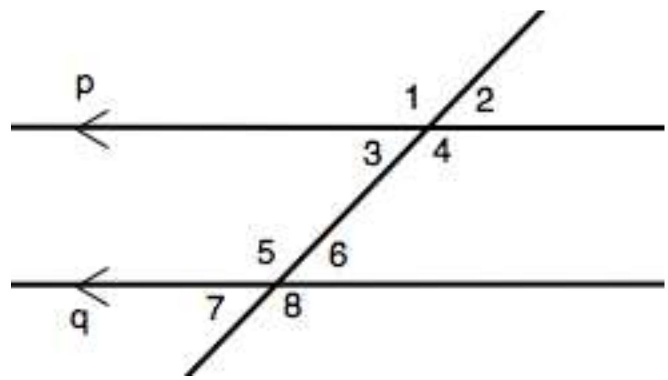
21. $\angle 2$ *and* $\angle 6$

22. $\angle 2$ *and* $\angle 8$

23. $\angle 4$ *and* $\angle 6$

24. $\angle 3$ *and* $\angle 5$

25. $\angle 1$ *and* $\angle 3$



GO

Topic: Complementary and supplementary angles

Find the complement and the supplement of the given angles. It is possible for the complement or supplement not to exist.

26. 37°

27. 59°

28. 89°

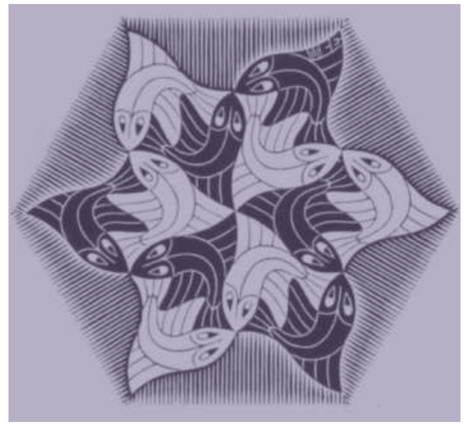
29. 111°

30. 3°

31. 90°

5.6 Justification and Proof

A Practice Understanding Task



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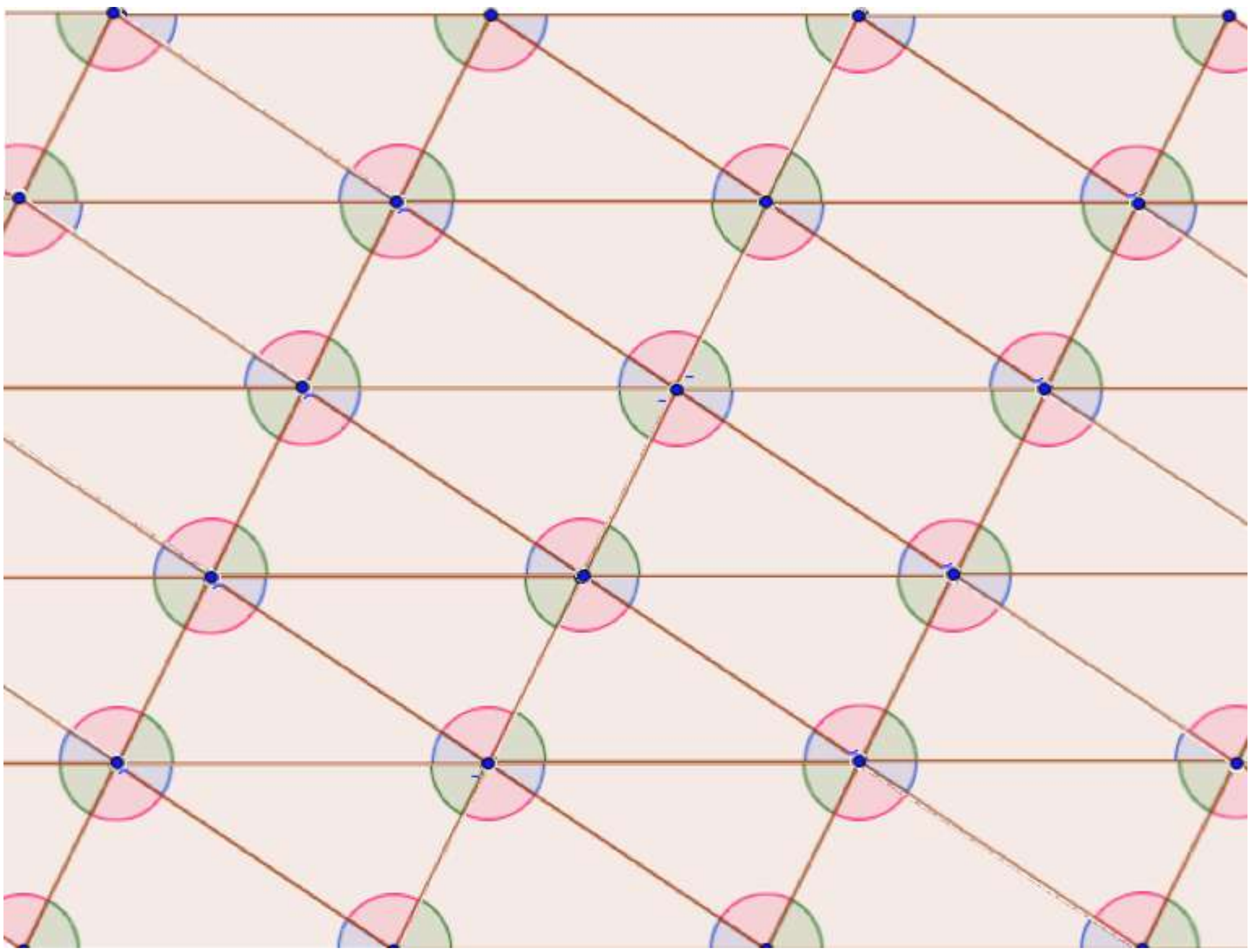
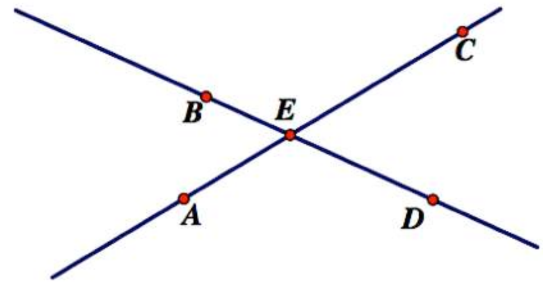


Using this diagram, you have made some conjectures about lines, angles and triangles. In this task you will write proofs to convince yourself and others that these conjectures are always true.

Vertical Angles



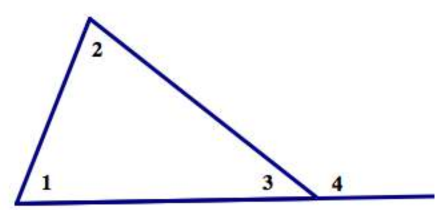
When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram, $\angle AEB$ and $\angle CED$ form a pair of vertical angles.

1. Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ intersect at E.
   Prove: $\angle AEB \cong \angle CED$

[Note: For each of the following proofs you may use any format you choose to write your proof: a flow proof diagram, a two-column proof, or a narrative paragraph.]

Exterior Angles of a Triangle

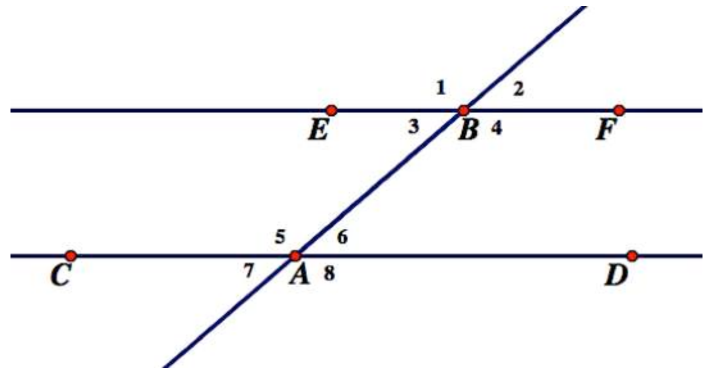
When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, $\angle 4$ is an exterior angle, and $\angle 1$ and $\angle 2$ are the two remote interior angles for this exterior angle.



2. Given: $\angle 4$ is an exterior angle of the triangle
   Prove: $m\angle 4 = m\angle 1 + m\angle 2$

Parallel Lines Cut By a Transversal

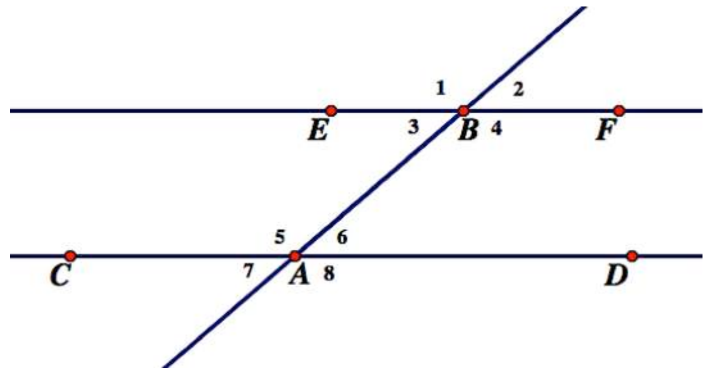
When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram, ∠ 1 and ∠ 5 are called *corresponding angles*, ∠ 3 and ∠ 6 are called *alternate interior angles*, and ∠ 3 and ∠ 5 are called *same side interior angles*.



3. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Corresponding angles ∠ 1 and ∠ 5 are congruent

4. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Alternate interior angles ∠ 3 and ∠ 6 are congruent

5. Given: $\overleftrightarrow{BF} \parallel \overleftrightarrow{AD}$
   Prove: Same-side interior angles $\angle 3$ and $\angle 5$ are supplementary



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6. Given: Alternate interior angles $\angle 3$ and $\angle 6$ are congruent
   Prove: $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$

READY, SET, GO! Name Period Date

READY

Topic: Recalling features of the rigid-motion transformations

Complete each statement

1. When I use line segments to connect the corresponding points of a pre-image and the image in a translation, the line segments are ________________________ and ___________________________
because ____

2. When I use line segments to connect the corresponding points of a pre-image and the image in a reflection, the line of reflection is the ________________________________ of the segments because
____

3. In a rotation, the corresponding points of the pre-image and the image are the same
____________________ from the center of rotation because ___________________________________

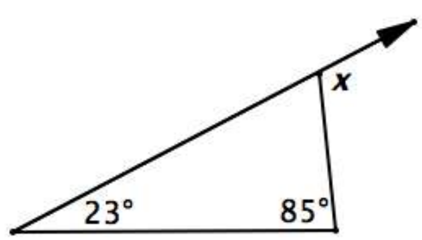
4. Translations, rotations, and reflections are rigid motion transformations because
____

SET

Topic: Solving for missing angles

Use what you know about vertical angles, exterior angles, and the angles formed by parallel lines and transversals to find the value of x in each of the diagrams.

5.



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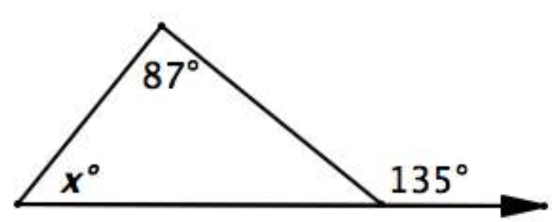

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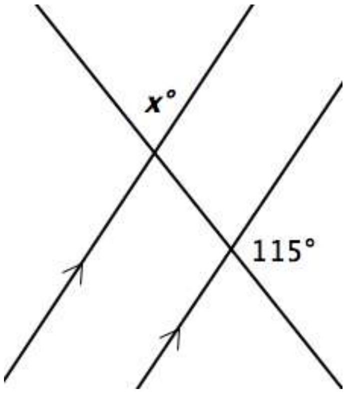

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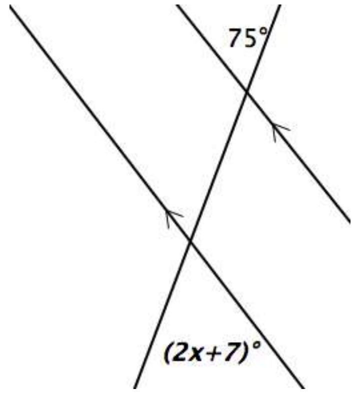
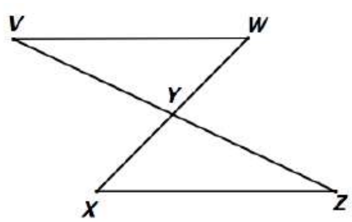
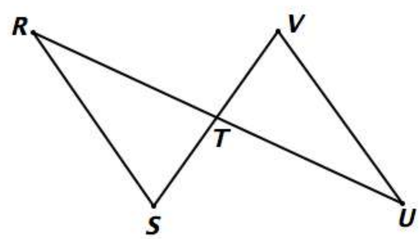


Prove each of the following.

9. Given: Y is the midpoint of $\overline{VZ}$ and $\overline{XW}$.
   Prove: $\Delta VYW \cong \Delta ZYX$



10. Given $\angle R \cong \angle U$ and $\overline{ST} \cong \overline{VT}$.
    Prove: $\Delta SRT \cong \Delta VUT$

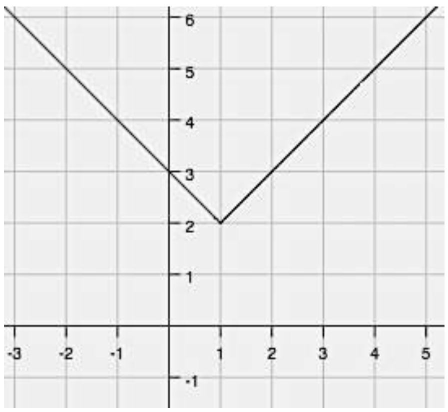


GO

Topic: Connecting a piecewise defined equation with the corresponding absolute value equation

The graph of an absolute value function is given. A) Write the equation using absolute value notation. B) Then write the equation as a piecewise defined function.

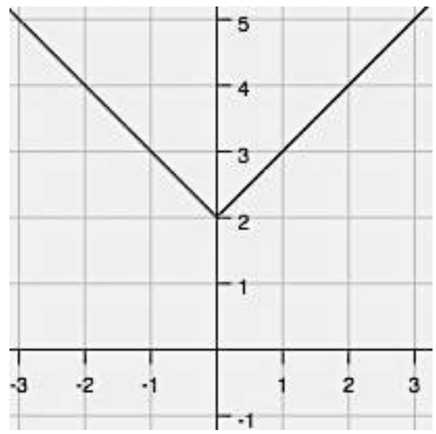
15.



A.

B.

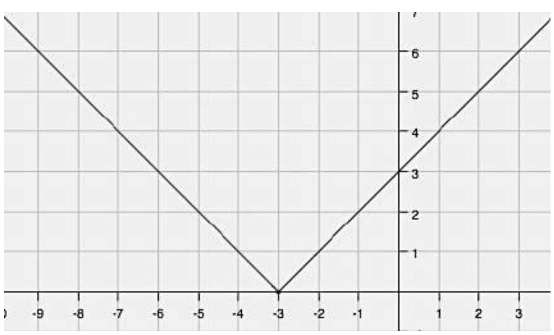
16.



A.

B.

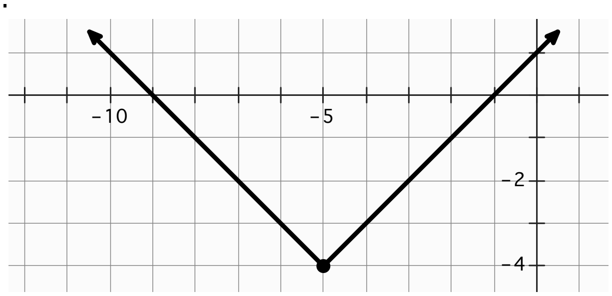
17.



A.

B.

18.



A.

B.