5.1 How Do You Know That?

A Develop Understanding Task

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You probably know that the sum of the interior angles of any triangle is 180°. (If you didn't know that, you do now!) But an important question to ask yourself is, "How do you know that?"

We know a lot of things because we *accept it on authority*—we believe what other people tell us; things such as the distance from the earth to the sun is 93,020,000 miles or that the population of the United States is growing about 1% each year. Other things are just defined to be so, such as the fact that there are 5,280 feet in a mile. Some things we accept as true based on experience or repeated experiments, such as the sun always rises in the east, or "I get grounded every time I stay out after midnight." In mathematics we have more formal ways of deciding if something is true.

Experiment #1

1. Cut out several triangles of different sizes and shapes. Tear off the three corners (angles) of the triangle and arrange the vertices so they meet at a single point, with the edges of the angles (rays) touching each other like pieces of a puzzle. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

   They (the angles) link back together to form a straight line.

2. Since you and your classmates have performed this experiment with several different triangles, does it guarantee that we will observe this same result for *all* triangles? Why or why not?

   No, 30 people tried out of ∞
   Seems to work for most

Experiment #2

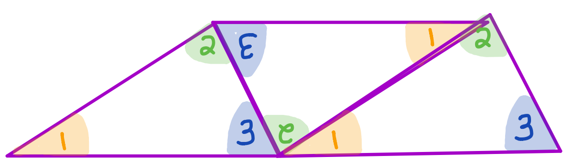
Perhaps a different experiment will be more convincing. Cut out another triangle and trace it onto a piece of paper. It will be helpful to color-code each vertex angle of the original triangle with a different color. As new images of the triangle are produced during this experiment, color-code the corresponding angles with the same colors.

- Locate the midpoints of each side of your cut out triangle by folding the vertices that form the endpoints of each side onto each other.

- Rotate your triangle 180° about the midpoint of one of its sides. Trace the new triangle onto your paper and color-code the angles of this image triangle so that corresponding image/pre-image pairs of angles are the same color.

- Now rotate the new "image" triangle 180° about the midpoint of one of the other two sides. Trace the new triangle onto your paper and color-code the angles of this new image triangle so that corresponding image/pre-image pairs of angles are the same color.

3. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?



They form a line = 180°

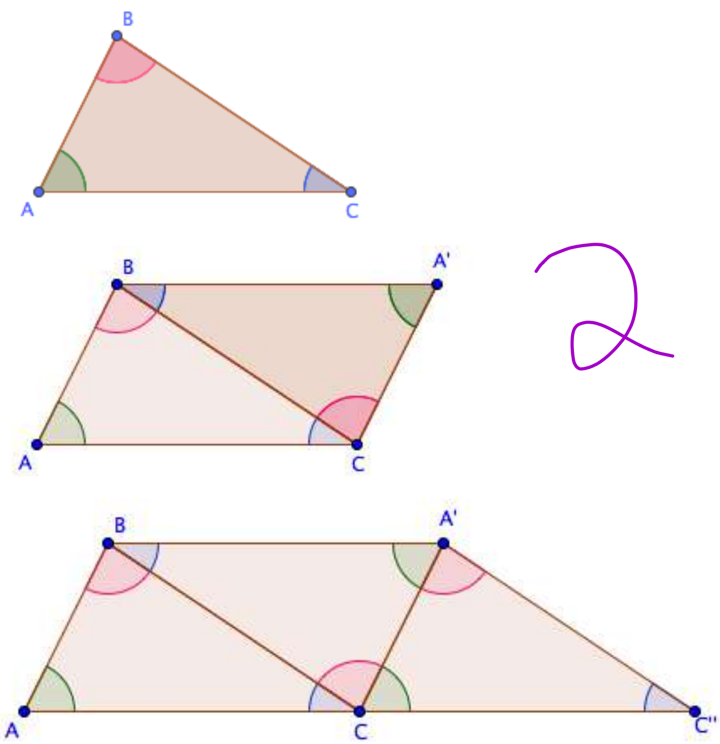
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

4. Do you think you can rotate *all* triangles in the same way about the midpoints of its sides, and get the same results? Why or why not?

yes

Examining the Diagram

Experiment #2 produced a sequence of triangles, as illustrated in the following diagram.



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Here are some interesting questions we might ask about this diagram:

5. Will the second figure in the sequence always be a parallelogram? Why or why not?

↳ opposite sides are parallel

Yes, when you rotate you will get opp. ∠'s ≅

6. Will the last figure in the sequence always be a trapezoid? Why or why not?

One pair of opposite sides parallel

yes

READY, SET, GO! Name Period Date

READY

Topic: Geometric Figures

One of the cool things about geometric figures is that our world is filled with them. For instance, my bathroom mirror is a perfect rectangle and the tiles on my floor are squares. Plus, the edges of these shapes are straight lines or line segments which are pieces of lines, since theoretically a line goes on forever.

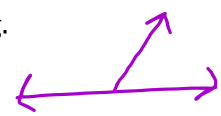
1. Look around your world and make a list of the things you see that have a geometric shape. Here are some shapes to begin with. Think of all you can and be prepared to share your lists with the class.

Triangle **Trapezoid** **Parallelogram** **Cube** **Perpendicular lines**

SET

Topic: Linear Pairs

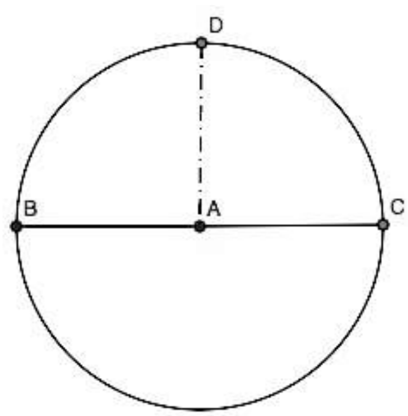
2. Fold a piece of paper, making a smooth crease. Open the paper and examine the shape that you made. Is it a line? Will it always be a line? Justify your thinking.



3. Look at a wall where it meets the ceiling. How would you describe the intersection of the wall and the ceiling?

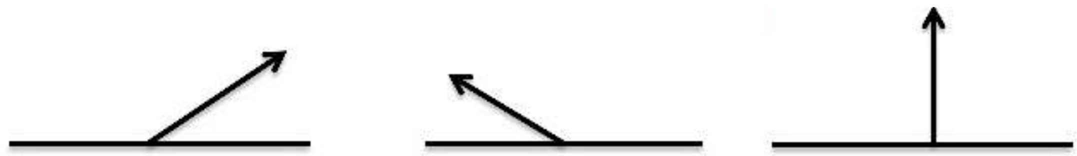
Imagine folding a circle exactly in half so that the fold passes through the center of the circle. This fold is called the diameter of the circle. It is a line segment with a length, but it is also a special kind of angle called a **straight angle***.*

In order to "see" the angle, think of the center of the circle. That point is the vertex of the angle. Either side of the vertex is a radius of the circle. Whenever you draw 2 radii of the circle you make an angle. When the two radii extend in exactly opposite directions and share a common endpoint (the center), they make a line or a **straight angle***.*

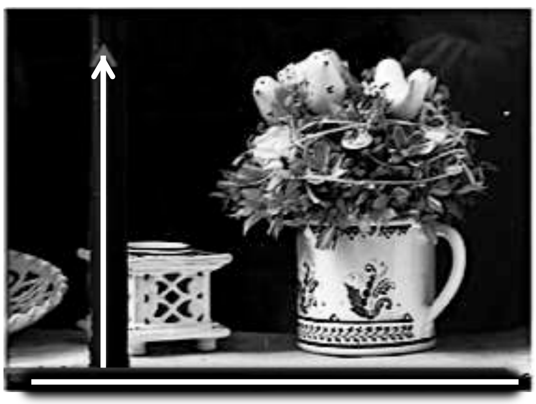


14. How many degrees do you think are in a *straight angle*?
Use features of the diagram to justify your answer.

If two angles share a vertex and together they make a straight angle, then the two angles are called a **linear pair**. *(Below are 3 examples of* **linear pairs**.*)*



Examples of linear pairs in real life:



http://www.flicker.com/photos/angle_dore/63650608

http://www.flicker.com/photos/truthlying/3845031/siz

5. Draw at least 2 diagrams of a real life linear pair.

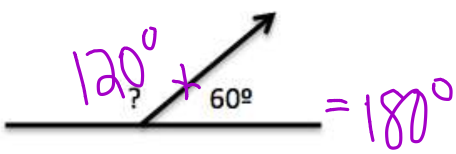
GO

Topic: Algebra of Linear Pairs

For 2 angles to be a **linear pair**, *they must share a vertex and a side, and the sum of their measures must equal 180°.*

Find the measure of the missing angle.

6.



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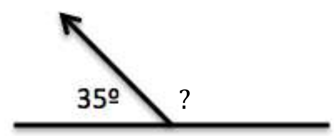

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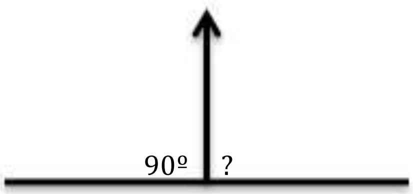

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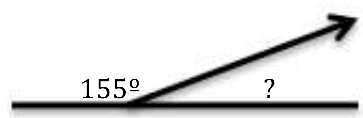

10. Linear pairs could be defined as being **supplementary angles** because they always add up to 180°. Are all supplementary angles linear pairs? Explain your answer.

180   90° → complementary

Find the supplement of the given angle. Then draw the two angles as linear pairs. Label each angle with its measure.

11. m∠ABC = 72° B will be the vertex.

$72 + x = 180$
$-72 \quad -72$
$x = 108°$

12. m∠GHK = 113° H will be the vertex.

13. m∠XYZ = 24° Y will be the vertex

14. m∠JMS = 168° M will be the vertex