NC Math 2

UNIT 6

Square Root and Inverse Variation Functions

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

Developed by CHCCS and WCPSS

2018

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Square Root and Inverse Variation Functions

6.1 Watch Out For That Wave!

A Develop Understanding Task

https://pixabay.com/en/photos/wave/

Family Kingdom Amusement Park in Myrtle Beach is a family friendly carnival style attraction right on the beach in SC. It is a great place to take a break from laying on the beach. However, with its proximity to the ocean, there are certain hazards that need to be accounted for.

One such ocean hazard is a tsunami. A tsunami is a long high sea wave caused by an earthquake, ocean floor landslide, or other underwater disturbance. These waves grow higher the closer to land they travel and can cause devastating damage. Many tsunamis are caused by seismic activity which is closely monitored by the US Geological Survey. When an earthquake is recorded in the ocean, they will send out warnings to the communities in the path of potential waves. The speed of a wave during a tsunami can be calculated with the formula $s = \sqrt{9.81d}$ where s represents speed, d represents the depth of the water where the earthquake takes place, and 9.81 m/s^2 is the acceleration due to gravity.

1. Let's take a look at this function! Create a table and graph of this function.

2. Describe the domain of the function. Explain.

3. What is the range of the function? Explain.

4. Describe the rate of change of the function. How does the rate of change of this function compare to the rate of change of other functions that we have encountered?

5. In the formula, 9.8 is the acceleration due to gravity and is measured in m/s^2. If the depth is measured in meters, what will the unit of measure be for the speed, s, that we find? Explain your thinking.

6. We can detect earthquakes even when they happen under the ocean. There are monitoring stations all over the globe. An earthquake is detected at 22° 27’ 06” N and 54° 02’47” W. This is off the coast of South Carolina. The ocean at that point has a depth of 5150 m, so how fast would you expect the wave to be traveling?

   Which representation of the function did you use to find the speed of the wave? Explain how you used this representation find your answer.

7. If a wave is detected traveling at 185 m/s how deep was the epicenter that created it?

   Which representation of the function did you use to find the depth of the epicenter? Explain how you used this representation find your answer.

READY, SET, GO! Name ________________

Period ______

Date __________

READY

Topic: Direct Variation.

Recall that direct variation should have a constant of variation *k*, such that $y = kx$. Determine which of the following represent direct variation relationships. If it is a direct variation relationship, identify the constant of variation.

1. $y = 9x$

2. $y = \frac{1}{2}x + 3$

3. $y = -\frac{3}{4}x$

4. $2y = x - 6$

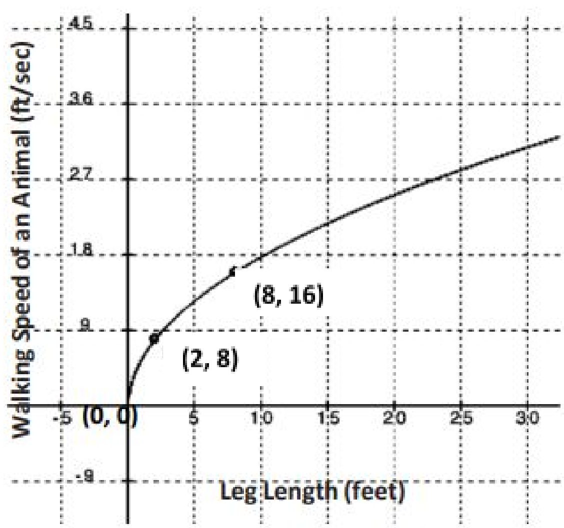
5. $3y = -7x$

6. $8x - 9y = 0$

SET

Topic: Square Root Functions in context.

The relationship between the length of one of the legs, in feet, of an animal, and its walking speed, in feet per second, can be modeled by the graph below. Use this graph to answer questions 7-10.



7. What units are involved in this problem? Define the quantities and variables you would use to model this situation.

8. What type of function does the graph represent? How can you tell?

9. Which of the following is the correct function for this graph? Explain your reasoning.

   a. $l(x) = \sqrt{x + 64}$ b. $l(x) = \sqrt{32x}$ c. $l(x) = 2x^2$

10. A T-Rex's leg length is 20 feet! What would the T-Rex's walking speed be in feet per second?

GO!

Topic: Simplifying Square Roots

Simplify the following radicals.

11. $\sqrt{32}$ 12. $\sqrt{18}$ 13. $\sqrt{30}$

14. $\sqrt{27}$ 15. $\sqrt{120}$ 16. $\sqrt{300}$

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6.2 Time's Running Out!

A Develop Understanding Task

https://pixabay.com/en/photos/wave/

Last lesson we were introduced to Family Kingdom Amusement Park in Myrtle Beach and how they needed to be prepared for the chance of a Tsunami. We learned how we could determine how fast a wave was moving, but how can we use this to help them prepare?

You may remember from science that $r = \frac{d}{t}$ where *d* is distance, *t* is time, and *r* is rate. The earthquake that we detected was at 22° 27' 06" N and 54° 02'47" W which is 27,393.82 km from Family Kingdom Amusement Park. This means we can model the time vs. rate with the equation $r = \frac{27393.82}{t}$.

1. Let's take a look at this function! Create a table and graph of this function.

2. Describe the domain of the function. Explain.

3. What is the range of the function? Explain.

4. Describe the rate of change of the function. How does the rate of change of this function compare to the rate of change of other functions that we have encountered?

5. In the model, the distance 27,393.82 is measured in km. What are logical units for *t* and *r*? Explain your thinking.

6. In the Watch Out for That Wave! task, we determined that the wave was traveling toward Family Kingdom Amusement Park at 224.66 m/s. Determine how long it will take the wave to reach the Family Kingdom Amusement Park. (Note that the units for the wave speed involve meters, not kilometers.) Explain your thinking.

   Which representation of the function did you use to find the time it takes for the wave to reach the Amusement Park? Explain how you used this representation find your answer.

7. Why is this important information for the park? How could they use this to help them prepare?

READY, SET, GO! Name ________________ Period ______ Date __________

READY

Topic: Predict whether a relationship is direct or inverse variation.

For each of the following scenarios, as the first quantity increases, predict whether the second quantity increases or decreases.

1. number of people at the beach; ice cream sales

2. speed of a car; time it takes to travel 100 miles

3. number of hours Thomas works each week; the amount of money he makes

4. number of people working on a project; time it takes to complete the project

5. number of lottery jackpot winners; amount of money per winner

SET

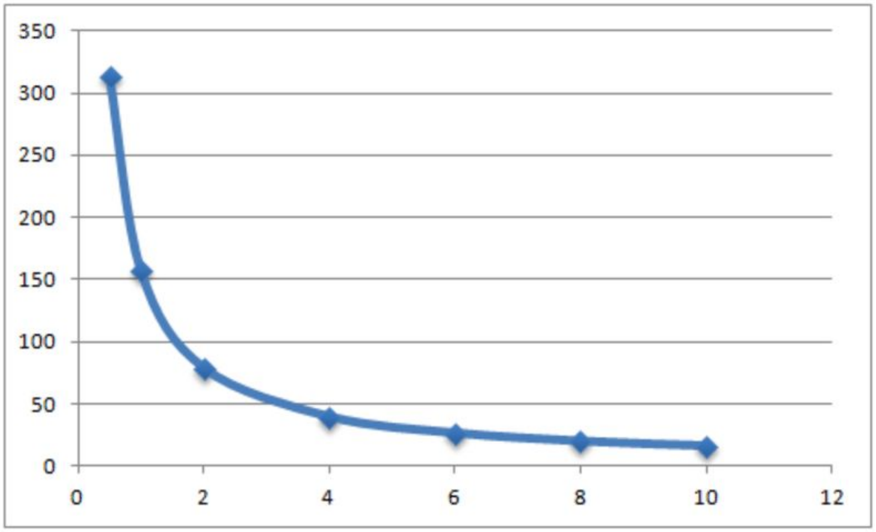
Topic: Inverse variation functions in context.

Boyle's Law states that as the pressure exerted on a gas increases, the volume of a gas decreases. A pump used to put air in a bicycle tire is a great example of Boyle's Law in action. When you push down on the pump, the volume inside the bike pump decreases, and the pressure of the air increases so that it's pushed into the tire.

If the volume inside of a bicycle pump is 8.2 cubic inches, and the pressure is 19.1 psi (pounds per square inch), the equation that represents this situation is:

$$V = \frac{156.62}{P}$$

A graph of this function is below.



6. What units are involved in this problem? Define the quantities and variables you would use to model this situation. Label them on the graph.

7. What type of function does this relationship represent? How can you tell?

8. If a pressure of 4 psi is exerted on the gas in the pump, what would be the volume of the gas?

GO!

Topic: Properties of exponents.

Simplify the following expressions.

9. $n^6 \cdot n^3$

10. $(x^5)^2$

11. $(3a^2b^{\frac{1}{3}})^3$

12. $\frac{x^8}{x^6}$

13. $\frac{x^5}{x^7}$

14. $\frac{x^4}{x^4}$

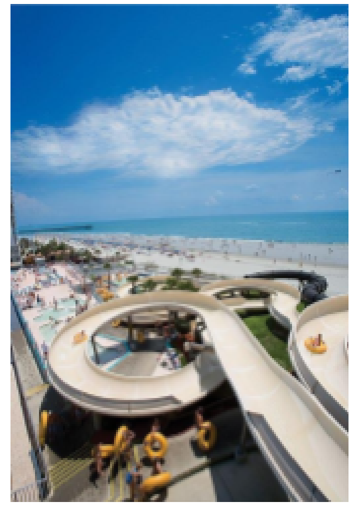
15. $(x^3y^6)^{-4}$

16. $8^{\frac{2}{3}}$

17. $x^{-\frac{1}{3}}$

6.3 Ramp it Up!

A Solidify Understanding Task



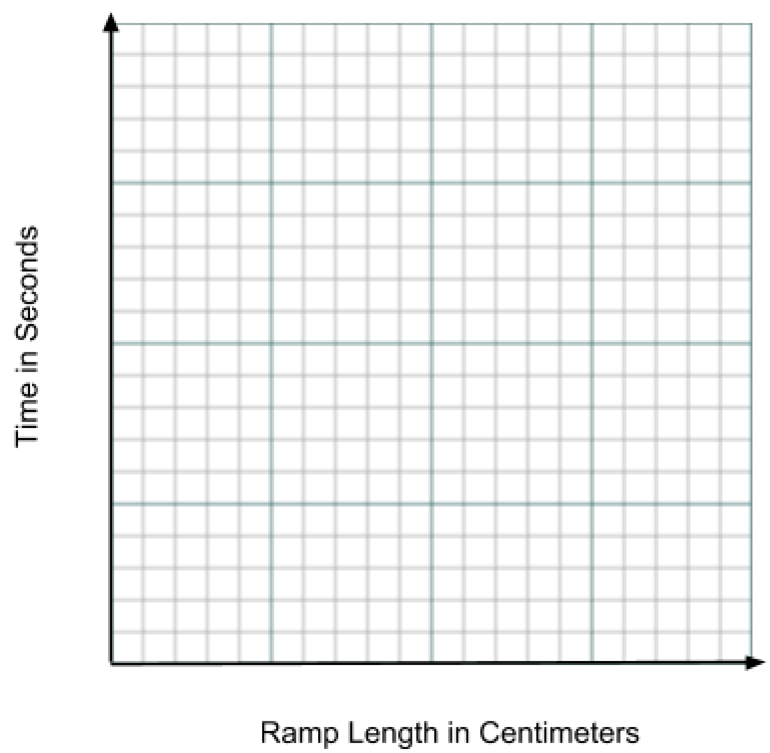
By Fk714 [CC BY-SA 3.0], from Wikimedia Commons

Family Kingdom Amusement Park is looking into adding another speed slide to their water park. To create a safe and exhilarating experience for everyone on this water slide it is important for the engineers and architects to understand the relationships between the height of the water slide, the length of the water slide, the time it will take for a person to go down the slide, and the speed the person will be traveling.

With so many variables to consider, let's first look at the relationship between the length of the slide and the time it will take for a person to go down the slide. To simulate this relationship you need several different lengths of ramp all set to the same height. Then, you need to release a ball from the top of each ramp and record how long it takes for the ball to reach the bottom of the ramp. Record your data in the table below and then create a scatter plot to represent the data.

Ramp Height : ___________

Ramp Length (centimeters)						
Time (seconds)						

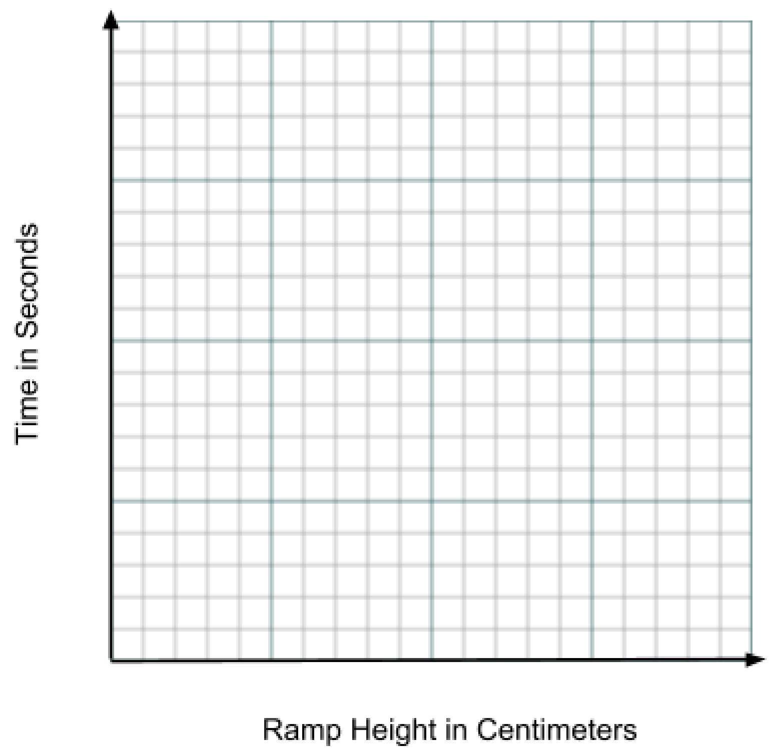


1. What happens to the time as the ramp length increases? Is this an example of a direct variation or inverse variation relationship? How do you know?

Now let's look at the relationship between ramp height and time. For this investigation we will need to keep the length of the ramp consistent so that we can make sure that our changes in time are dependent only on the changes in height.

Ramp Length : ___________

Ramp Height (centimeters)						
Time (seconds)						



2. What happens to the time as the ramp height increases? Is this an example of a direct variation or inverse variation relationship? How do you know?

3. How do the rates of change for the two situations compare?

4. Consider the following two sets of data. One is a direct variation relationship and the other is an inverse variation relationship. The constant of variation is the same for the two relationships. Which one is which? How do you know?

x	1	2	3	4	5	6
y	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

x	1	2	3	4	5	6
y	2	4	6	8	10	12

5. How are the x and y values related to the constant of proportionality in each case?

Use what you have learned about direct and inverse variation relationships to solve the following problems:

6. The time it takes to paint a house varies inversely with the number of painters. If 3 people can paint a house in 7 hours, how long will it take 5 people?

7. The number of gallons of fuel used on a trip varies directly with the number of miles traveled. If a trip of 270 miles requires 12 gallons of fuel, how many gallons are required for a trip of 400 miles?

READY, SET, GO! Name ____________________ Period ______ Date __________

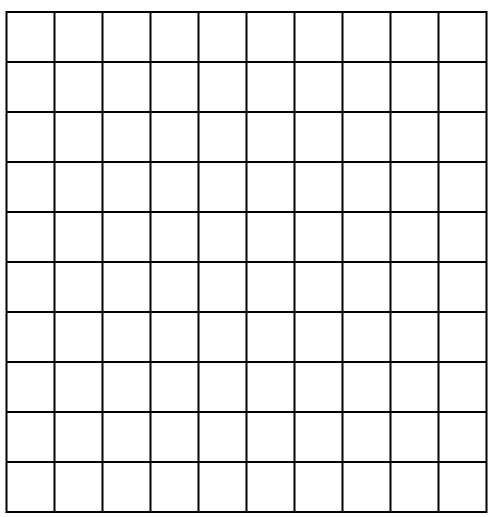
READY

Topic: Quadratic inequalities.

1. Name one solution to the inequality $y < x^2 + 6x + 5$.

2. Solve $x^2 + 7x - 3 > 5$.

3. Graph the solution set for $y > -x^2 + 4x - 6$



4. Name three solutions to the system:

$$y \geq x^2 - 2x + 2$$

$$y = x + 1$$

SET

Topic: Differentiating between direct and inverse variation relationships and using them to solve problems.

5. Determine whether each of the following relationships represents a direct variation, an inverse variation function, or neither

	Direct	**Inverse**	**Neither**
A group of friends goes on a 180 mile road trip to the beach. The time it will take for the group of friends to reach the beach is a function of the speed they are driving.			
A car is traveling at an average rate of 45 miles per hour. The distance the car travels is a function of the time the car travels.			
A soccer player kicked a soccer ball so that it was in the air for 5 seconds. The height of the ball is a function of the time it was in the air.			
Ohm's law states that current equals voltage divided by resistance. The standard voltage in a wall outlet in the United States is 120 volts. The current flowing through a phone charger that is plugged into a wall outlet in the US is a function of resistance.			

6. Write an equation for each of the direct variation functions described in question 5. Then, write a statement describing the relationship between the variables in the form: __________________ is directly proportional to _________________ with a constant of proportionality _______.

7. Write an equation for each of the inverse variation functions described in question 5. Then, write a statement describing the relationship between the variables in the form: __________________ is inversely proportional to _________________ with a constant of proportionality _______.

8. Write an equation of the function given in the table below:

x	-3	-2	-1	0	1	2	3
f(x)	-4	-6	-12	undefined	12	6	4

9. Your distance from lightning varies directly with the time it takes you to hear the thunder. If you hear thunder 10 seconds after you see lightning, you are about 2 miles from the lightning. If you hear thunder 4 seconds after seeing lightning, how far away is the lightning?

10. The time it takes to fly from Los Angeles to New York varies inversely as the speed of the plane. If the trip takes 6 hours at 900 km/h, how long would it take at 800 km/h?

11. When a person walks, the pressure P on each boot sole varies inversely with the area A of the sole. Denise is walking through deep snow, wearing boots that have a sole area of 29 square inches each. The boot-sole pressure is 4 pounds per square inch when she stands on one foot. The constant of variation is Denise's weight in pounds. What is her weight? If Denise wears snowshoes, each with an area 3.5 times that of her boot soles, what is the snowshoe pressure when she stands on one foot?

https://en.wikipedia.org/wiki/Snowshoe

GO!

Topic: Key features of quadratic functions.

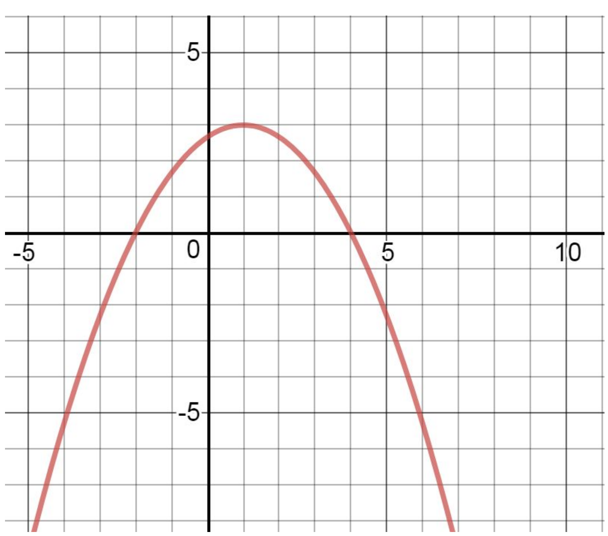
For each function, identify the domain, range, intercepts, vertex, and intervals where the function is increasing or decreasing.

12. $h(x) = \frac{1}{2}(x-5)^2$

13.

x	y
-5	-4
-4	0
-3	2
-2	2
-1	0
0	-4
1	-10

14.



6.4 Tools for Solving

A Practice Understanding Task

Image Source: https://pixabay.com/en/toolbox-green-box-grey-closed-575405/

Square root and inverse variation functions can be represented with an equation, graph, table or with words describing the relationship. In this task you will be presented with problems that involve square root and inverse variation functions. For each of the following problems...

- Write an inverse variation or square root equation or inequality that represents the context.
- Create a table that represents the situation and label where the solution to the problem is found on the table.
- Create a graph that represents the situation and label where the solution to the problem is found on the graph.
- Write a statement that includes the answer to the problem in the context of the problem using the correct units of measure.

1. The relationship between rate, distance and time can be calculated with the equation $r = \frac{d}{t}$ where r is the rate (speed), d represents the distance traveled, and t represents the time. If the speed of a wave from a tsunami is 150 m/s and the distance from the disturbance in the ocean to the shore is 35 kilometers, how long will it take for the wave to reach the shore?

2. The speed of a wave during a tsunami can be calculated with the formula $s = \sqrt{9.81d}$ where s represents speed in meters per second, d represents the depth of the water in meters where the disturbance (for example earthquake) takes place, and 9.81 m/s^2 is the acceleration due to gravity. If the speed of the wave is 150 m/s, what is depth of the water where the disturbance took place?

3. The distance a person can see to the horizon can be approximated by using the function $d(h) = \sqrt{1.5h}$, where d represents the distance in miles and *h* represents the height the person is above sea level in feet. Jacob is standing at the top of Mount Mitchell, which is the highest peak in the US east of the Mississippi. The top of Mt. Mitchell is 6684 ft above sea level and Jacob's eye level is 6 feet from the ground. How far can Jacob see?

4. Tamara is looking to purchase a new outdoor storage shed. She sees an advertisement for a custom built shed that fits into her budget. In this advertisement, the builder offers a 90 square foot shed with any dimensions. Tamara would like the shed to fit into a corner of her backyard, but the width will be restricted by a tree. She remembers the formula for the area of a rectangle is $l \cdot w = a$ and solves for the width to get $w = a/ l$. She then measures the restricted width to be 12 feet. What will be the length of her shed?

5. In kickboxing, it is found that the force, f, needed to break a board, varies inversely with the length, l, of the board. If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will break a board that is 6 feet long?

6. To be considered a ‘fuel efficient’ vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. How many gallons of fuel can a car use on this test run and be considered ‘fuel-efficient’?

7. The centripetal force F exerted on a passenger by a spinning amusement park ride is related to the number of seconds t the ride takes to complete one revolution by the equation $t = \sqrt{\frac{155\pi^2}{F}}$. What is the centripetal force exerted on a passenger when it takes 12 seconds for the ride to complete one revolution?

READY, SET, GO! Name ____________________ Period ______ Date __________

READY

Topic: Transformations of Quadratic Functions.

For each of the following quadratic functions, describe the transformations that have occured from the parent graph $f(x) = x^2$

1. $g(x) = -(x-2)^2$
2. $h(x) = \frac{1}{2}(x+3)^2$
3. $m(x) = (x-1)^2 + 5$
4. $w(x) =- 2(x-6)^2 - 1$
5. $d(x) = x^2 + 4x + 10$
6. $p(x) = (x+6)(x-4)$

SET

Topic: Interpreting and solving problems involving square root and inverse variation relationships.

Use multiple representations to solve each problem.

7. As you have learned in this unit, the amount of time it takes to travel a given distance is inversely proportional to how fast something travels.
   a. In dry air at a temperature of 68°F, sound travels at the speed of 767 mph or 1,125 ft/sec. How many seconds does it take sound to travel 1 mile (5280 ft) in air?
   b. Sound travels faster in liquids than in air. In water, sound travels at a speed of 3,320 mph or 4869 ft/sec. How many seconds would it take sound to travel 1 mile in water? How many times faster does sound travel in water vs. air?

c. Sound travels even faster in solids than liquids or air. It takes sound about .31 seconds to travel 1 mile through iron. How fast does sound travel through iron in ft/s? mph?

8. At a swimming pool, there is a large water slide. Darius is getting ready to go down the water slide. The function $h(t) = -2\sqrt{x} + 12$ models Darius' height above the pool in meters as a function of the time elapsed in seconds since he started down the slide. Make a table and graph this function.

a. What is the initial height of the slide? What is the mathematical name for this point?

b. How long does it take before Darius hits the pool? What is the mathematical name for this point?

c. How many seconds does it take him to reach a height of 4 meters above the pool?

d. What is the practical domain for this function? What is the theoretical domain?

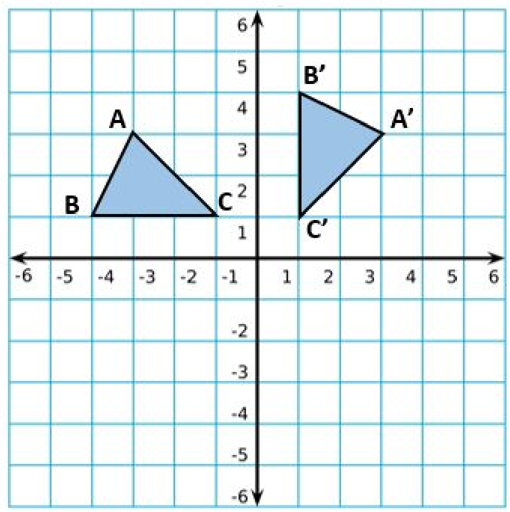
e. What is the practical range for this function? What is the theoretical range?

GO!

Topic: Transformations of Geometric Figures.

For questions 9-11 write the transformation of triangle ABC in function notation.

9.



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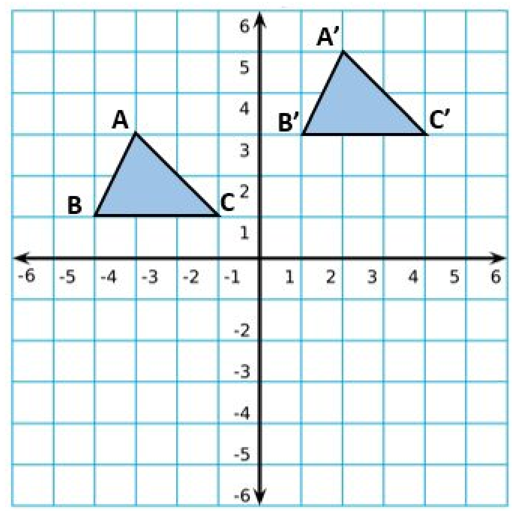
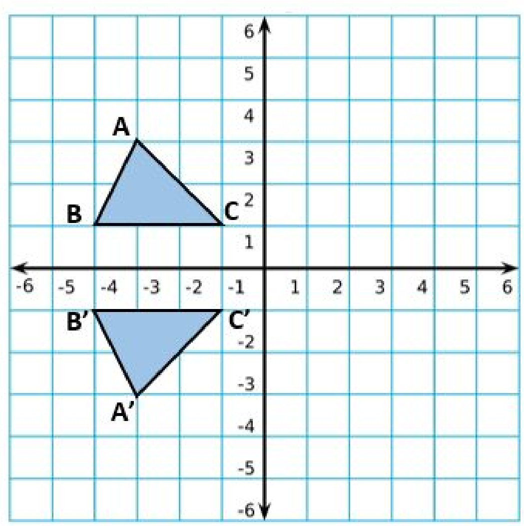


11.



For questions 12 - 15 describe the transformation that has occured given the function notation.

12. $f(x,y) \rightarrow (x+1,\ y-4)$

13. $f(x,y) \rightarrow (-y, x)$

14. $f(x,y) \rightarrow (x-2,\ y+5)$

15. $f(x,y) \rightarrow (-x, y)$

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6.5 Let the Games Begin!

A Develop Understanding Task

http://2.bp.blogspot.com/-7KOA3j5Hyho/UghXeGJGuMI/AAAAAAAAHE8/AwoR9-VHJIY/s1600/carnival-basketball-hoop.jpg

Part 1: A Shot Down Memory Lane?

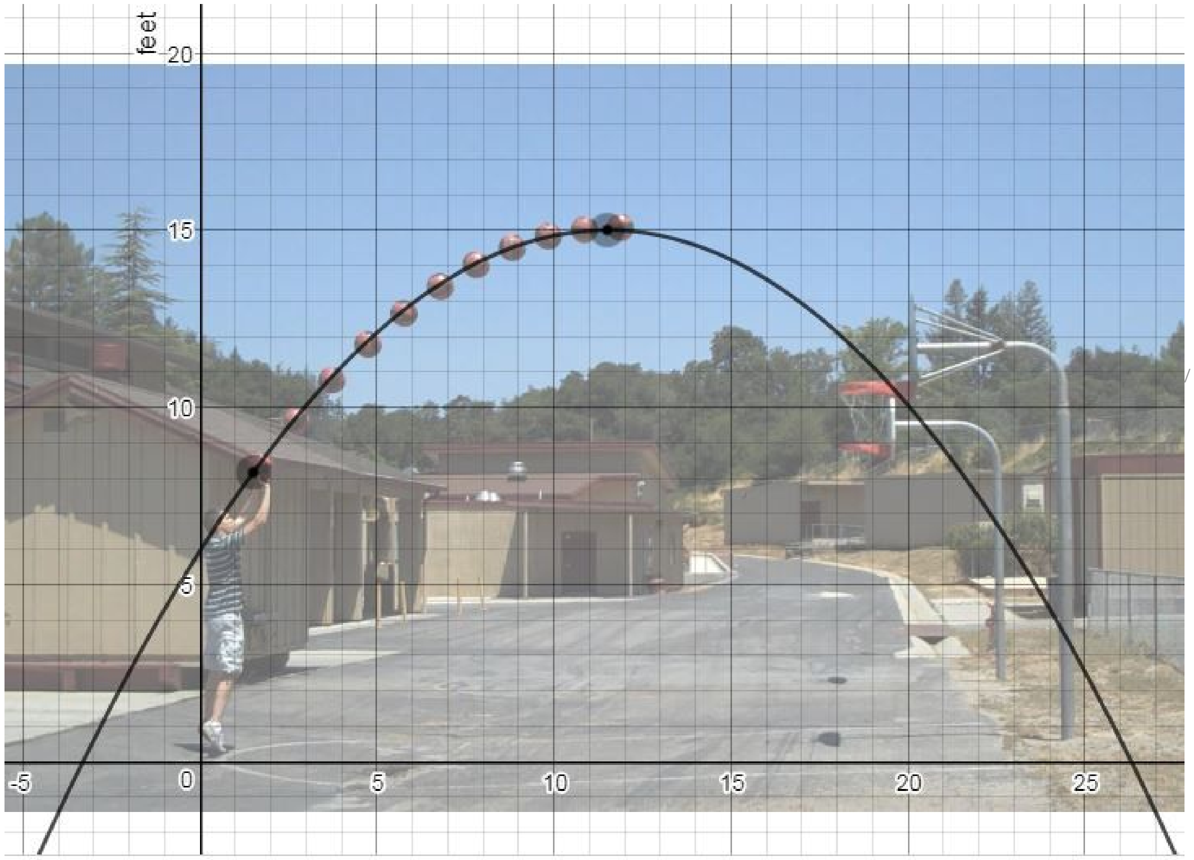
Mr. Gordon has been asked to make a game for the carnival section of Family Kingdom Fun Park. He was thinking of doing a basketball game but he wants to do a little research first. He doesn't want to make an impossible game, but he doesn't want it to be too easy either. He gets his son, Jeff, to help gather some data on shots.

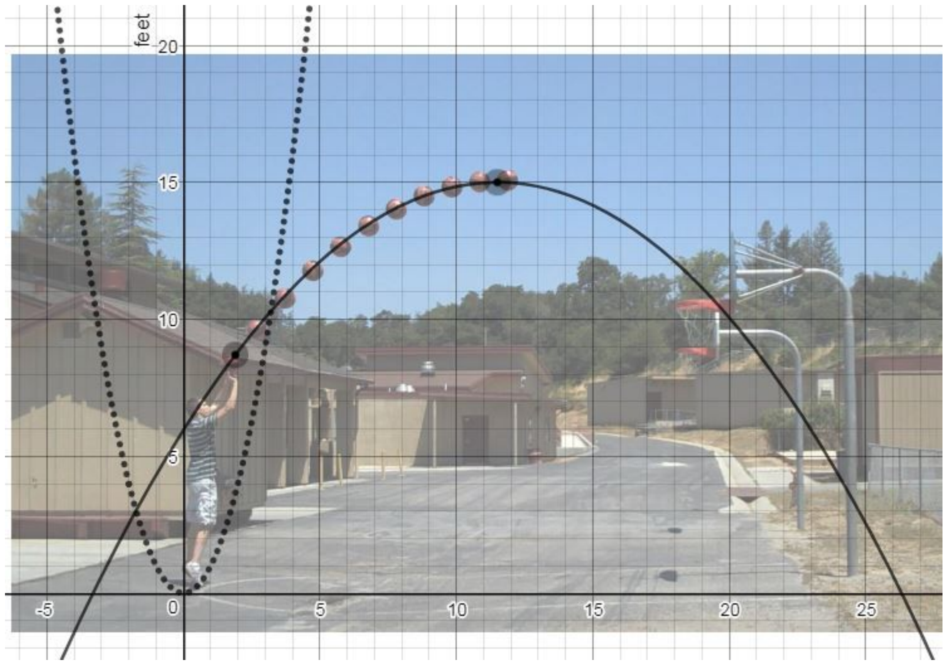
Shot #1

Jeff took a time-lapse picture of his dad shooting a basketball. Do you think this shot is going to go in?

Which function have we studied that could help us model this situation? Why?

Jeff drew a parabola over the image of the ball moving to try to predict the path the ball would take. He then overlaid a coordinate system on the picture. Identify all the key features (domain, range, max/min, intercepts, intervals where the function is increasing or decreasing, rate of change) of the parabola and describe them in context.





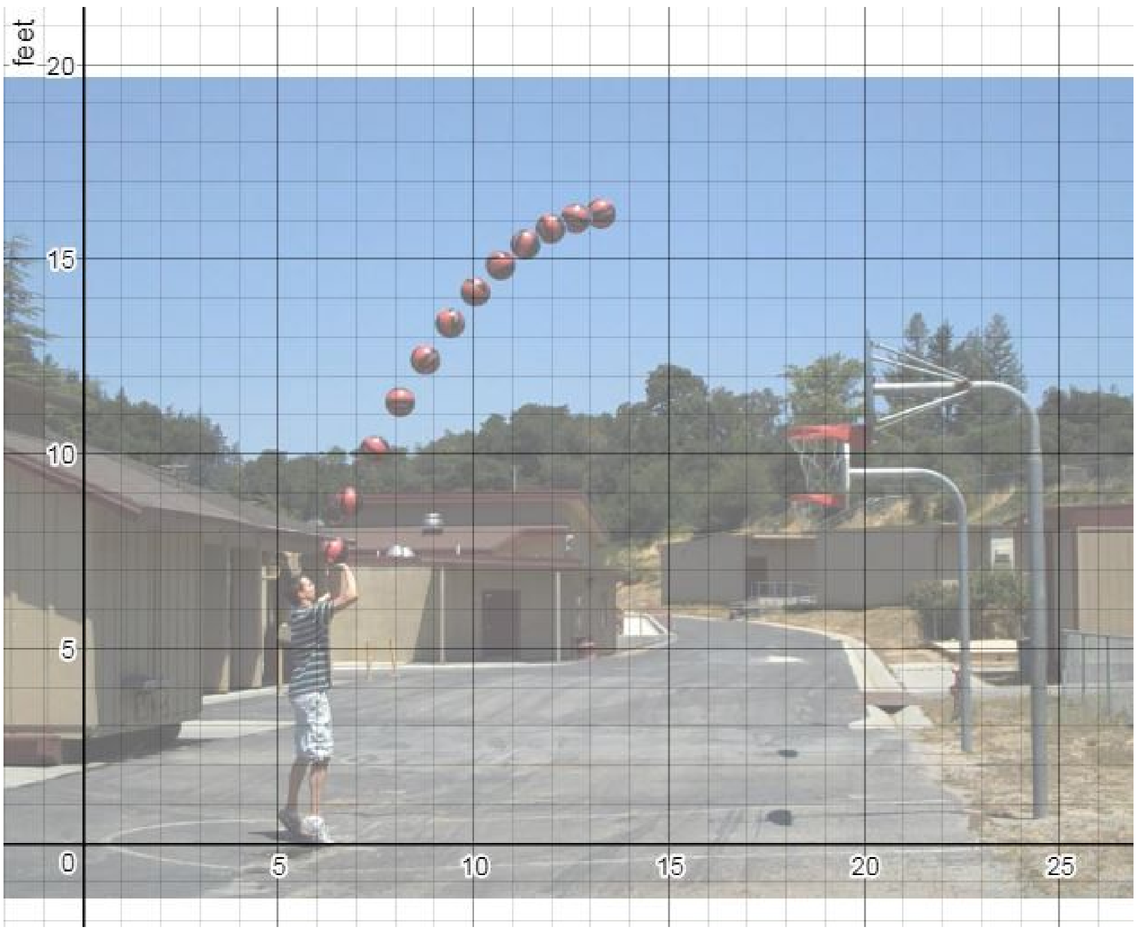
The dotted line shows the parent graph of the parabola $y = x^2$.

Describe the **transformations** needed to match the parent graph onto Jeff's parabola.

The equation of Jeff's graph is $y = -0.068(x - 11.5)^2 + 15$. How are the transformations you described above reflected in this equation?

Let's look at another shot. Do you think this shot is going to go in?

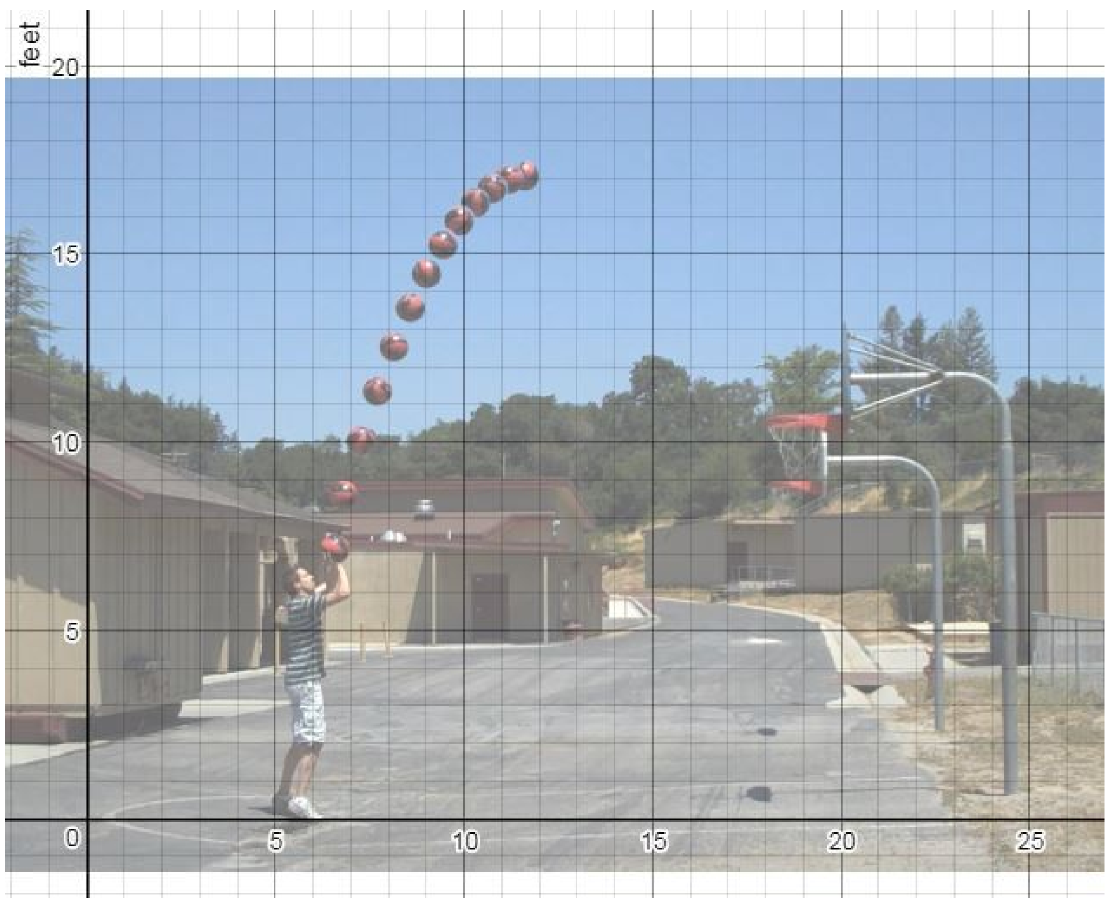
Draw a parabola to model the path of the shot. Then identify its key features.



Use the key features to write a model for the parabola using your observations from the last example.

One more shot. Do you think this shot is going to go in?

Draw a parabola to model the path of the shot. Then identify its key features.



Use the key features to write a model for the parabola using your observations from the last example.

Part 2: More Parent Graphs

In Unit 4: Structures of Expressions, you studied transformations of quadratic functions. Describe what happens to the quadratic parent graph $y = x^2$ for each of the following transformations:

$y = x^2 + k$, where $k > 0$

$y = x^2 - k$, where $k > 0$

$y = (x+k)^2$, where $k > 0$

$y = (x-k)^2$, where $k > 0$

$y = k \cdot x^2$, where $0 < k < 1$

$y = k \cdot x^2$, where $k > 1$

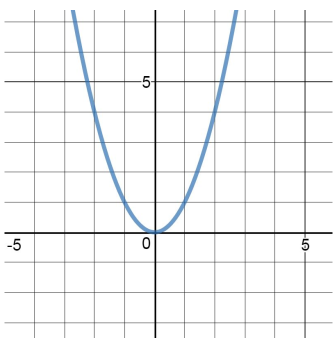
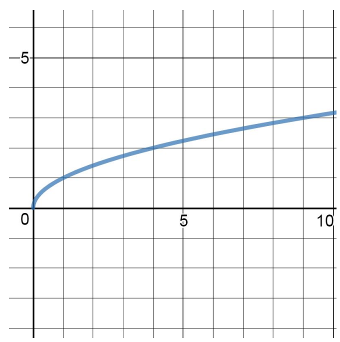
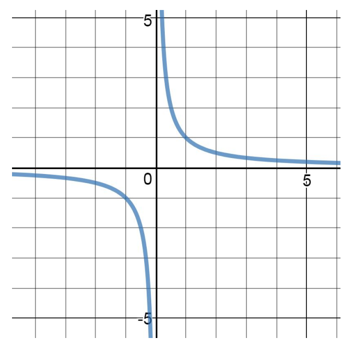
$y = k \cdot x^2$, where k is a negative number

In this unit, you have been introduced to two new types of functions - the square root function and the inverse variation function. Each of these functions also has a parent graph:

Square root function parent graph: $y = \sqrt{x}$

Inverse variation function parent graph: $y = \frac{1}{x}$

Complete the table below to compare the key features of these three parent functions:

<table>
<tr><th>Feature</th><th>Quadratic</th><th>Square Root</th><th>Inverse Variation</th></tr>
<tr><td>Equation</td><td>$y = x^2$</td><td>$y = \sqrt{x}$</td><td>$y = \frac{1}{x}$</td></tr>
<tr><td>Graph</td><td>
</td><td>
</td><td>
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<tr><td>Table of Key Points - mark these on the graphs above</td>
<td><table>
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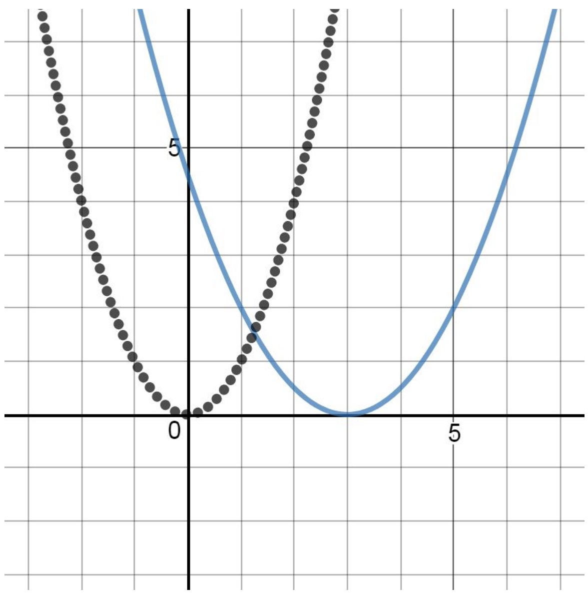
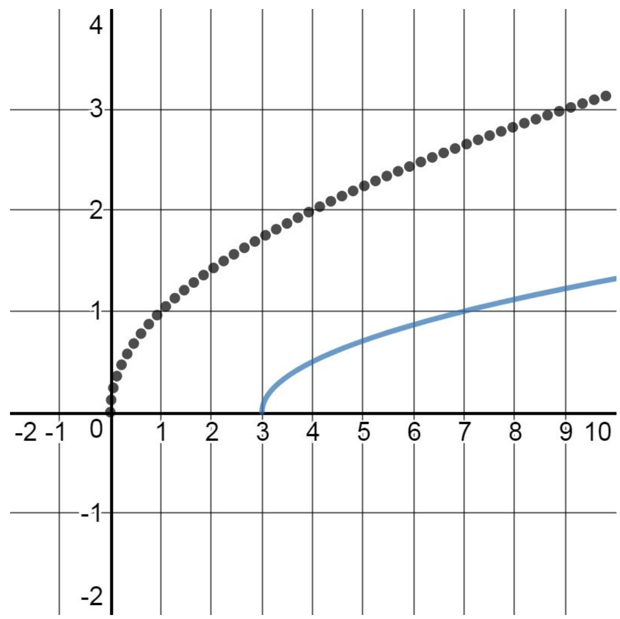
Domain			
Range			
Description of Rate of Change			
Intercepts			
Intervals Where Increasing or Decreasing			
Maximum or Minimum			
Symmetry			
End Behavior			

Jeff graphed the following pairs of functions to see if transformations with square root functions behave in the same way that transformations with quadratic functions do. What do you think? (The parent graph is the dotted curve.)

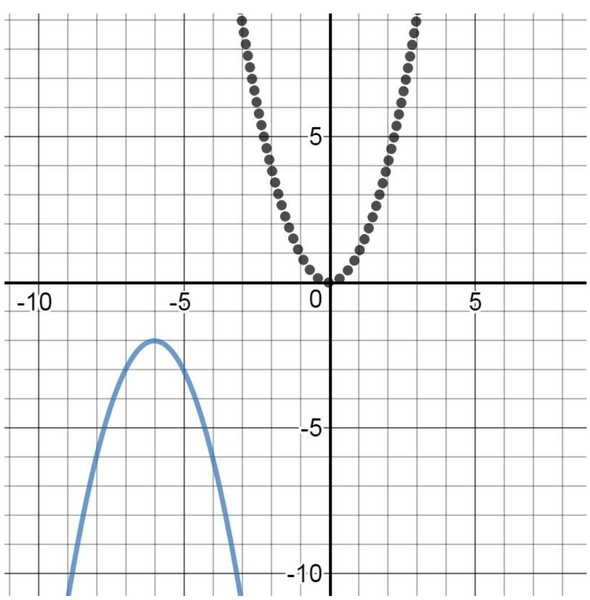
$y = \frac{1}{2}(x-3)^2$



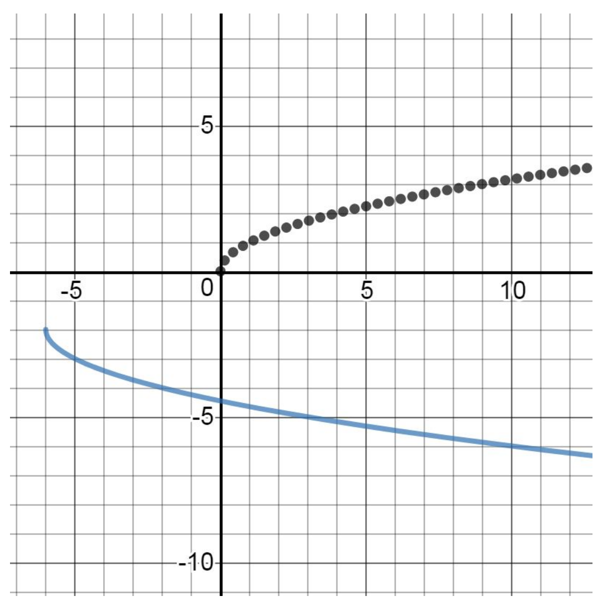
$y = \frac{1}{2}\sqrt{x-3}$



$y = -(x+6)^2 - 2$



$y = -\sqrt{x+6} - 2$



READY, SET, GO! Name ____________________ Period ______ Date __________

READY

Topic: Making Tables.

Fill in the table for the given functions.

1. $f(x) = 2\sqrt{x} + 3$

x	f(x)
0	
1	
4	
9	

2. $f(x) = 2\sqrt{x+4}$

x	f(x)
-4	
-3	
0	
5	

3. $f(x) = 0.5\sqrt{x+1} - 5$

x	f(x)
-1	
0	
3	
8	

4. $f(x) = \frac{1}{(x-2)}$

x	f(x)
0	
1	
3	
4	

5. $f(x) = \frac{1}{x} + 10$

x	f(x)
-2	
-1	
1	
2	

6. $f(x) = \frac{6}{(x+1)}$

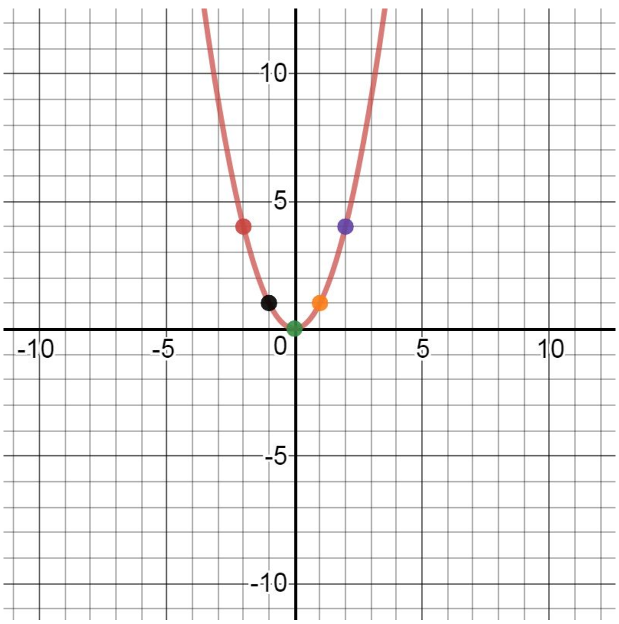
x	f(x)
-7	
-5	
-4	
-3	

SET

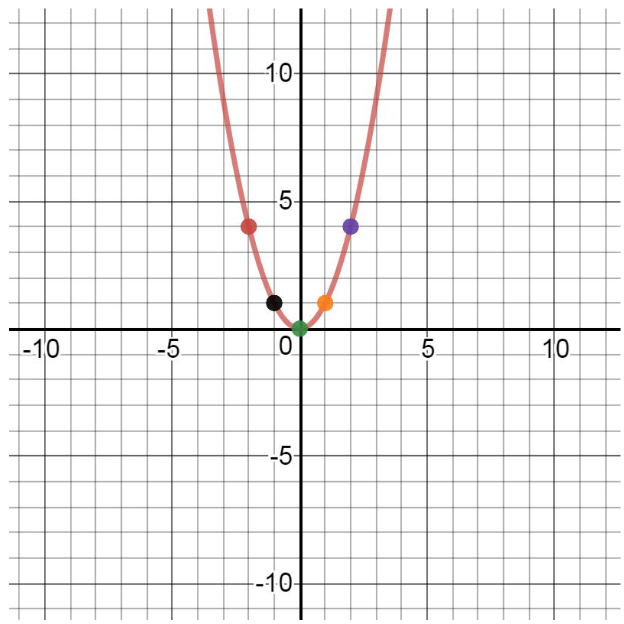
Topic: Graphing Transformations of Quadratic Functions.

Given the equation or description, graph the transformation of the parent quadratic function shown in each graph.

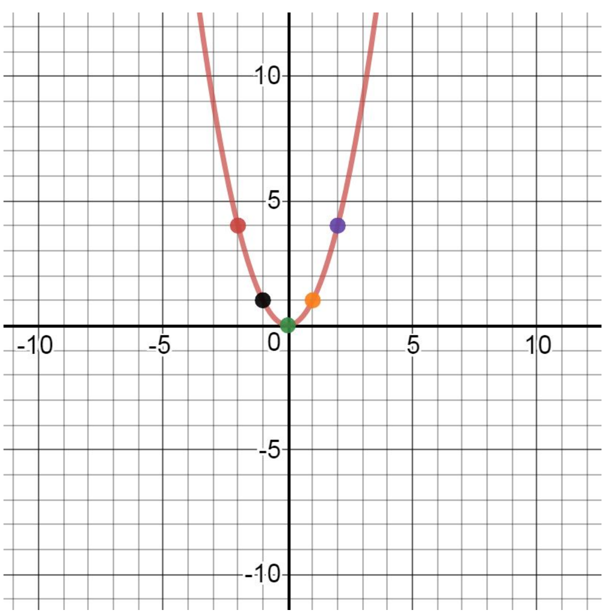
7. The graph has been stretched by a factor of three, shifted four units to the right, and two units down.



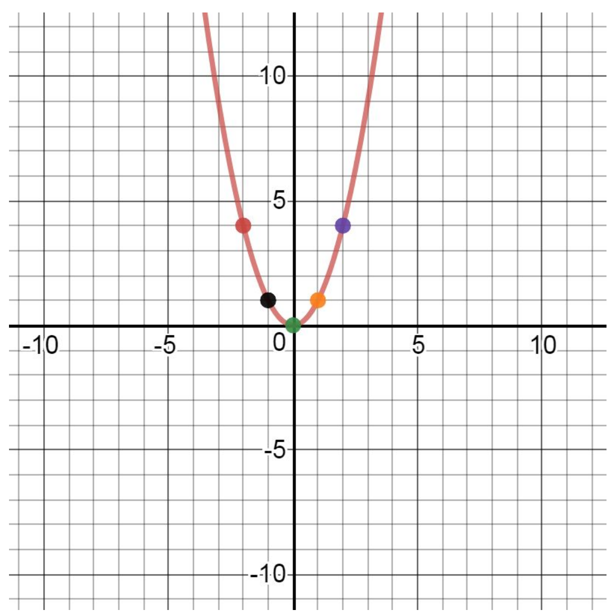
8. The graph has been reflected over the x-axis and shifted 5 units up.



9. $f(x) = 2(x-3)^2$



10. $f(x) = -0.5x^2 - 1$



Looking back at questions 9 and 10, what transformations do you think have occured to the parent graph $f(x) = \sqrt{x}$ if the new functions are:

11. $f(x) = 2\sqrt{x-3}$

12. $f(x) = -0.5\sqrt{x} - 1$

GO!

Topic: Direct or Inverse Variation.

Fill in each blank with varies “directly” or varies “inversely”. Find the constant of variation for each scenario.

13. The electric current I, is amperes, in a circuit varies __________ as the voltage V. When 12 volts are applied, the current is 4 amperes. When 18 volts are applied, the current is 6 amperes.

14. The volume V of gas varies __________ to the pressure P. The volume of a gas is 200 cm^3 under pressure of 32 kg/cm^2. The volume of a gas is 160 cm^3 under pressure of 40 kg/cm^2.

15. The number of kilograms of water in a person’s body varies __________ as the person’s mass. A person with a mass of 90 kg contains 60 kg of water. A person with a mass of 45 kg contains 30 kg of water.

16. On a map, distance in km and distance in cm varies __________, so that 25 km are represented by 2cm, and 87.5 km are represented by 7cm.

17. The time it takes to fly from Los Angeles to New York varies __________ as the speed of the plane. One trip takes 6 hours at 900 km/h, at 600 km/h the trip will take 9 hours.

6.6 Transformation Exploration

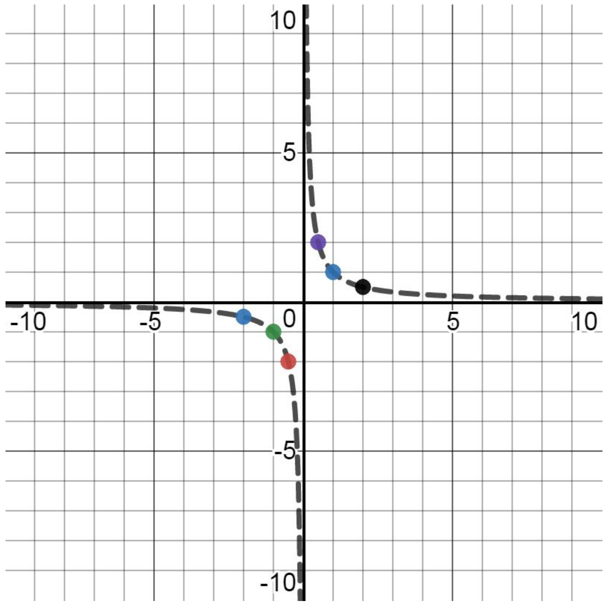
https://commons.wikimedia.org/wiki/File:Christopher_Columbus3.jpg

A Solidify Understanding Task

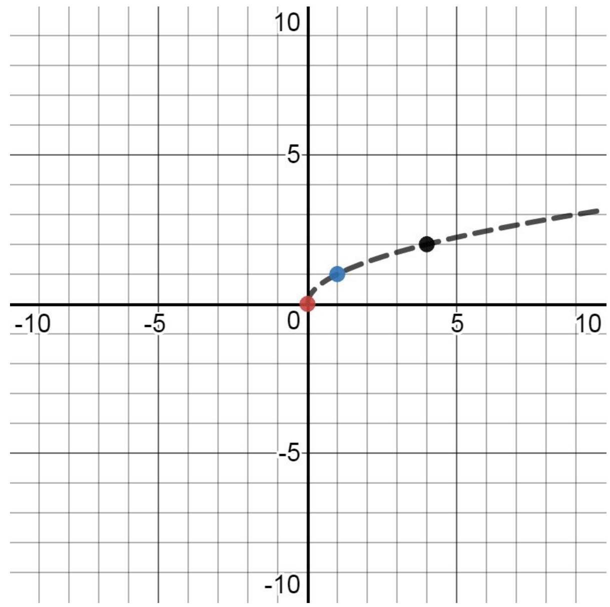
In *6.5 Let the Games Begin!* you were introduced to two new parent functions, the square root function $y = \sqrt{x}$ and the inverse variation function $y = \frac{1}{x}$. Jeff was intrigued by your insights about transformations with these two functions and he wants to explore further. You will be given a set of cards with various graphs and equations of transformed functions. Your task is to match each graph with its equation and to group together cards that show similar transformations. What conclusions do you have to share with Jeff?

Graph the following transformations. Be sure to show the key points of the transformed function. The parent function with key points has been provided for you.

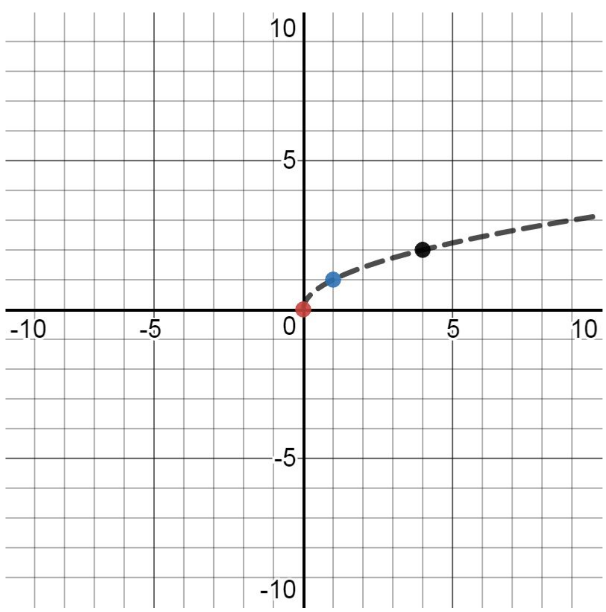
1. $y = \frac{1}{(x-4)} + 2$



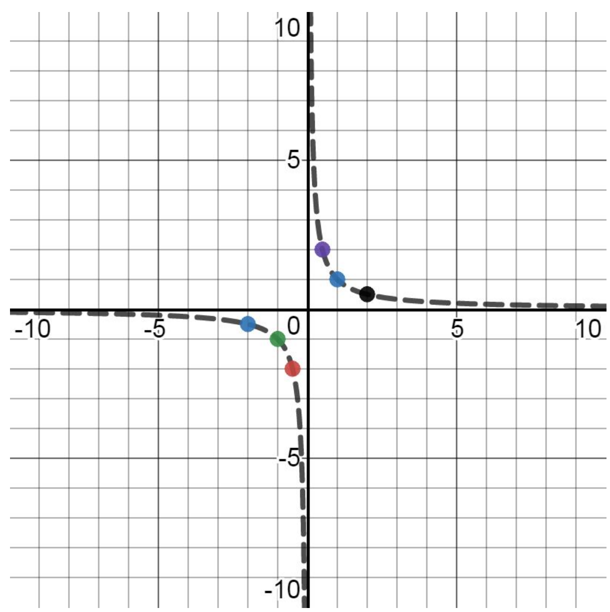
2. $y = -\sqrt{x+5}$



3. $y = \sqrt{x-2} - 6$



4. $y = -\frac{1}{x} - 3$



READY, SET, GO! Name ____________________ Period _______ Date __________

READY

Topic: Solving Systems of Linear and Quadratic Equations.

Find any points of intersection for the two equations in each problem.

1. $\begin{cases} y = x^2 + x - 2 \\ y = -x + 1 \end{cases}$

2. $\begin{cases} y + x = 5 \\ y = x^2 - 6x + 9 \end{cases}$

3. $\begin{cases} y = 3x + 4 \\ y = -x^2 \end{cases}$

4. $\begin{cases} y = x^2 + 11 \\ y = -12x \end{cases}$

5. $\begin{cases} y = 3x^2 + 21x - 5 \\ -10x + y = -1 \end{cases}$

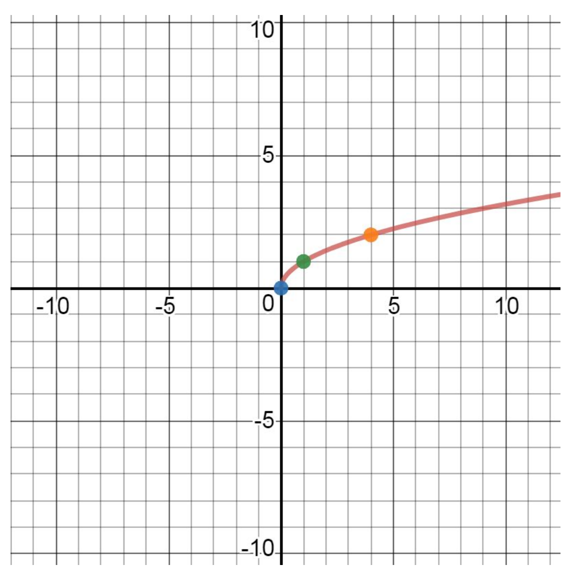
6. $\begin{cases} y = x^2 - 11x - 20 \\ y = 25(4 - x) \end{cases}$

SET

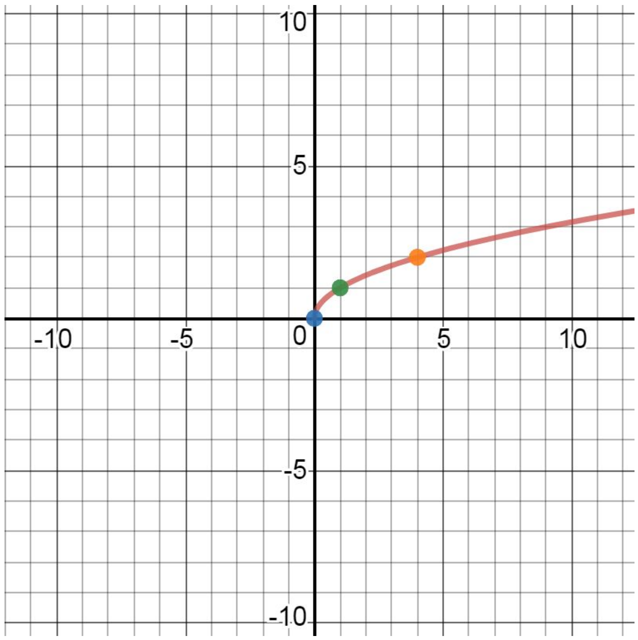
Topic: Graphing Transformations.

Given the equation or description, graph the transformation of the parent function shown in each graph.

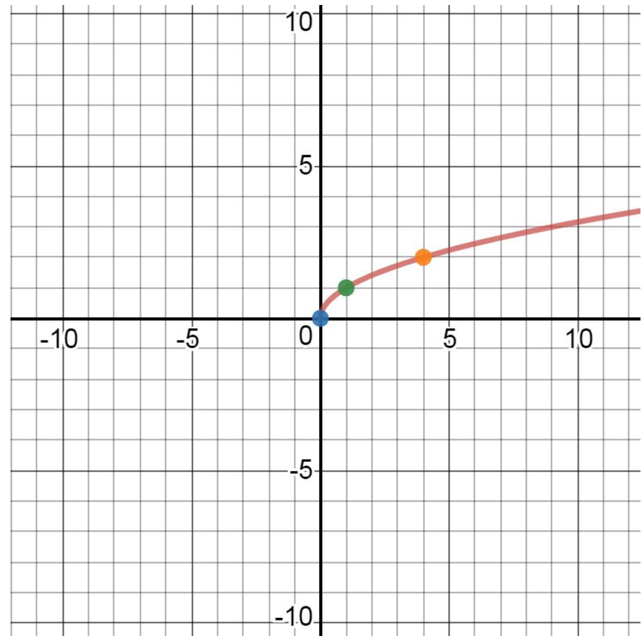
7. The graph has been reflected over the x-axis and shifted three units down, and two units right.



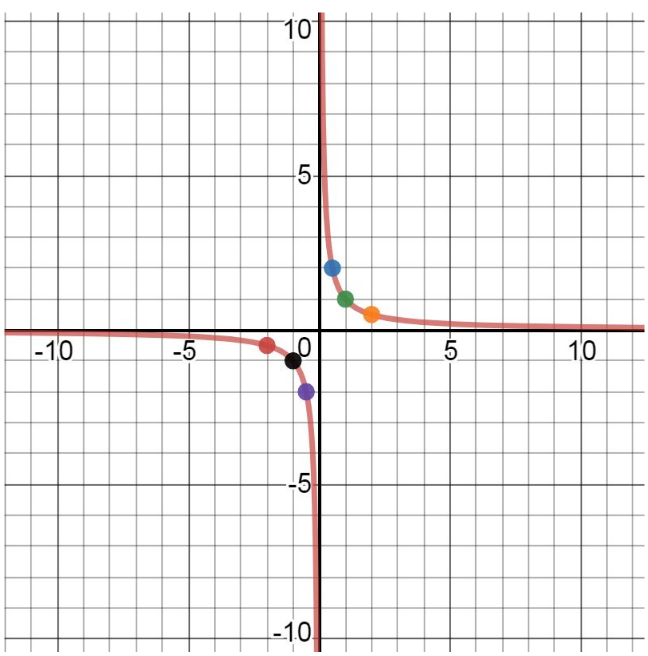
8. The graph has been stretched by a factor of 2, and shifted 5 units to the left.



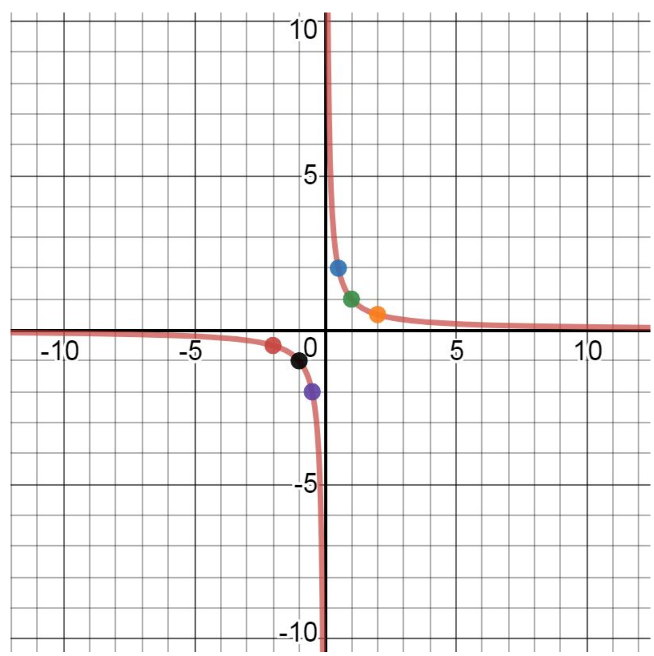
9. $f(x) = -\sqrt{x} + 7$



10. The graph has shifted six units left, and two units down.



11. $f(x) = \frac{8}{x} + 1$



GO!

Topic: Identify the type of function.

Each table is a model for a different type of function. Determine if each table is linear, exponential, quadratic, square root, or inverse variation.

12.

x	f(x)
0	0.25
1	0.50
2	1
3	2
4	4

13.

x	g(x)
1	5
5	1
10	$\frac{1}{2}$
15	$\frac{1}{3}$

14.

x	h(x)
0	-2
1	2
2	8
3	16
4	26

15.

x	m(x)
16	5
9	4
4	3
1	2

16.

x	p(x)
-1	6
-2	10
-3	14
-4	18
-5	22

17.

x	v(x)
1	12
2	6
3	4
4	3

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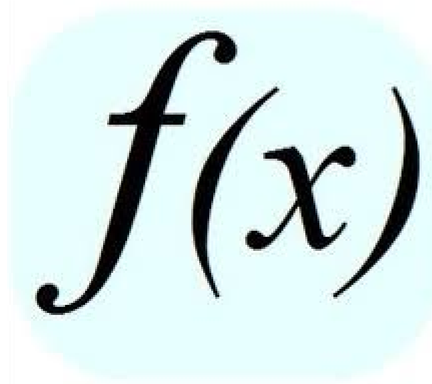
6.7 Let's Make a Function

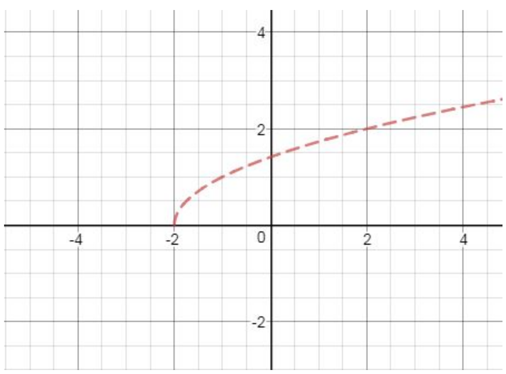
A Practice Understanding Task



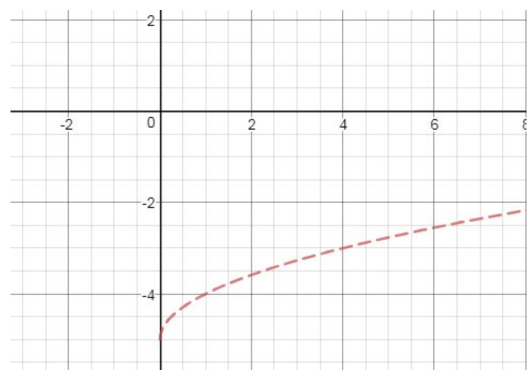
For this activity, you will be using what you have learned about radical functions and inverse variation functions and transformations to write the equation of a transformed function given a graph.

In the following graphs, a transformed function is represented with dashed lines. For each graph, determine whether the parent function is a square root function , $y = \sqrt{x}$, or an inverse variation function, $y = \frac{1}{x}$, and which transformation(s) have occurred. Then write the equation of the transformed function. (graphs created using Desmos)

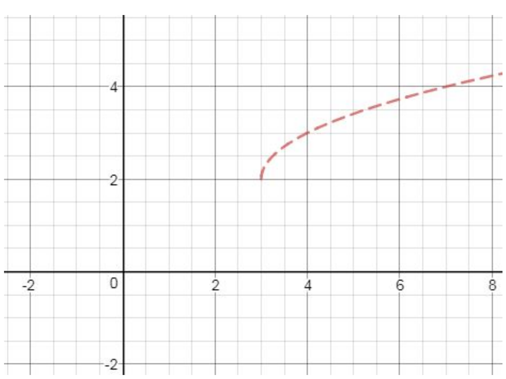
1. ______________________________



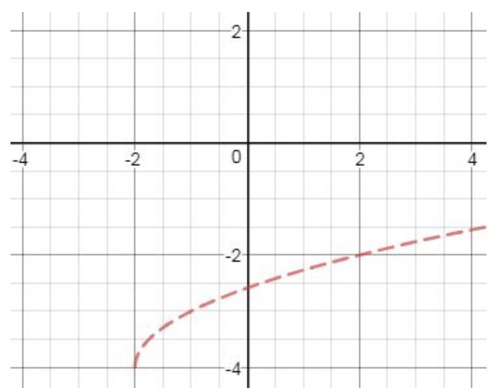
2. ______________________________



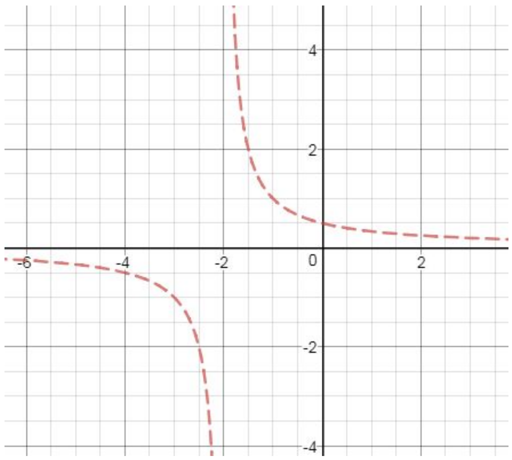
3. ______________________________



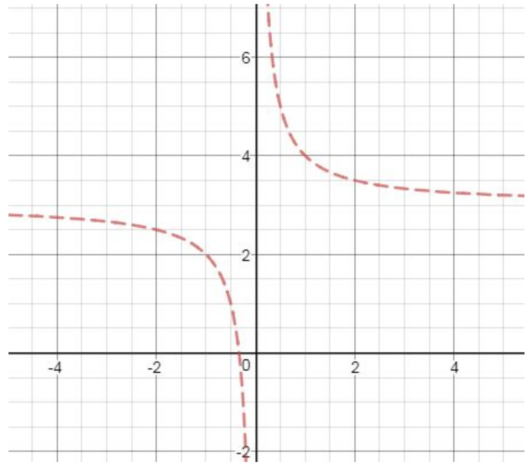
4. ______________________________



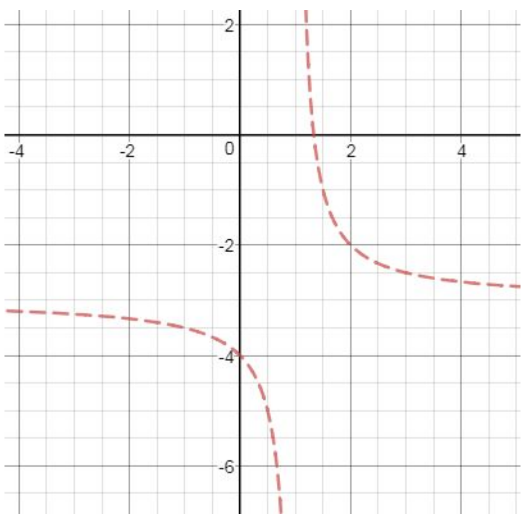
5. ___________________________________



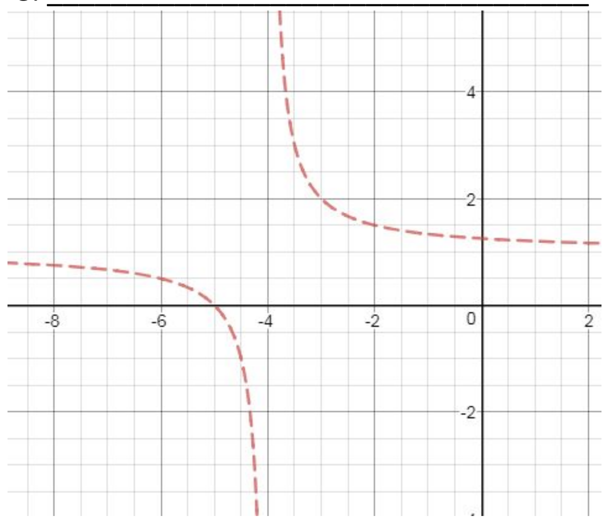
6. ___________________________________



7. ___________________________________



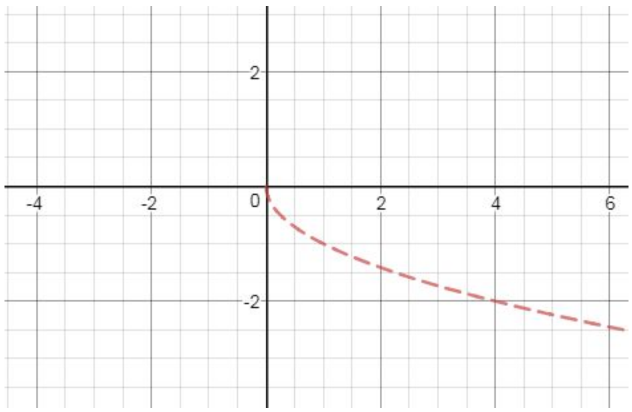
8. ___________________________________



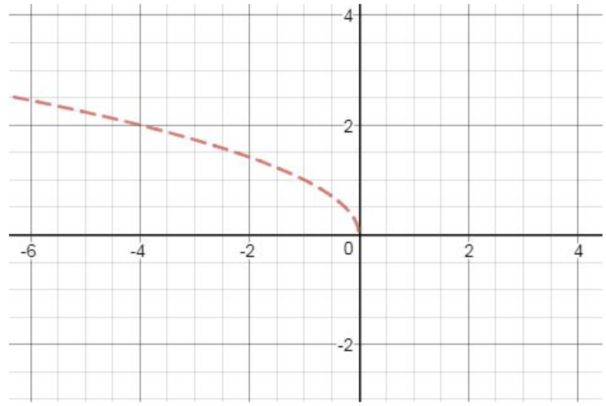
Which type of transformation occured in problems 1-8?

For the next few problems, a different type of transformation has been applied. You may also still see some of the previous type of transformation as well. Watch out as you write the equations!

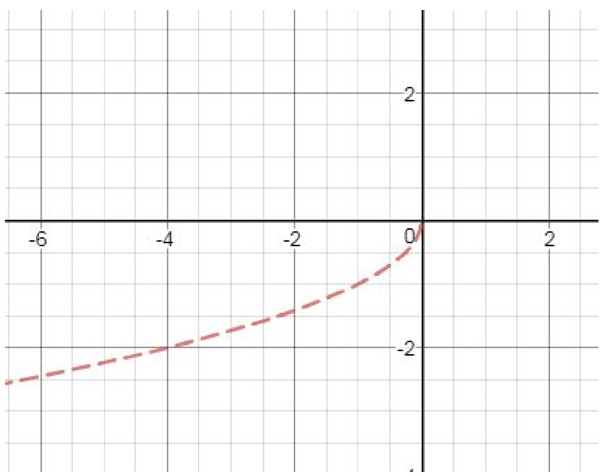
9. ________________________________



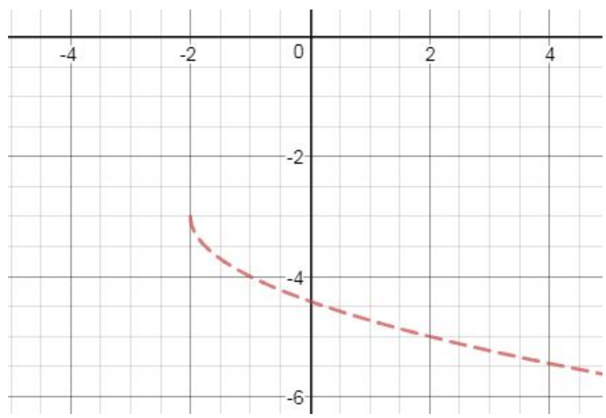
10. ________________________________



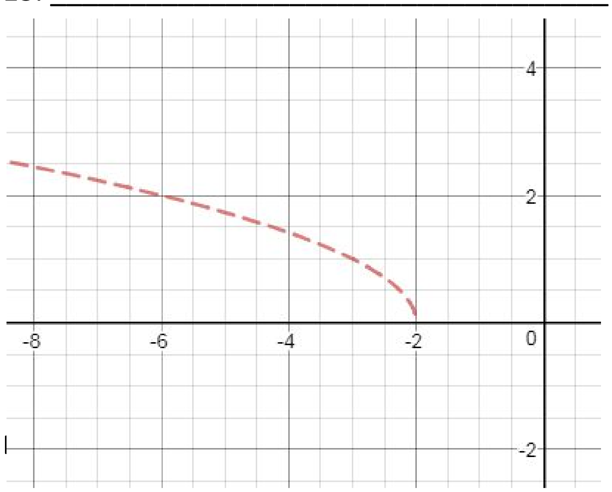
11. ________________________________



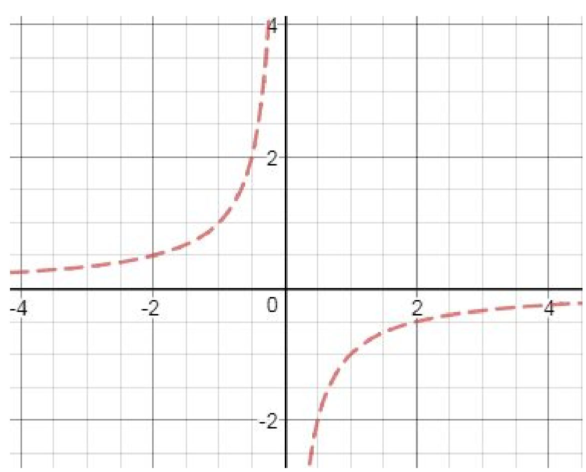
12. ________________________________



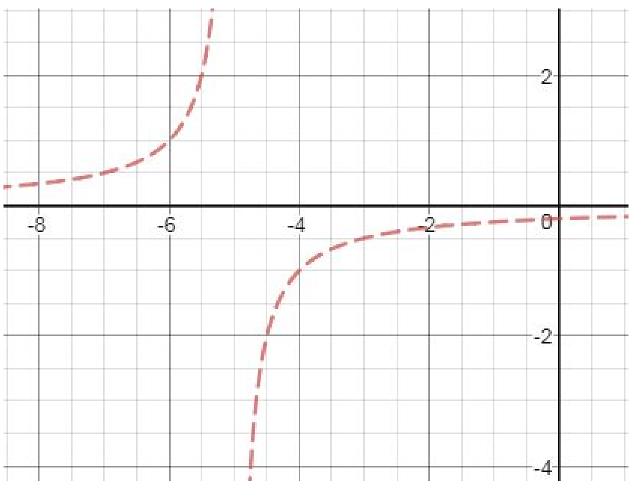
13. ________________________________



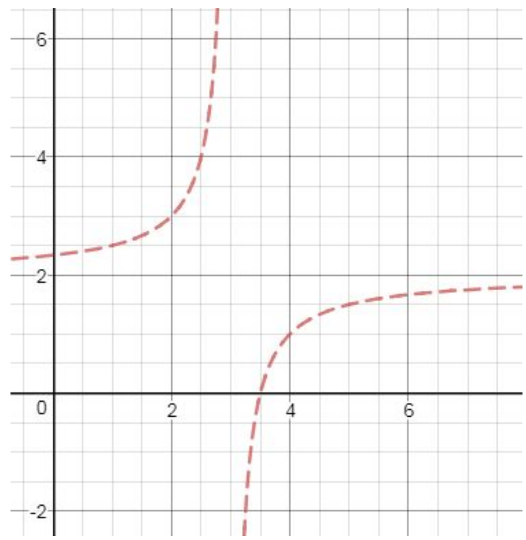
14. ________________________________



15. ______________________________________



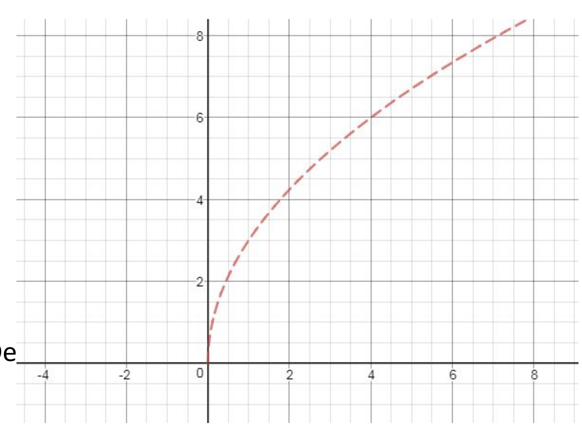
16. ____________________________________



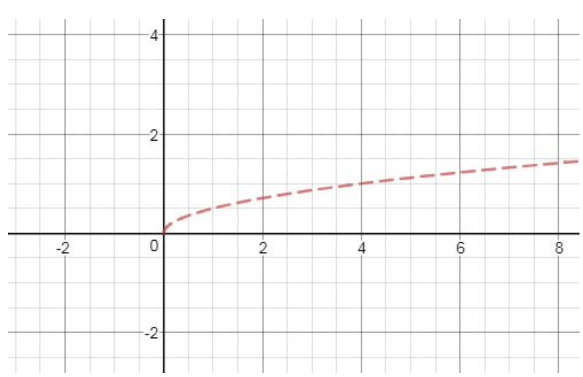
Which type of transformation was added in problems 9-16? How is this type of transformation different for radical functions as compared to inverse variation functions? Explain.

For the next few problems, yet another type of transformation has been applied. You may also still see some of the previous types of transformations as well. Be careful!

17. ____

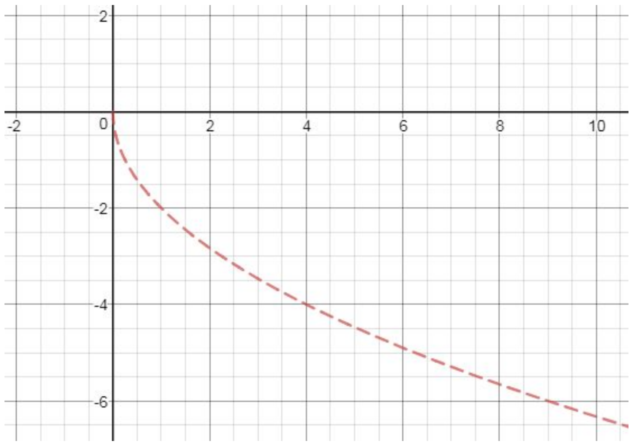


18. ____________________________________

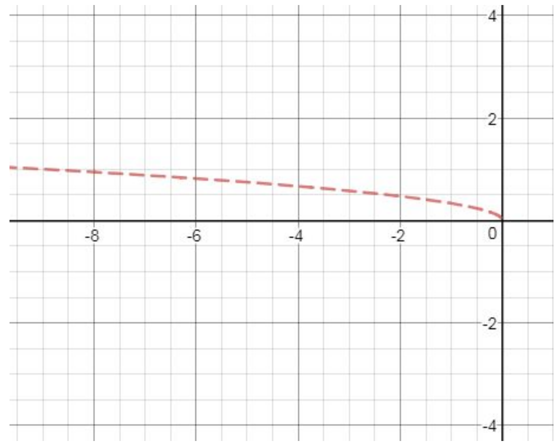


De_

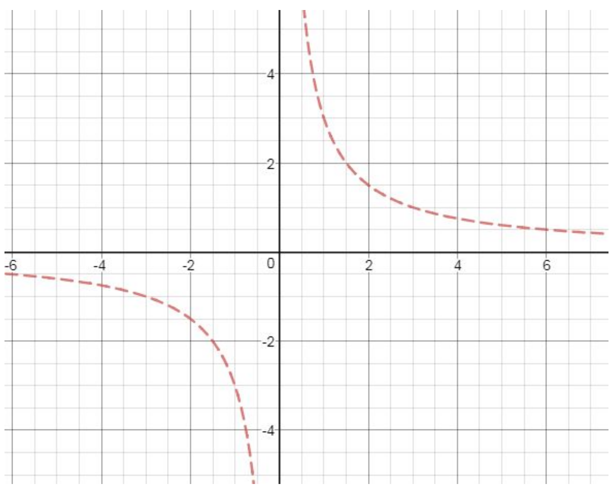
19. ___________________________________



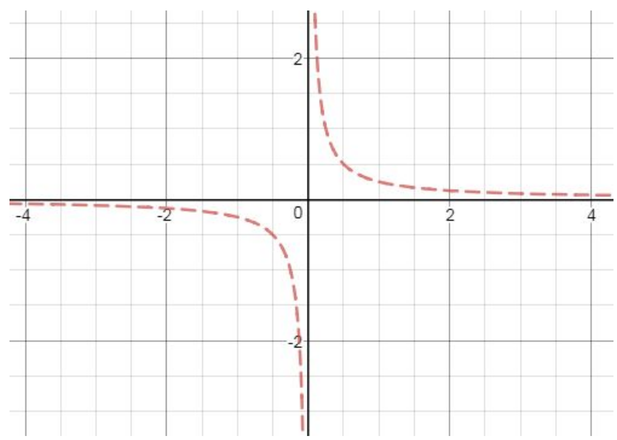
20. ___________________________________



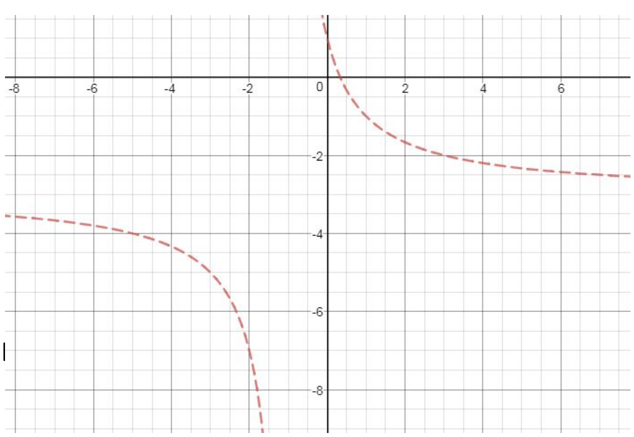
21. ___________________________________



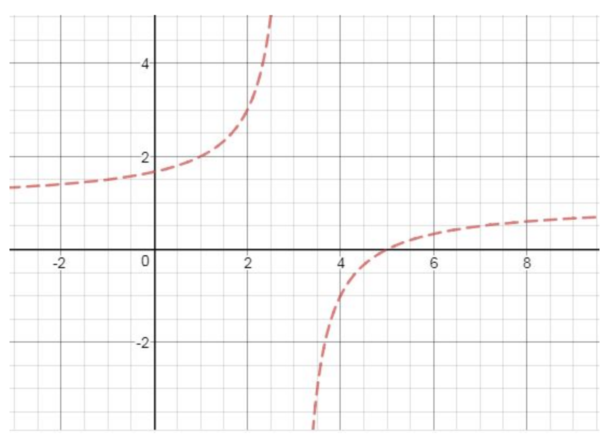
22. ___________________________________



23. ___________________________________



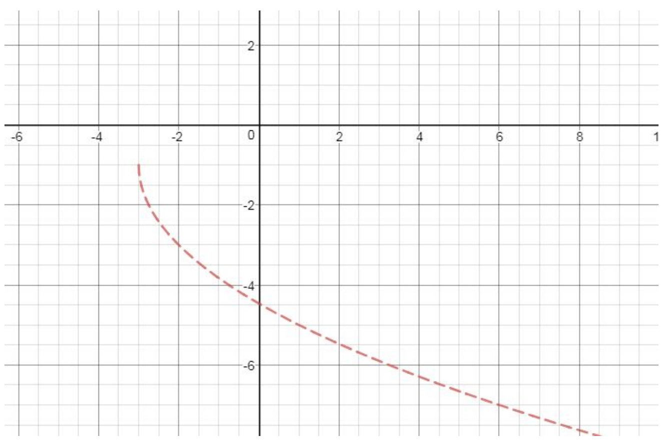
24. ___________________________________



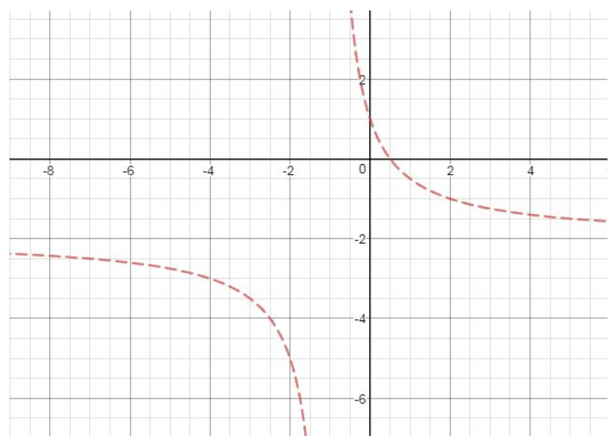
When applying a dilation to a function, the equation of the parent function is multiplied by a coefficient. Describe the effect different types of coefficients have on the shape of a function.

For these last few problems, all types of transformations are combined. You can do this!

25. ___________________________________

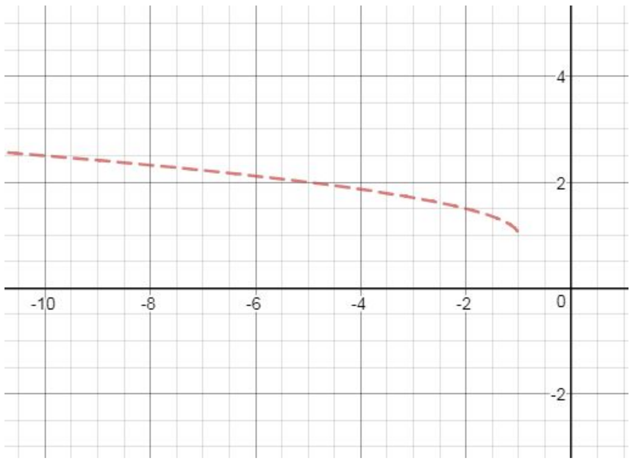


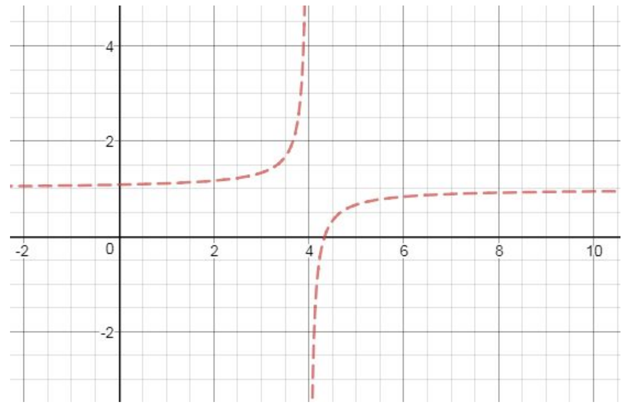
26. ___________________________________



27. ___________________________________

28. ___________________________________


4
2
-10
-8
-6
-4
-2
0
-2


4
2
-2
0
2
4
6
8
10
-2

READY, SET, GO! Name ________________ Period ______ Date __________

READY

Topic: Solving Proportions.

Find the value of x in each equation.

1. $\frac{2}{x} = \frac{8}{20}$

2. $24 = \frac{36}{x}$

3. $\frac{9}{16} = \frac{6}{x}$

4. $\frac{x+1}{20} = \frac{1}{2}$

5. $\frac{35}{2x-3} = \frac{70}{34}$

6. $\frac{10}{x} = 8$

SET

Topic: Transformations of Functions.

Given the following descriptions of transformations, write the new function of the transformed graph.

7. From the parent graph $f(x) = x^2$ the graph has been shifted 3 units to the right and 4 units up.

8. From the parent graph $f(x) = \sqrt{x}$ the graph has been shifted 2 units to the left and one unit down.

9. From the parent graph $f(x) = \frac{1}{x}$ the graph has been shifted six units to the right and five units down.

10. From the parent graph $f(x) = x^2$ the key points of (0, 0), (1, 1), and (2, 4) are now (0, 0), (1, 0.5), and (2, 2).

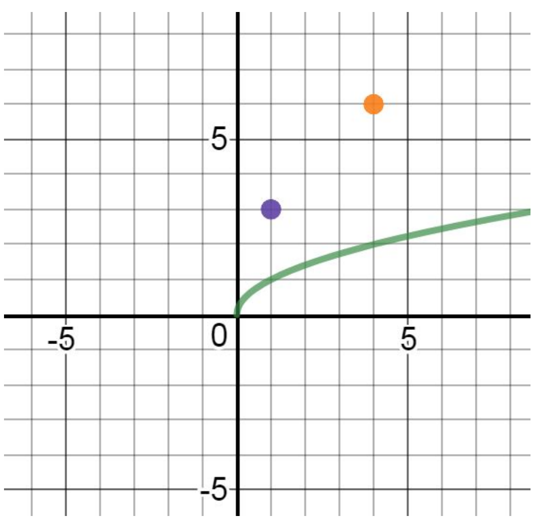
11. From the parent graph $f(x) = \sqrt{x}$ to $g(x)$ given in the table below.

x	f(x)
0	0
1	1
4	2
9	3

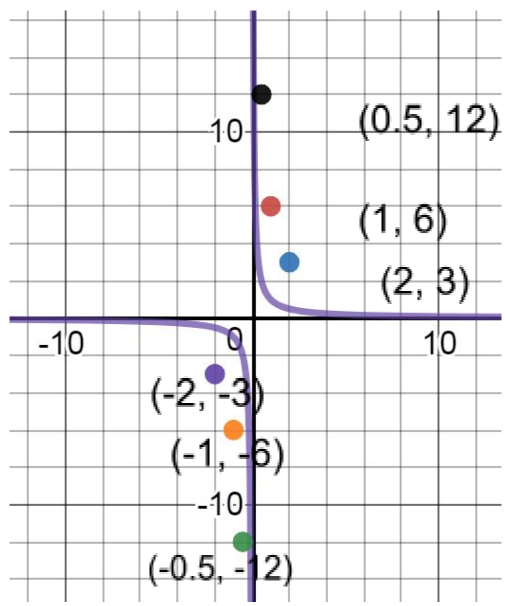
x	g(x)
0	0
1	4
4	8
9	12

Given the following parent graphs, write the new function of the transformed graph that will go through the given ordered pairs.

12.

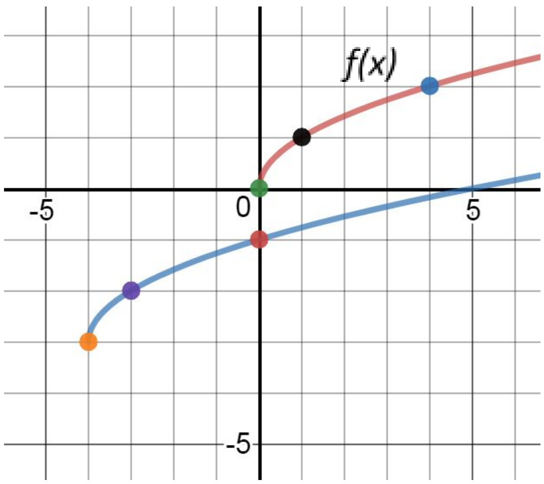


13.

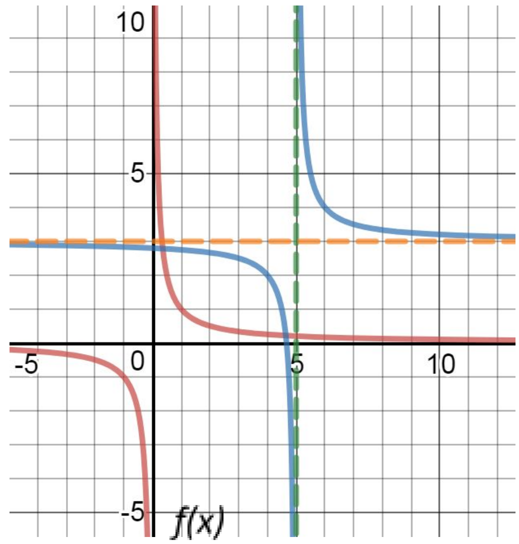


Given the following parent graphs $f(x)$, write the new function of the transformed graph that is shown.

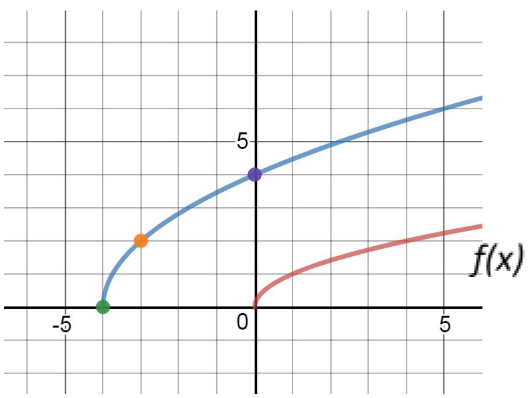
14.



15.



16.



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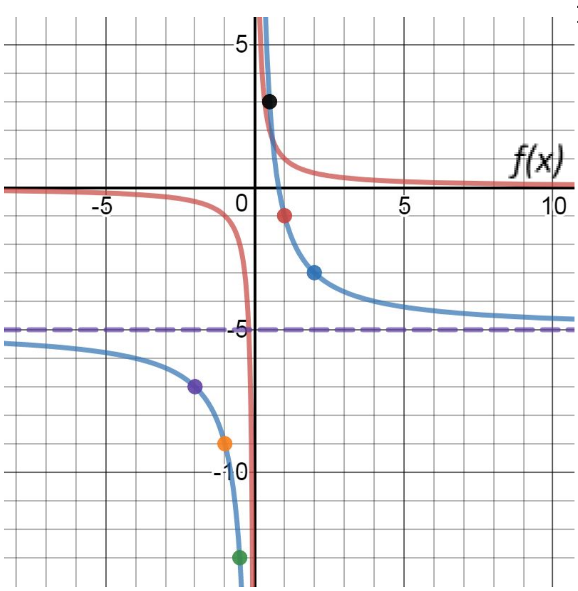

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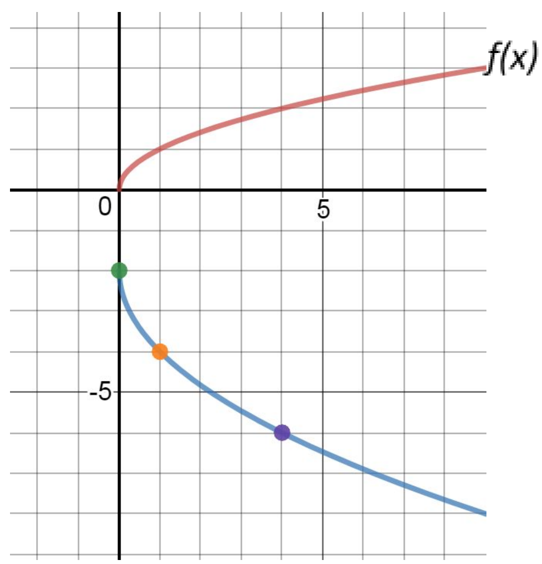

GO!

Topic: Different forms of a quadratic function.

19. Given the quadratic function: $y=(x-4)(x+5)$, re-write the function in standard form.

20. Given the quadratic function: $y=x^2-2x-48$ re-write the function in factored form.

21. Given the quadratic function: $y=x^2+16x+71$ re-write the function in vertex form.

22. Given the quadratic function: $y=(x+2)^2-4$ re-write the function in standard form.

23. Given the quadratic function: $y=(x-3)^2-4$ re-write the function in factored form.

24. Given the quadratic function: $y=(x-5)(x-3)$ re-write the function in vertex form.

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6.8 Extraneous Events

A Developing Understanding Task

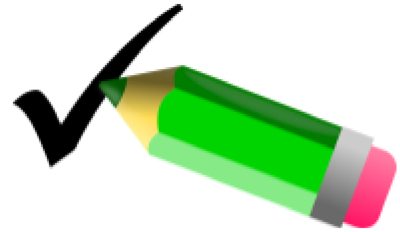
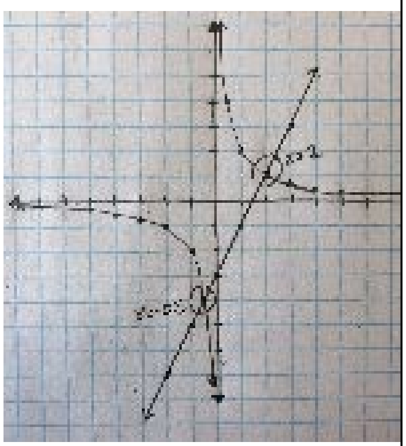


Image Source: https://pixabay.com/en/check-mark-check-tick-tick-off-35600/

Jasmine, Kevin and Francisco were working in a group that was asked to solve the equation $\frac{2}{x} = 2x - 3$. Jasmine used a graph to solve the equation, Kevin used algebraic methods to solve, and Francisco combined the two methods together to solve the problem. Below are pictures of their work. Explain the steps that each student took to get their answer.

<table>
<tr><th>Jasmine's Method</th><th>Kevin's Method</th><th>Francisco's Method</th></tr>
<tr>
<td>
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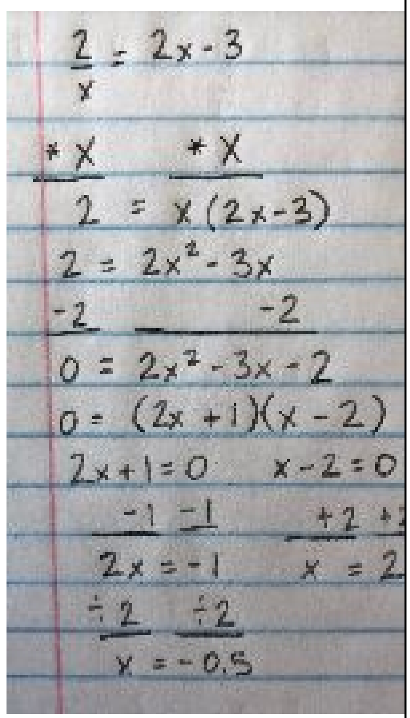
<td>
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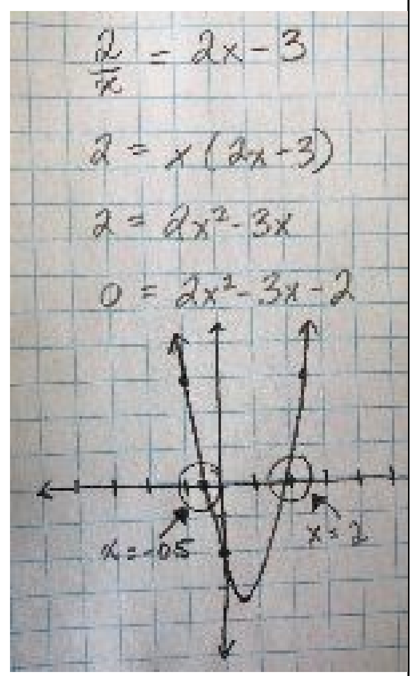
<td>
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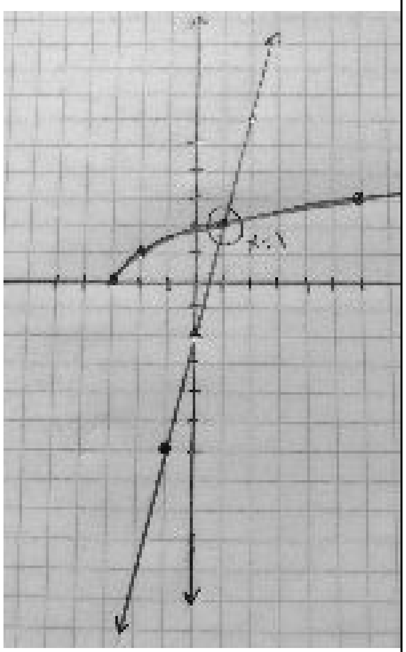
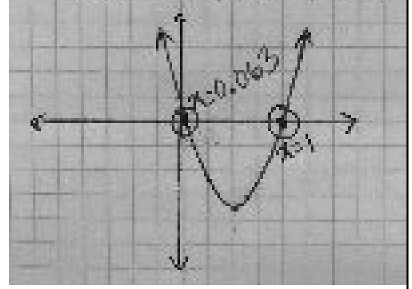
</table>

The next equation that the group was given to solve was $4x - 2 = \sqrt{x + 3}$. Each student still had their own preferred way of solving so Jasmine used a graph, Kevin solved with algebra, and Francisco solved with a combination of algebra and graphing. However, this time Jasmine's answer was slightly different. Explain each step of each student's work.

Jasmine's Method	**Kevin's Method**	**Francisco's Method**
 	$4x-2=\sqrt{x+3}$ $(4x-2)^2=(\sqrt{x+3})^2$ $(4x-2)(4x-2)=x+3$ $16x^2-8x-8x+4=x+3$ $16x^2-16x+4=x+3$ $-x \quad -3 \quad -x \quad -3$ $16x^2-17x+1=0$ $(16x-1)(x-1)=0$ $16x-1=0 \qquad x-1=0$ $+1 \quad +1 \qquad x=1$ $16x=1$ $\div 16 \quad \div 16$ $x=1/16=0.0625$	$(4x-2)^2=(\sqrt{x+3})^2$ $(4x-2)(4x-2)=x+3$ $16x^2-8x-8x+4=x+$ $16x^2-16x+4=x+3$ $16x^2-17x+1=0$ 

Why is Jasmine's answer different? Is Jasmine's answer correct or are Kevin and Francisco's answers correct? Explain your reasoning.

Apply the methods that Jasmine, Kevin, and Francisco used to solve the following problems. Identify any extraneous solutions and justify why they are extraneous.

1. $0.5x + 1.5 = \sqrt{2x + 3}$
2. $\sqrt{x + 7} = x + 5$

3. $\sqrt{3x+1} = x+1$

4. $3x - 2 = \sqrt{2x-5}$

READY, SET, GO! Name ____________________ Period ______ Date __________

READY

Topic: Determine if a given value or point is a solution to an equation.

For each of the equations below, determine if x = -2 is a solution. Justify your answer.

1. $5 = \sqrt{x+6} + 3$

2. $x^2 - 9x - 22 = 0$

3. $6x - 10 = 5x + 15$

4. $4 = \frac{x^2}{x+2}$

5. $\sqrt{x-7} = -3$

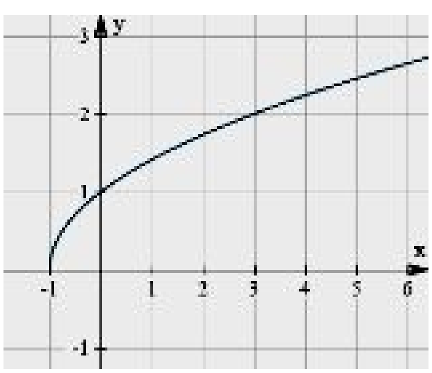
6. $x^2 + 5 = 9$

For each of the functions below, determine if the point (3, 4) is a solution. Justify your answer.

7. $y = (x-1)(x-5)$

8. $y = \frac{1}{2}x + \frac{5}{2}$

9. $y = \sqrt{x+5}$

10. 

11.

x	g(x)
-1	-6
0	-5
1	-3
2	0

12. $y = 2^x$

SET

Topic: Solve square root and inverse variation equations.

Solve each equation. Remember to look for extraneous solutions.

13. $-x = \frac{1}{x}$

14. $\sqrt{-10 + 7x} = x$

15. $\sqrt{2x - 7} = x - 3$

16. $\frac{2}{x} = x + 1$

17. $\sqrt{9x - 5} = x + 1$

18. $\sqrt{x - 1} = 3$

GO!

Topic: Solve problems involving direct and inverse variation relationships.

19. If y varies inversely as x, and y = 32 when x = 3, find x when y = 15.

20. If y varies directly as x, and y = 8 when x = 2, find y when x = 5.

21. The frequency of vibration of a guitar string varies inversely with the length of the guitar string. Suppose a guitar string is 0.65 meters long, and vibrates 4.3 times per second. At what frequency would a string that is 0.5 meters long vibrate?

22. The amount of calories a person burns varies directly with the amount of miles that they run. Sonya ran 2 miles on a treadmill. The display reported that she burned 220 calories. She wants to treat herself with a hot fudge sundae after her workout. A hot fudge sundae has 380 calories. How far does Sonya have to run to burn off that many calories?

23. The current in a simple electrical circuit is inversely proportional to the resistance. If the current is 80 amps when the resistance is 50 ohms, find the current when the resistance is 22 ohms.

24. The amount of money you earn varies directly with amount of time that you work. If you work 6.5 hours, you will make $66.95. If you made $97.85, how many hours did you work?

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6.9 Seasonal Systems

A Develop Understanding Task

Image Source: https://pixabay.com/en/carnival-summer-concession-stand-2456911/

Family Kingdom Amusement Park is planning to hire more support staff to work during the summer months. They have budgeted $12,000 per week to go towards the pay for these seasonal workers.

1. If Family Kingdom Amusement Park hired 60 people, how much would each person make per week? Justify your answer.

2. Create an equation, table, and graph that models the relationship between the number of people that Family Amusement Park can hire and the weekly pay per person.

From previous years the owners at Family Kingdom Amusement Park have realized that the more money they advertise for pay per week, the more applicants they will receive. Two years ago the owners advertised this seasonal part-time work as having a weekly salary of $205 and they received 40 job applications. Last year they advertised a weekly salary of $220 and they received 50 job applications.

3. Create an equation, table and graph to model the relationship between the weekly salary advertised and the number of job applications the owners will receive.

4. How much should the owners at Family Kingdom Amusement Park advertise as the weekly pay if they want for the number of job applications to be the same as the number of people they can hire? If the owners choose to advertise at this amount of pay per week, how many applications will they receive? Justify your answer.

5. The owners at Family Kingdom Amusement Park want to have more applicants than people that they can actually hire, so that they can choose the best workers from the pool of applicants. Write an inequality that can be solved to find the weekly salary that the owners should advertise to make sure that they receive more applications than people that they can hire. Then, use tables and graphs to solve the inequality so that the owners know how much they can advertise to ensure that they receive more applicants than people they can actually hire.

6. Make a recommendation to the owners about how much they should advertise as the weekly salary. Explain why your recommendation is the owner's best option. Make sure that you let them know how many job applications they should expect, how many people they can hire, and the total cost per week for salaries of all seasonal workers.

7. You just solved a system of equations that involved a linear equation and an inverse variation equation, both graphically and using tables. How would you solve the following system?

$$y = \sqrt{x}$$

$$x - y = 6$$

READY, SET, GO! Name ____________________ Period ______ Date __________

READY

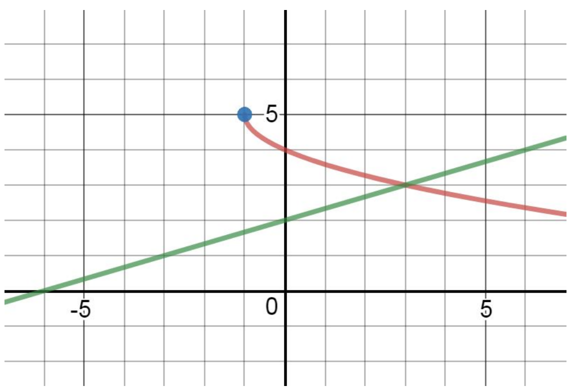
Topic: Solving Systems of Equations.

For each of the systems below, determine whether the ordered pair (3, 3) is a solution.

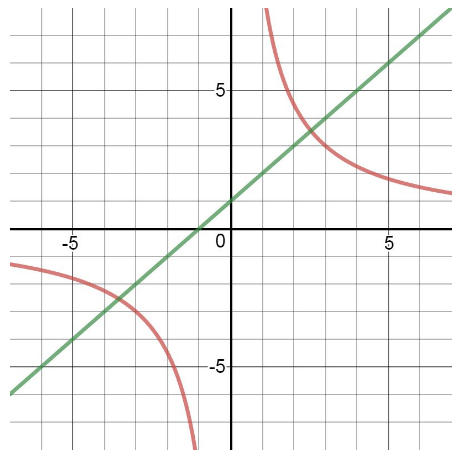
1. $y = \sqrt{x+6}$ and $y = -2x + 9$

2. $y = \frac{6}{x} + 1$ and $y = \frac{2}{3}x + 1$

3.



4.



5.

x	f(x)
0	0
1	3
4	6
9	9

x	g(x)
-4	-8
-3	-6
-2	-4
-1	-2

6. $h(x) = 2\sqrt{x+1} - 1$ and

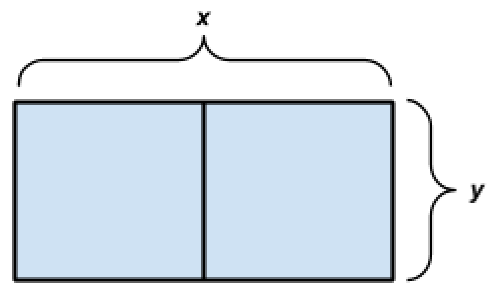
x	f(x)
-8	-8
-6	-6
-4	-4
0	0

SET

Topic: Solve systems of equations involving square root and inverse variation relationships.

Write a system of equations or inequalities to solve each problem.

7. You have 36 yards of fencing to build the enclosure pictured to the right. Some of this fencing is to be used to build an internal divider. If you'd like to enclose 54 square yards, what are the dimensions of the enclosure?



8. $y = \sqrt{2x}$

   $x - y = 4$

9. The sum of two numbers is 10 and their product is 24. Find the numbers.

10. $y = \sqrt{3x - 2}$

    $y = \sqrt{x + 2}$

GO!

Topic: Solve quadratic equations.

11. $x^2 - 5x + 10 = 0$

12. $-2x^2 + 4x + 6 = 15$

13. $x^2 + 4x + 2 = 0$

14. $5x^2 - 5x + 2 = 3x^2 - 3x$

15. $3x - 2 = 5x^2$

16. $7 - 8x^2 = 6x + 16$

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6.10 Algebra and Functions

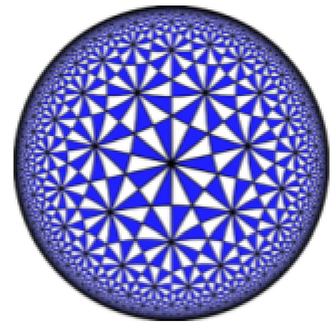


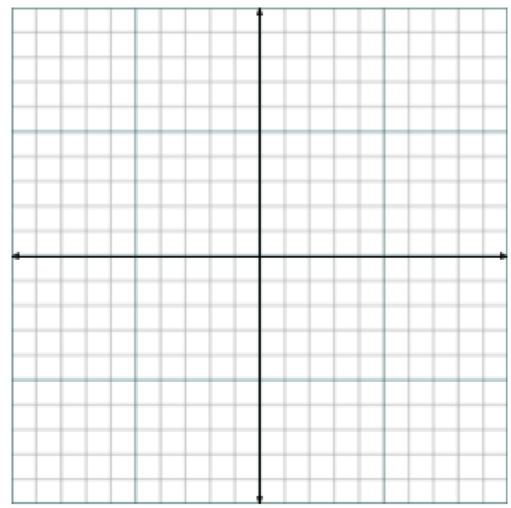
Image Source: https://en.wikipedia.org/wiki/Geometry

A Practice Understanding Task

Consider the equation $x - 1 = \sqrt{2(x+3)}$.
This equation is made up of a linear function, $f(x) = x - 1$,
and a square root function, $g(x) = \sqrt{2(x+3)}$.

1. Find the domain and range of each function and then graph both functions on the same coordinate plane.

$f(x) = x - 1$	$g(x) = \sqrt{2(x+3)}$
Domain:	Domain:
Range:	Range:



2. Algebraically solve the equation $x - 1 = \sqrt{2(x+3)}$.

3. Are the solutions elements of the domain in both the linear and square root functions?

4. Substitute the solutions back into the functions to see if the outputs are elements in the range of each function.

$f(x) = x - 1$	$g(x) = \sqrt{2x + 6}$

5. Are all of the outputs elements in the range of both functions? Explain why or why not.

6. If the square root function was reflected over the x-axis, so that it's equation was $h(x) = -\sqrt{2x + 6}$, what would be the solution to the equation $x - 1 = -\sqrt{2(x + 3)}$?

Solve the following equations algebraically. Be sure to check for extraneous solutions.

7) $\frac{2}{x} = 3x + 5$ 8) $2x + 3 = \frac{5}{x}$ 9) $\frac{6}{x} = 9 - 3x$

10) $4x - 7 = \frac{2}{x}$

11) $\frac{3}{5}x + 5 = \sqrt{2x - 1} + 5$

12) $4x - 2 = \sqrt{x + 3}$

13) $\sqrt{4x} = -2x + 4$

14) $0.5x - 8 = 2 - 2\sqrt{x + 1}$

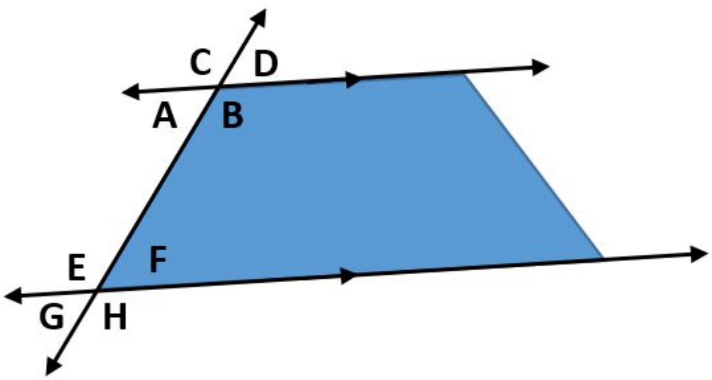
15) $x - 7 = -\sqrt{4x - 8}$

READY, SET, GO! Name ________________ Period ______ Date __________

READY

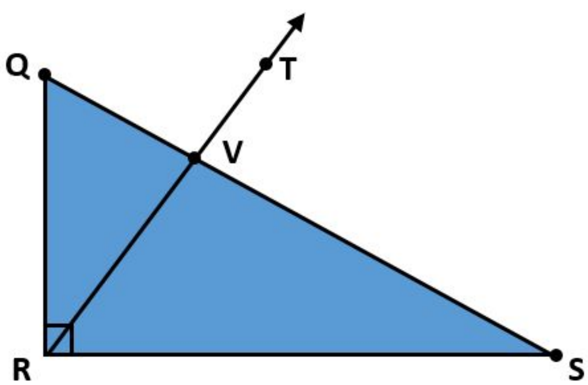
Topic: Angle Pairs and Geometry Vocabulary.

For questions 1-5 list a pair of angles for each of the following vocabulary words:



1. Alternate interior angles _______
2. Corresponding angles _______
3. Alternate exterior angles _______
4. Supplementary angles _______
5. Vertical angles _______
6. Identify the transversal line.
7. Identify the parallel lines.

Use the triangle below to answer questions 8-11.



8. List a pair of complementary angles _________
9. List a pair of supplementary angles _________
10. List a pair of vertical angles _________
11. List a pair of perpendicular line segments

SET

Topic: Solving Equations

Solve each equation algebraically. Remember to check for extraneous solutions.

12. $x - 7 = \sqrt{x+5}$

13. $\sqrt{8-x} = 2x - 1$

14. $\sqrt{x+10} + 2 = 0$

15. $\sqrt{-16+10x} = x$

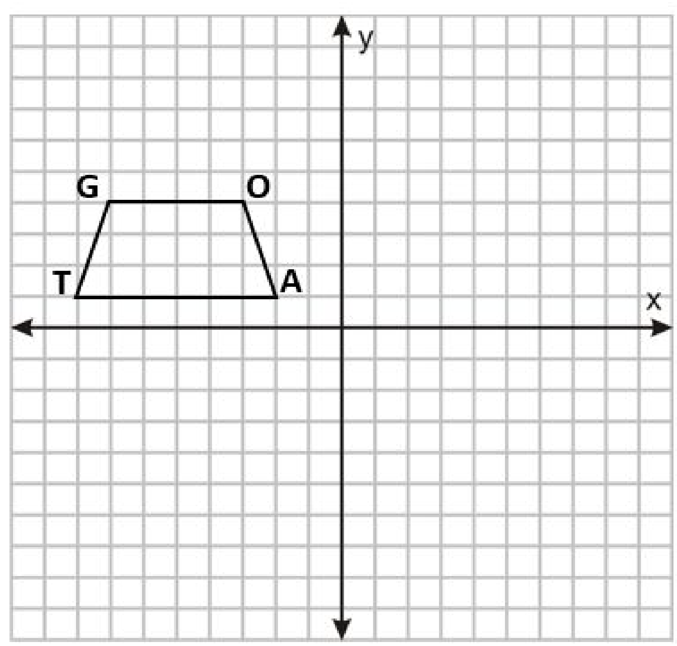
16. $x + 3 = \sqrt{4x+8}$

17. $\sqrt{7x-54} - x = -6$

GO!

Topic: Transformations of Geometric Shapes.

Transform the trapezoid GOAT based on the directions in each problem. Write the new ordered pairs of each transformation below.



18. g(x, y) $\rightarrow$ (y, -x)

19. g(x, y) $\rightarrow$ (x + 10, y - 7)

20. g(x, y) $\rightarrow$ (x, -y)

Transform the parallelogram FROG based on the directions in each problem. Write the new ordered pairs of each transformation below.

21. $f(x, y) \rightarrow (-x, -y)$

22. $f(x, y) \rightarrow (-x, y)$

23. $f(x, y) \rightarrow (x + 3, y - 5)$