6.10 Algebra and Functions

A Practice Understanding Task

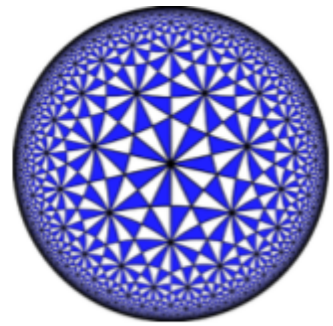
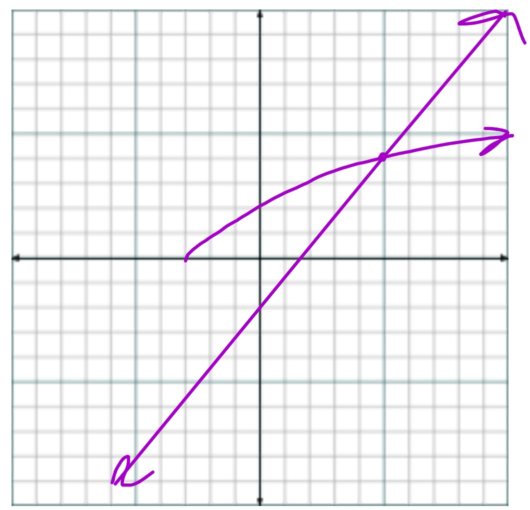


Image Source: https://en.wikipedia.org/wiki/Geometry

Consider the equation $x-1=\sqrt{2(x+3)}$.
This equation is made up of a linear function, $f(x)=x-1$,
and a square root function, $g(x)=\sqrt{2(x+3)}$.

1. Find the domain and range of each function and then graph both functions on the same coordinate plane.

$f(x)=x-1$	$g(x)=\sqrt{2(x+3)}$
Domain: $(-\infty,\infty)$	Domain: $[-3,\infty)$
Range: $(-\infty,\infty)$	Range: $[0,\infty)$



2. Algebraically solve the equation $(x-1)^2=\left(\sqrt{2(x+3)}\right)^2$

$x^2-2x+1=2x+6$
$-2x-6 \quad -2x-6$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

4 ✓ $x=5$ $\quad$ $\cancel{x=-1}$ +2

4 $\quad$ -2

3. Are the solutions elements of the domain in both the linear and square root functions?

Yes

4. Substitute the solutions back into the functions to see if the outputs are elements in the range of each function.

$f(x) = x - 1$	$g(x) = \sqrt{2x + 6}$
$f(5) = 4$ | $f(-1) = \cancel{-2}$	$g(5) = 4$ | $g(-1) = 2$

5. Are all of the outputs elements in the range of both functions? Explain why or why not.

No

6. If the square root function was reflected over the x-axis, so that it's equation was $h(x) = -\sqrt{2x + 6}$, what would be the solution to the equation $x - 1 = \cancel{-}\sqrt{2(x + 3)}$?

$-1 \qquad -1$

$(-x+1)^2 = (\sqrt{2(x+3)})^2$

$x^2 - 2x + 1 = 2x + 6$

$-2x - 6 \quad -2x - 6$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1)$

$x = \cancel{5}$ -4, 2.4

$x = -1$ -2, -2 ✓

Solve the following equations algebraically. Be sure to check for extraneous solutions.

7) $\frac{2}{x} = 3x + 5$

$2 = 3x^2 + 5x$

$0 = 3x^2 + 5x - 2$

$\frac{-5 \pm \sqrt{25 - 4(3)(-2)}}{6}$

$\frac{-5 \pm \sqrt{49}}{6}$

$\frac{-5 \pm 7}{6}$ $\frac{1}{3}$ ✓, -2 ✓

8) $2x + 3 = \frac{5}{x}$

$5 = 2x^2 + 3x$

$0 = 2x^2 + 3x - 5$

$0 = x^2 + 3x - 10$

$0 = (x + \frac{5}{2})(x - \frac{2}{2})$

$0 = (2x+5)(x-1)$

$x = \frac{-5}{2} \quad x = 1$

✓

9) $\frac{6}{x} = 9 - 3x$

$6 = 9x - 3x^2$

$3x^2 - 9x + 6 = 0$

$3(x^2 - 3x + 2) = 0$

$3(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

10) $4x - 7 = \frac{2}{x}$

11) $\frac{3}{5}x + \cancel{5} = \sqrt{2x-1} + \cancel{5}$

$\left(\frac{3}{5}x\right)^2 = \left(\sqrt{2x-1}\right)^2$

$\frac{9}{25}x^2 = 2x - 1$

$\frac{9}{25}x^2 - 2x + 1 = 0$

$\frac{2 \pm \sqrt{256}}{\frac{18}{25}}$ 5 .5

$\frac{+2 \pm \sqrt{4 - 4\left(\frac{9}{25}\right)(1)}}{2\left(\frac{9}{25}\right)}$

12) $4x - 2 = \sqrt{x+3}$

$16x^2 - 16x + 4 = x + 3$

$-x - 3 \quad -x - 3$

$16x^2 - 17x + 1 = 0$

$x^2 - 17x + 16 = 0$

$\left(x - \frac{16}{16}\right)\left(x - \frac{1}{16}\right) = 0$

$(x-1)(16x-1) = 0$

$x = 1 \quad x = \cancel{1/16}$

13) $\sqrt{4x} = -2x + 4$

14) $0.5x - 8 = 2 - 2\sqrt{x+1}$

$-2 \quad -2$

$\frac{.5x - 10}{-2} = \frac{-2\sqrt{x+1}}{-2}$

$\left(-\frac{1}{4}x + 5\right)^2 = \left(\sqrt{x+1}\right)^2$

$+\frac{1}{16}x^2 - 2.5x + 25 = x + 1$

$-x \quad -1 \quad -x - 1$

$\frac{1}{16}x^2 - 3.5x + 24 = 0$

$\frac{3.5 \pm \sqrt{12.25 - 4\left(\frac{1}{16}\right)(24)}}{\frac{1}{8}} = \frac{3.5 \pm \sqrt{6.25}}{\frac{1}{8}}$

$\cancel{48} \qquad 8$

15) $\frac{x - 7}{-1} = \frac{-\sqrt{4x-8}}{-1}$

$-x + 7 = \sqrt{4x-8}$