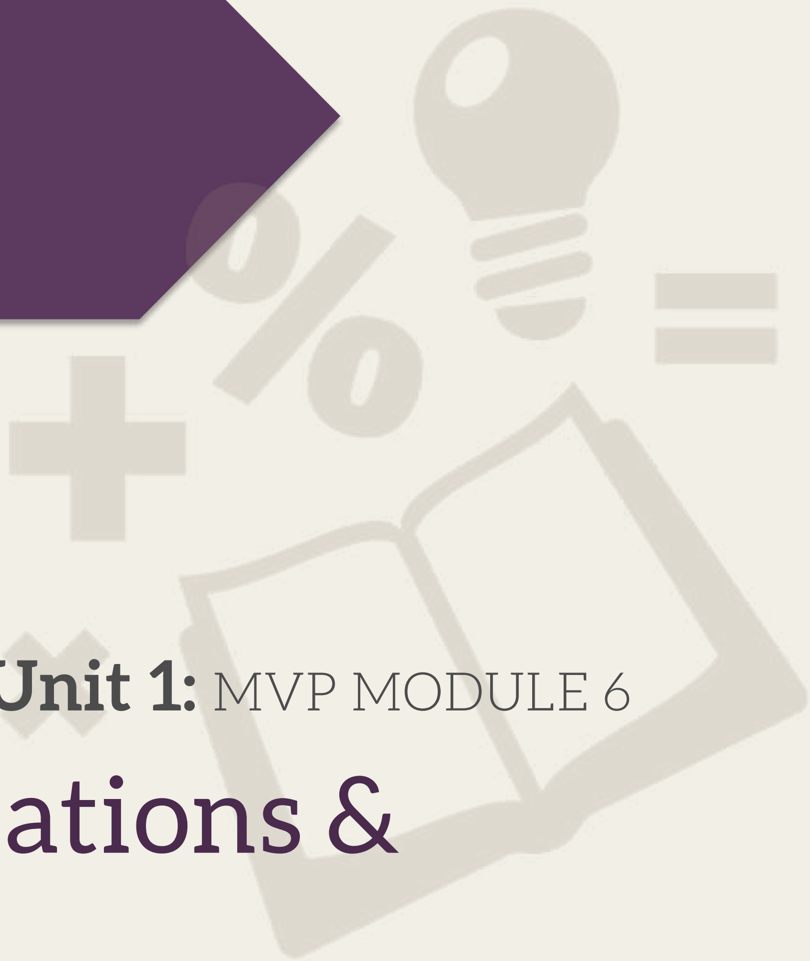
WAKE COUNTY
PUBLIC SCHOOL SYSTEM

mathematics
vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach



WCPSS Math 2 Unit 1: MVP MODULE 6

Transformations & Symmetry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

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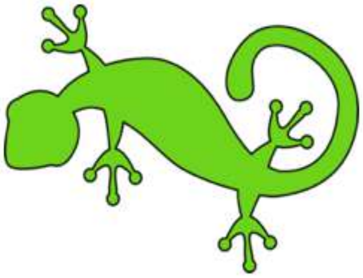
MODULE 6 - TABLE OF CONTENTS

TRANSFORMATIONS AND SYMMETRY

6. 1 Leaping Lizards!



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A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns), or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid, and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

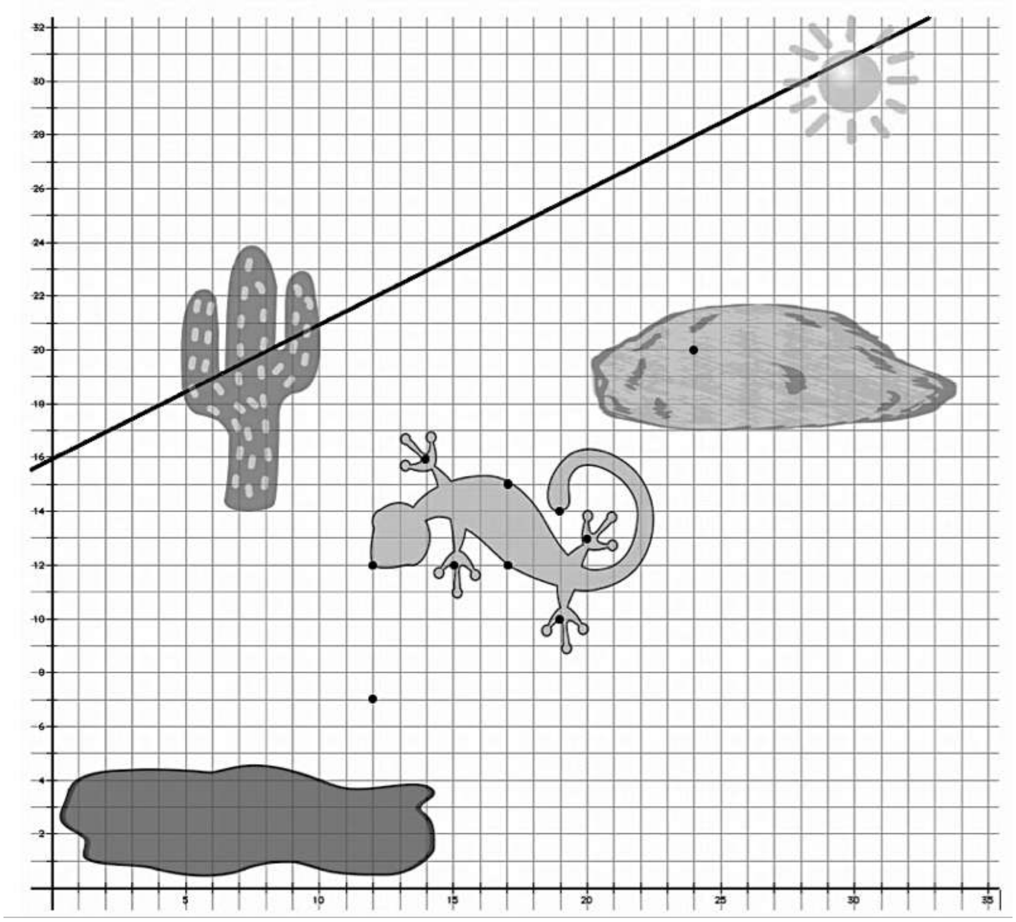
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

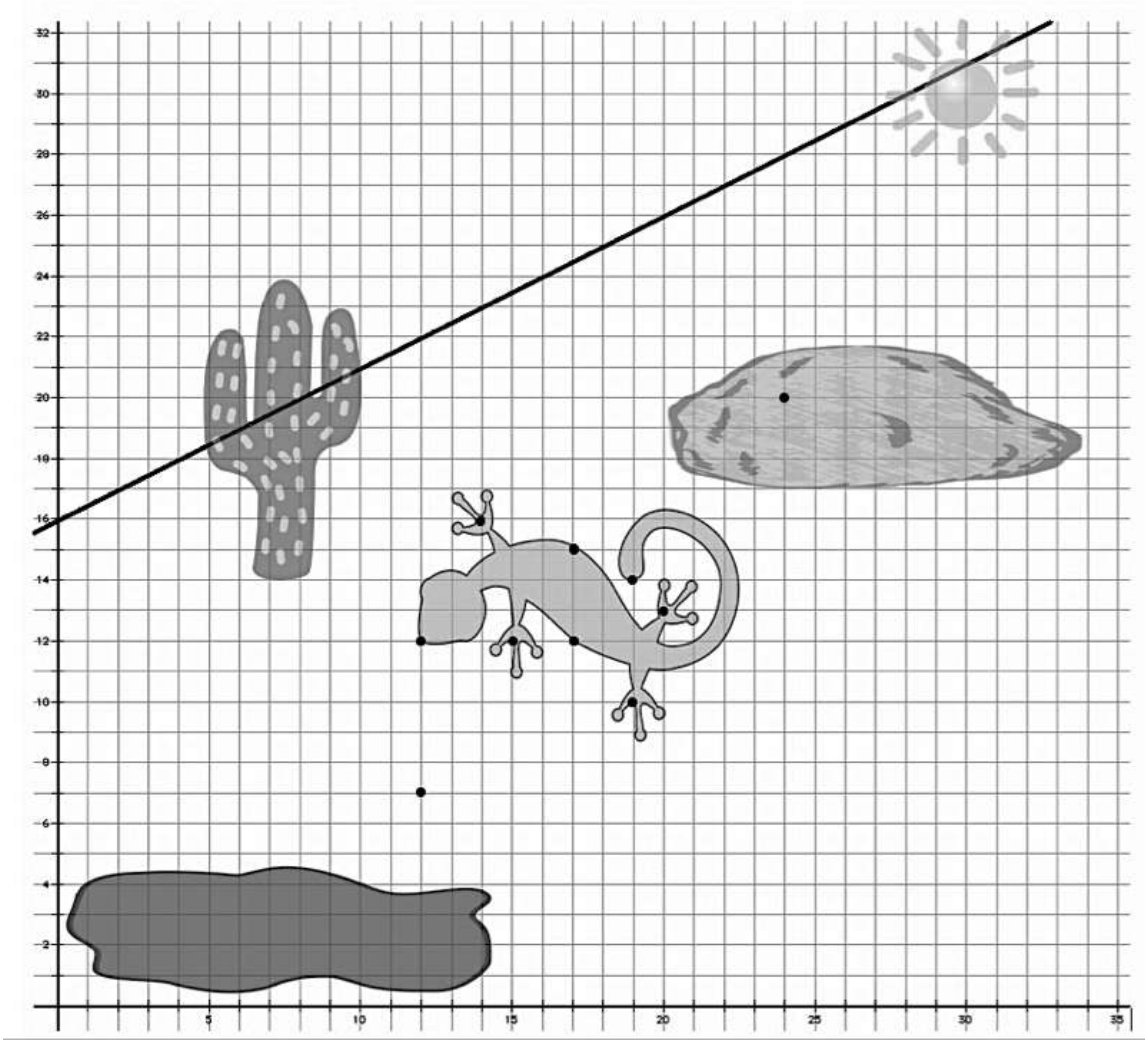
Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.

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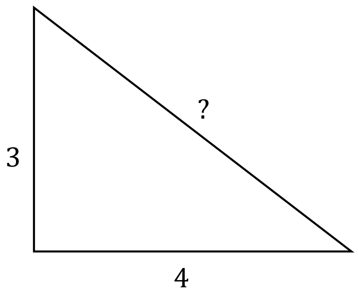
READY, SET, GO! Name Period Date

READY

Topic: Pythagorean Theorem

For each of the following right triangles determine the measure of the missing side. Leave the measures in exact form if irrational.

1.



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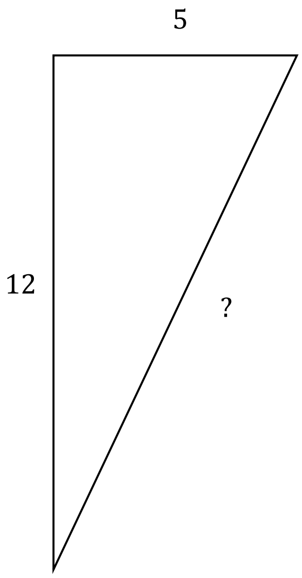

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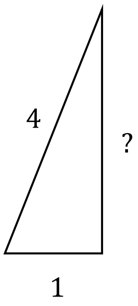

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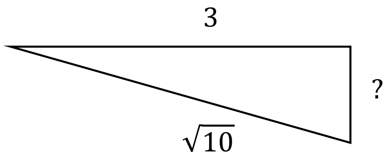

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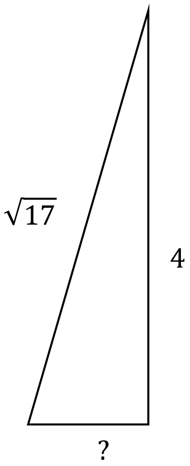

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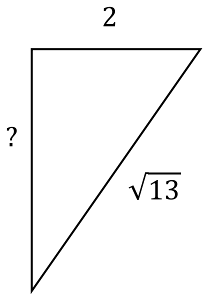


SET

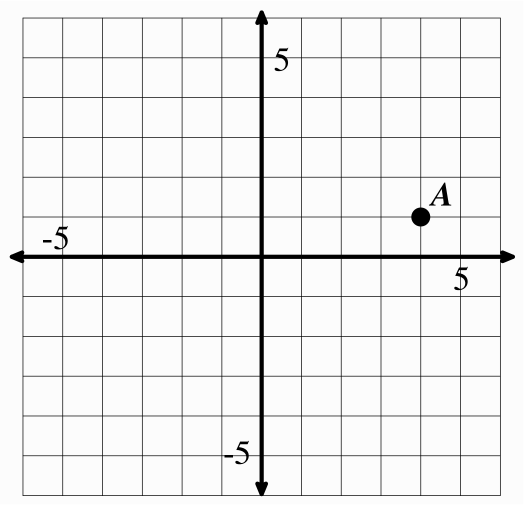
Topic: Transformations.

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A'

b. Reflect point A over x-axis, label as A''

c. Apply the rule $(x - 2, y - 5)$, to point A and label A'''

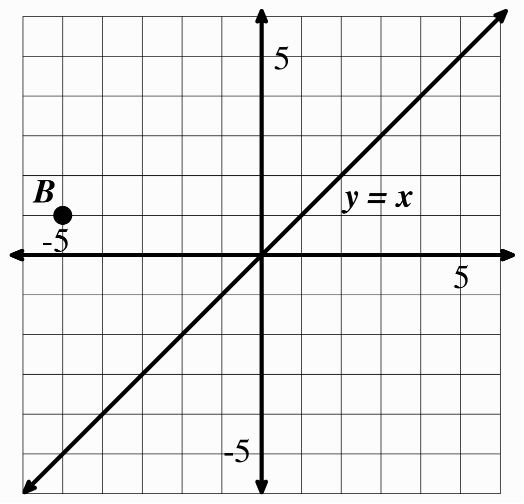


8a. Reflect point B over the line $y = x$, label as B'

b. Rotate point B 180° about the origin, label as B''

c. Translate point B the point up 3 and right 7 units,

label as B'''

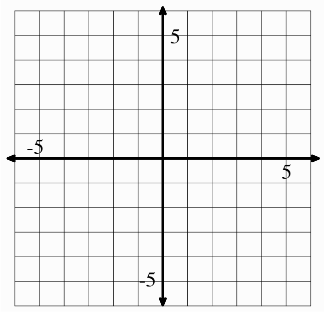


GO

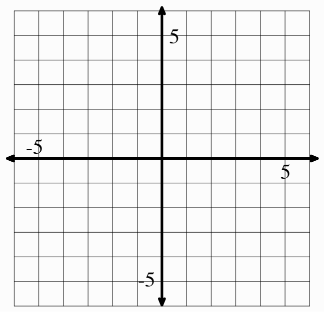
Topic: Graphing linear equations.

Graph each function on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $f(x) = 2x - 3$

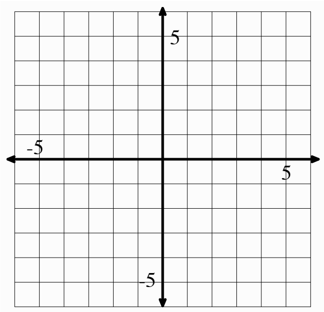


10. $g(x) = -2x - 3$

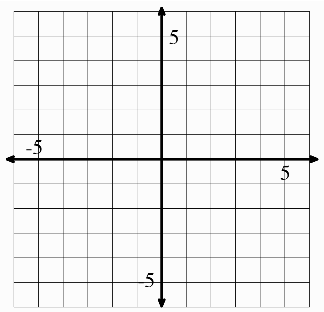


11. What similarities and differences are there between the functions *f(x)* and *g(x)*?

12. $h(x) = \frac{2}{3}x + 1$

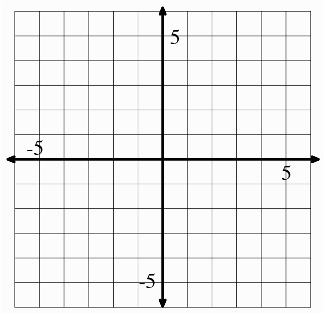


13. $k(x) = -\frac{3}{2}x + 1$

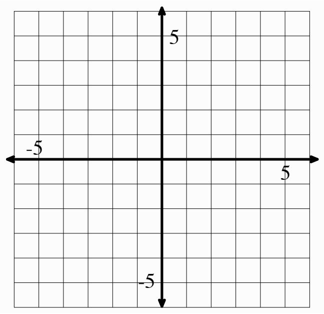


14. What similarities and differences are there between the equations *h(x)* and *k(x)*?

15. $a(x) = x + 1$



16. $b(x) = x - 3$



17. What similarities and differences are there between the equations *a(x)* and *b(x)*?

6. 3 Leap Frog

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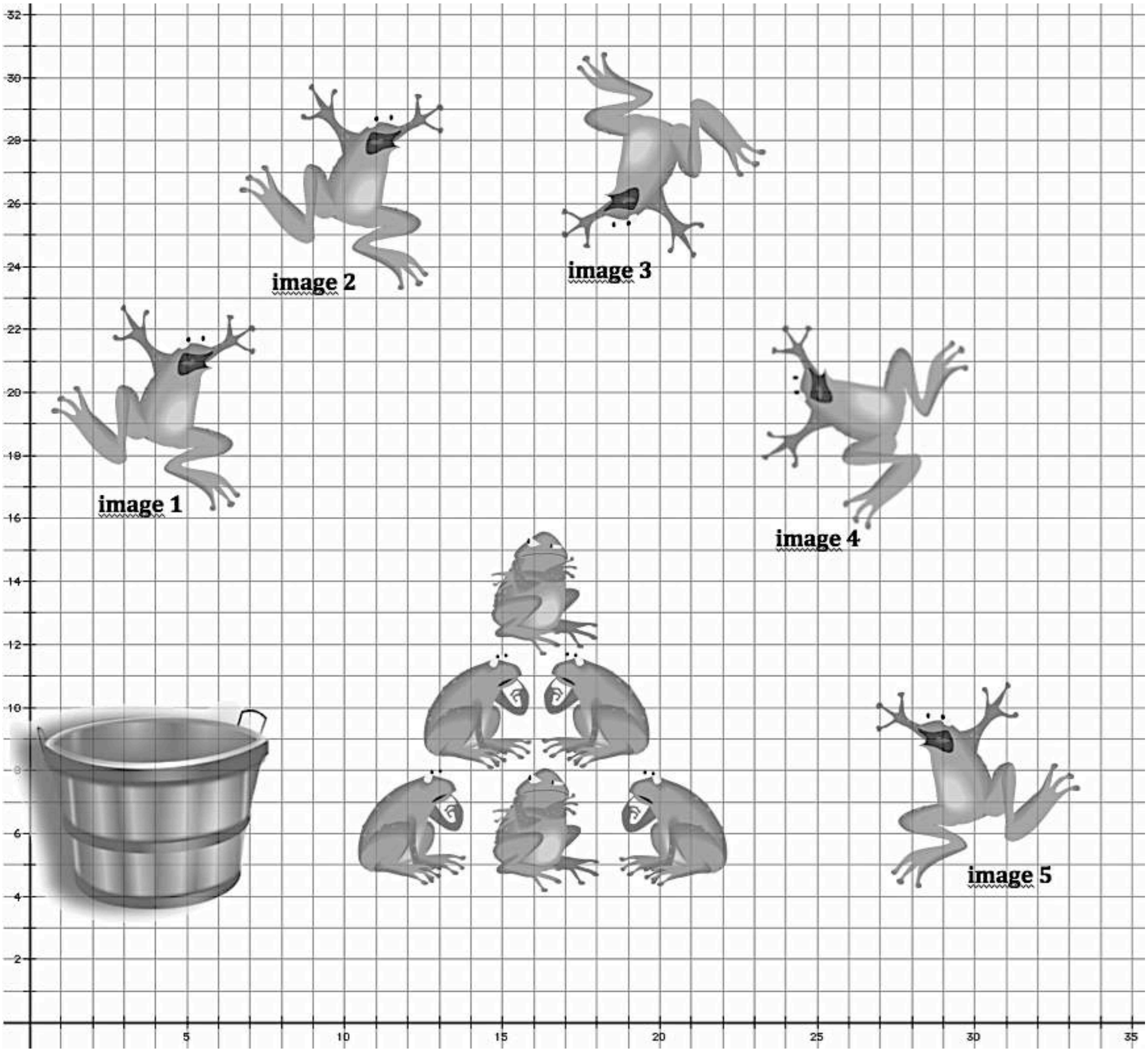
A Solidify Understanding Task

Josh is animating a scene in which a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.
- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.
- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.
- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	



images this page:
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CC0 http://openclipart.org/detail/33985/architetto
CC0 http://openclipart.org/detail/170806/hatalar205

READY, SET, GO! Name Period Date

READY

Topic: Rotations and Reflections of figures.

In each problem there will be a pre-image and several images based on the give pre-image. Determine which of the images are rotations of the given pre-image and which of them are reflections of the pre-image. If an image appears to be created as the result of a rotation and a reflection then state both. (Compare all images to the pre-image.)

1.


Pre-Image

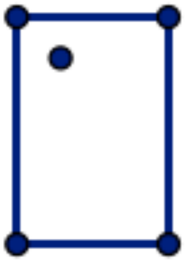


Image A

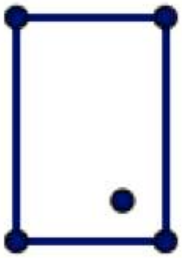


Image B

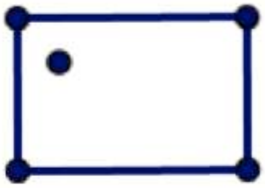


Image C

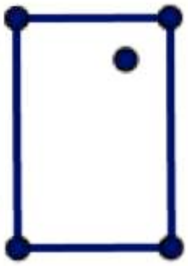


Image D

2.


Pre-Image


Image A

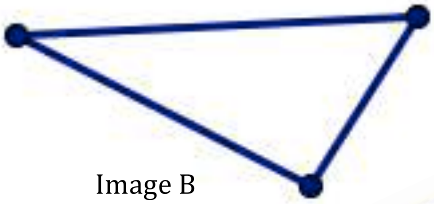


Image B

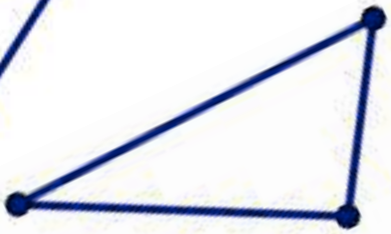


Image D

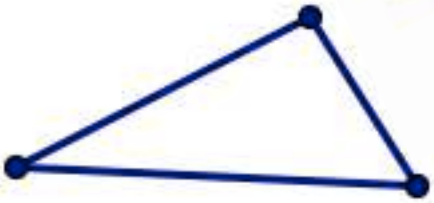


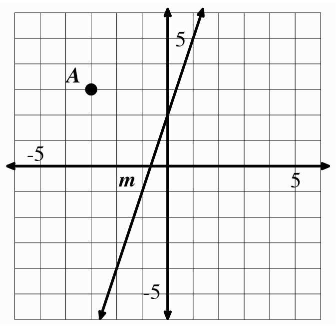
Image C

SET

Topic: Reflecting and rotating points.

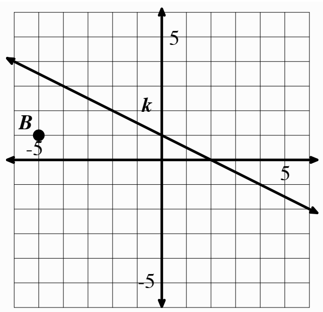
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection. (Hint: points reflect along paths perpendicular to the line of reflection. Use perpendicular slope!)

3.



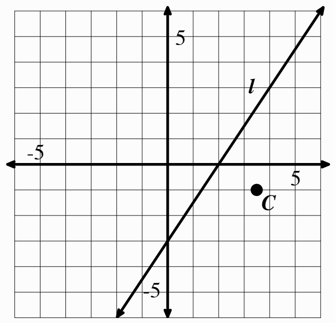
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



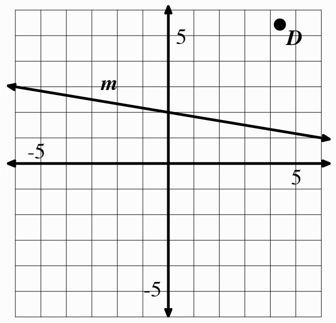
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

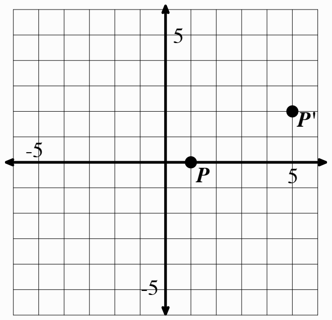
6.



Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.



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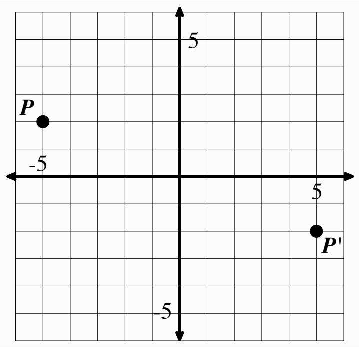
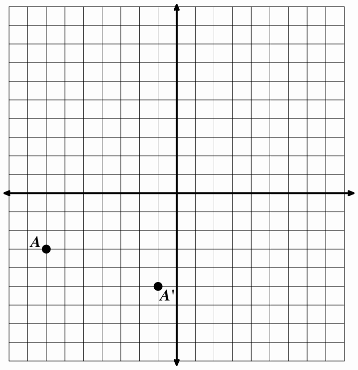


For each pair of point, A and A' draw in the line of reflection that would need to be used to reflect A onto A'. Then find the equation of the line of reflection.

9.



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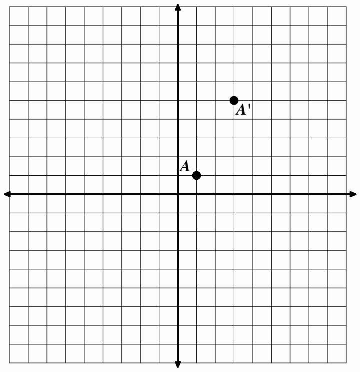


GO

Topic: Slopes of parallel and perpendicular lines and finding slope and distance between two points.

For each linear equation <u>write the slope</u> of a line parallel to the given line.

11. $y = -3x + 5$
12. $y = 7x - 3$
13. $3x - 2y = 8$

For **each linear equation <u>write the slope</u> of a line perpendicular to the given line.**

14. $y = -\frac{2}{7}x + 5$
15. $y = \frac{1}{5}x - 4$
16. $3x + 5y = -15$

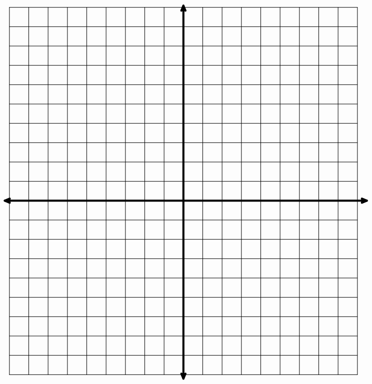
Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)

a. Slope: b. Distance:

18. (-7, 5) (-2, -7)

a. Slope: b. Distance:



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6. 4 Leap Year

A Practice Understanding Task

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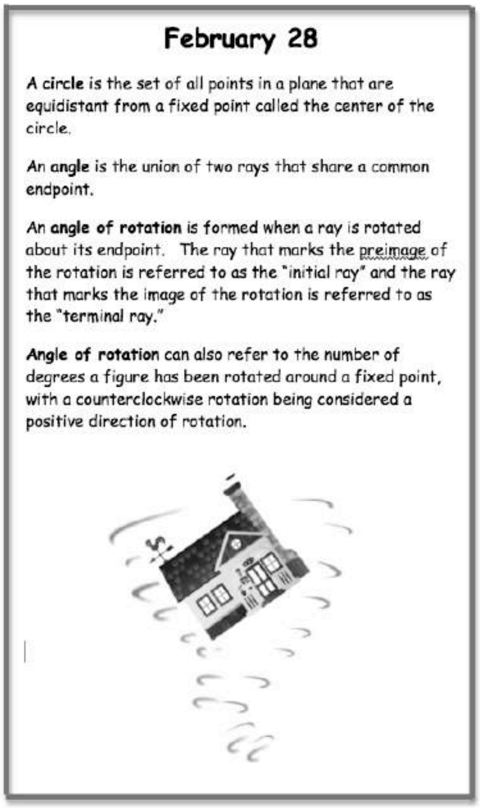
Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages." So now Carlos and Clarita have the dilemma of needing to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.



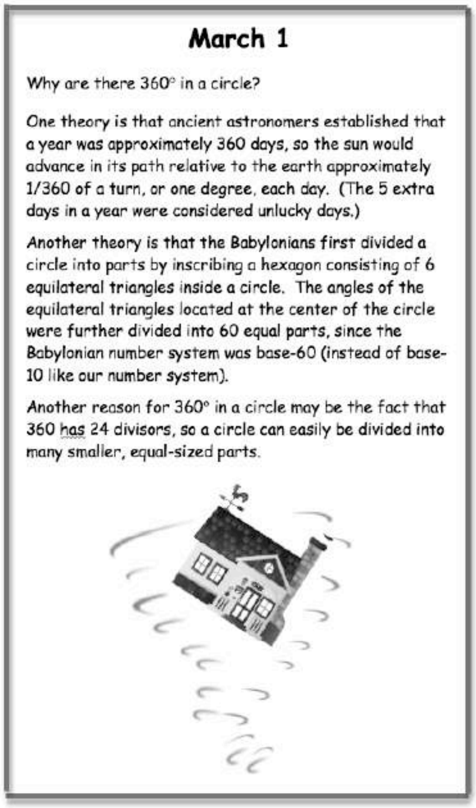
February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

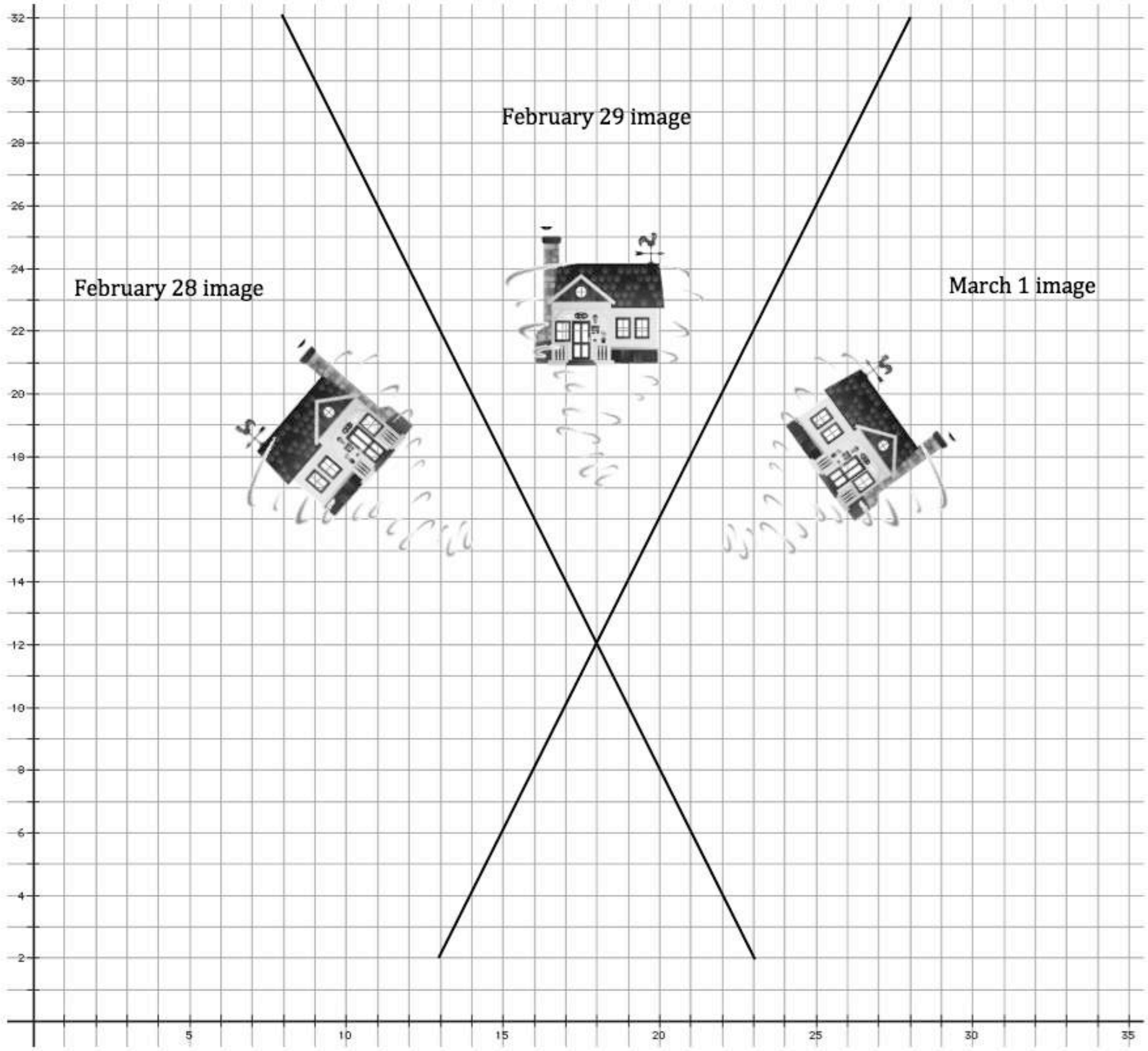
Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures within the shape. Revise your definitions so that they also use these words or phrases.

Part 2

In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?
- What convinces you that the March 1 image is a reflection of the February 29 image about the given line of reflection?
- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



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www.clker.com/clipart.tornado-gray

6.4

READY, SET, GO! Name Period Date

READY

Topic: Defining polygons and their attributes

For each of the geometric words below write a definition of the object that addresses the essential elements.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

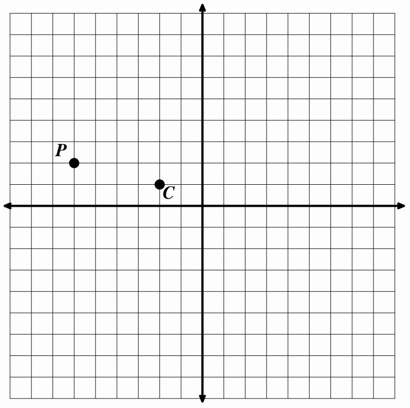
5. Rhombus:

6. Trapezoid:

SET

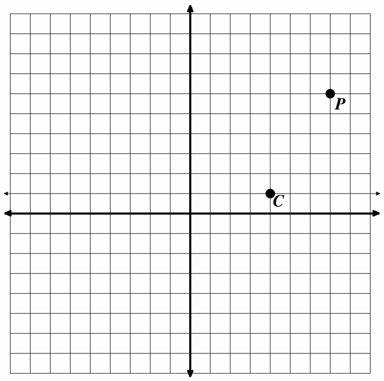
Topic: Reflections and rotations, composing reflections to create a rotation.

7.



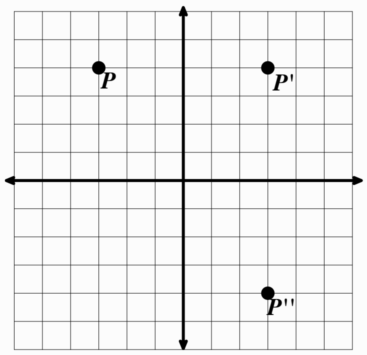
Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P***'. With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P***''.

8.



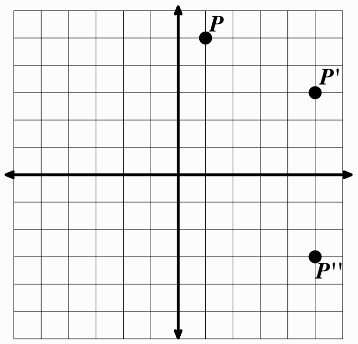
Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90⁰. Label the image ***P'***.
With point ***C*** as a center of rotation also rotate point ***P*** 180⁰. Label this image ***P''***.

9.



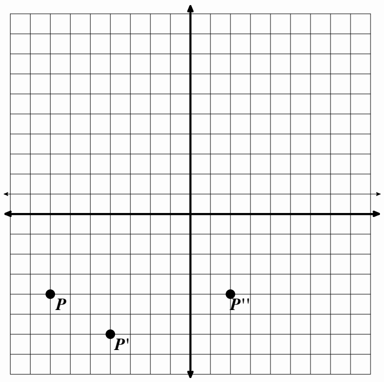
a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?
b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?
c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?
b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?
c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?
b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?
c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

GO

Topic: Rotations about the origin.

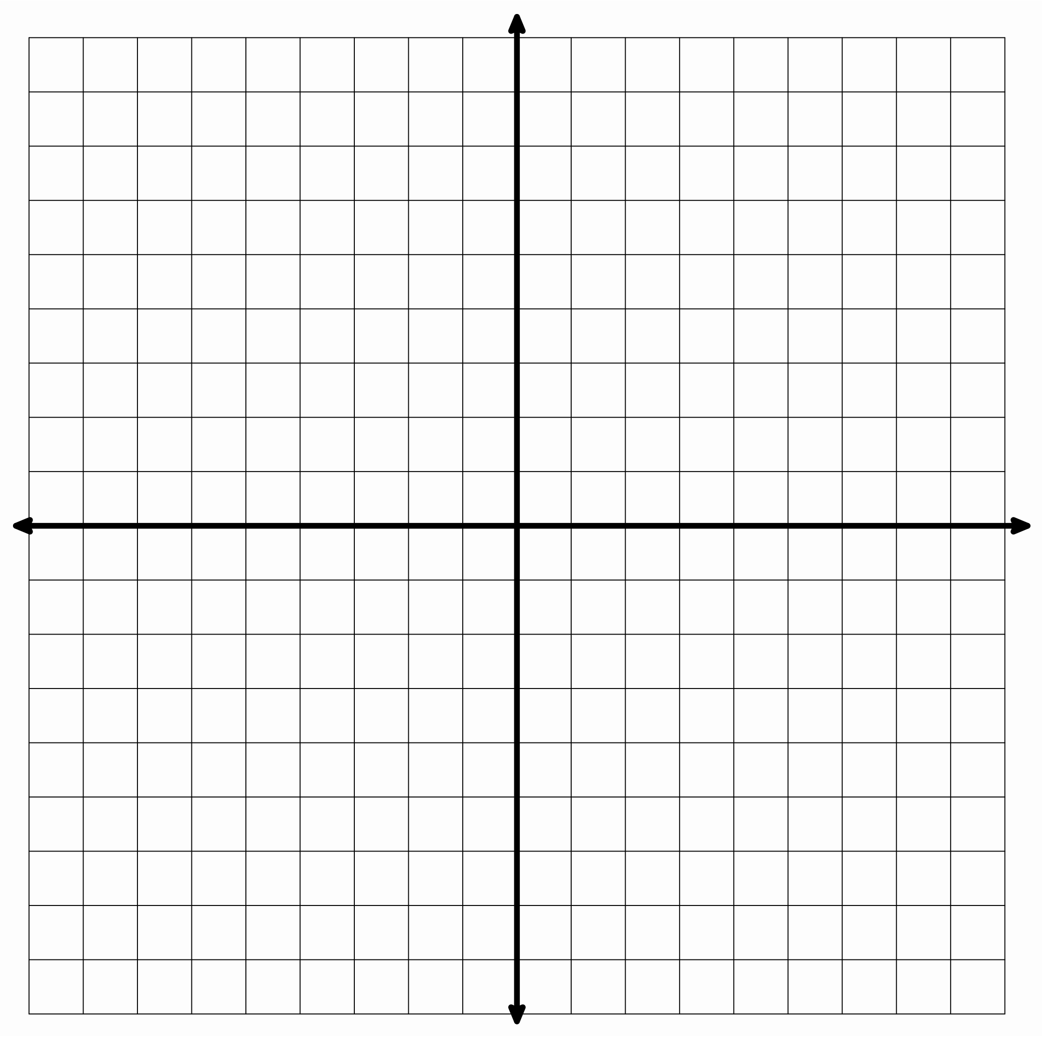
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
Coordinates for Point ***A'*** (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
Coordinates for Point ***B'*** (___ , ___)

14. Point ***C*** (-7, 3) rotate 180^0
Coordinates for Point ***C'*** (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
Coordinates for Point ***D'*** (___ , ___)



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6. 5 Symmetries of Quadrilaterals

A Develop Understanding Task

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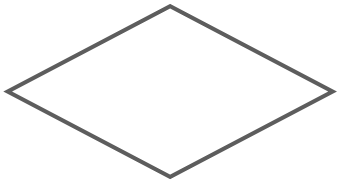
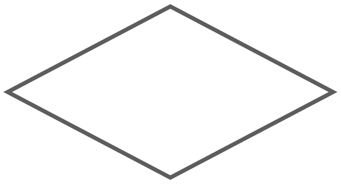
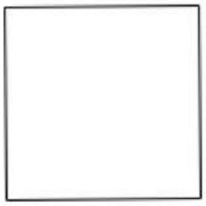
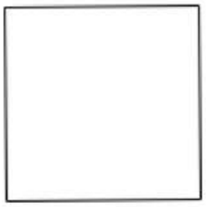
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. Some quadrilaterals are symmetric about their diagonals. Some are symmetric about other lines. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

For each of the following quadrilaterals you are going to try to answer the question, "Is it possible to reflect or rotate this quadrilateral onto itself?" As you experiment with each quadrilateral, record your findings in the following chart. Be as specific as possible with your descriptions.

Defining features of the quadrilateral	**Lines of symmetry that reflect the quadrilateral onto itself**	**Center and angles of rotation that carry the quadrilateral onto itself**
A **rectangle** is a quadrilateral that contains four right angles.		
A **parallelogram** is a quadrilateral in which opposite sides are parallel.		

A **rhombus** is a quadrilateral in which all sides are congruent.		
A **square** is both a rectangle and a rhombus.		

A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find:

- any lines of symmetry, or
- any centers of rotational symmetry,

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

READY, SET, GO! Name Period Date

READY

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

SET

Topic: Kites, Lines of symmetry and diagonals.

3. One quadrilateral with special attributes is a kite. Find the geometric definition of a kite and write it below along with a sketch. (You can do this fairly quickly by doing a search online.)

4. Draw a kite and draw all of the lines of reflective symmetry and all of the diagonals.

Lines of Reflective Symmetry **Diagonals**

5. List all of the rotational symmetry for a kite.

6. Are lines of symmetry also diagonals in a polygon? Explain.

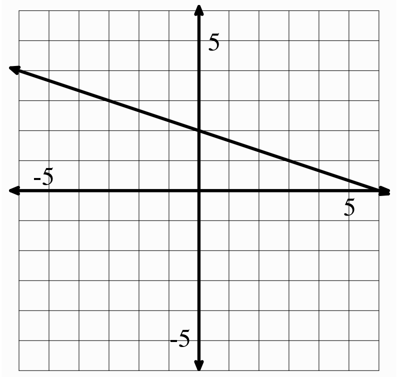
6. Are all diagonals also lines of symmetry in a polygon? Explain.

7. Which quadrilaterals have diagonals that are not lines of symmetry? Name some and draw them.

8. Do parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

GO

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated y-intercept.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated y-intercept.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1.$</td><td>a. Parallel line through point (0, -7):</td><td>b. Perpendicular to the line line through point (0 , -7):</td></tr>
<tr><td>10. Table of a line:
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>3</td><td>-8</td></tr>
<tr><td>4</td><td>-10</td></tr>
<tr><td>5</td><td>-12</td></tr>
<tr><td>6</td><td>-14</td></tr>
</table></td><td>a. Parallel line through point (0 , 8):</td><td>b. Perpendicular to the line through point (0 , 8):</td></tr>
<tr><td>11. Graph of a line:

</td><td>a. Parallel line through point (0 , -9):</td><td>b. Perpendicular to the line through point (0 , -9):</td></tr>
</table>

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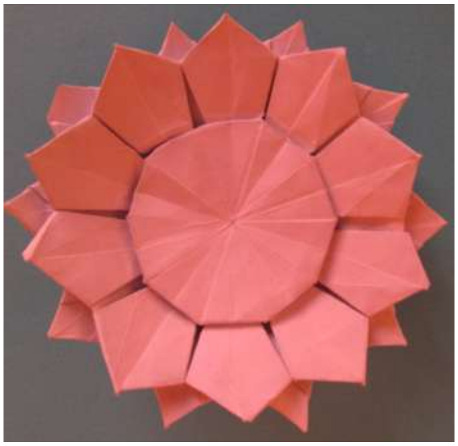
6.6 Symmetries of Regular Polygons

A Solidify Understanding Task



CC BY Jorge Jaramillo
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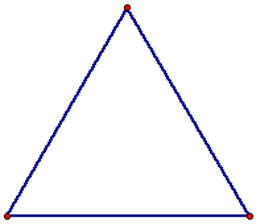
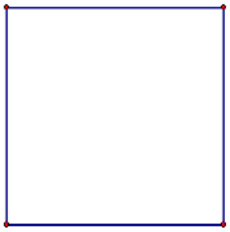
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

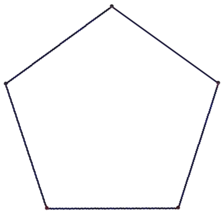
1. An equilateral triangle



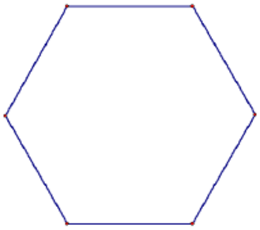
2. A square



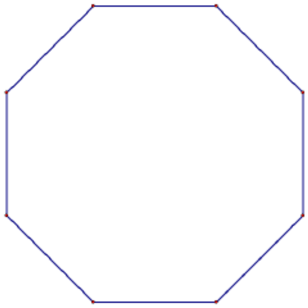
3. A regular pentagon



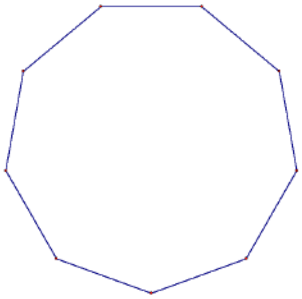
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

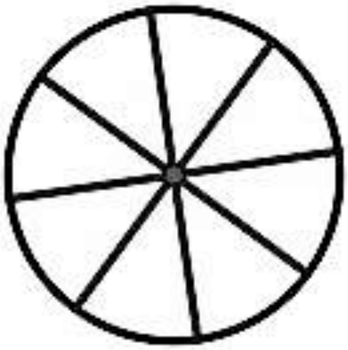
What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

READY, SET, GO! Name Period Date

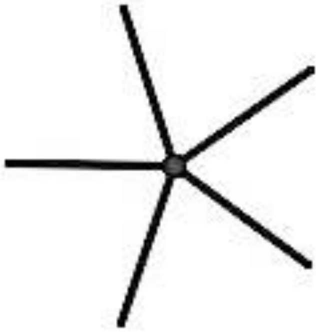
READY

Topic: Rotational symmetry, connected to fractions of a turn and degrees.

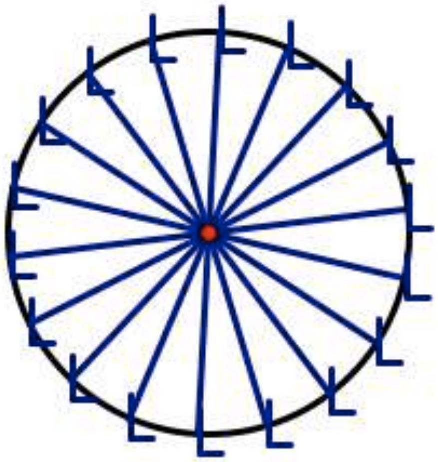
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



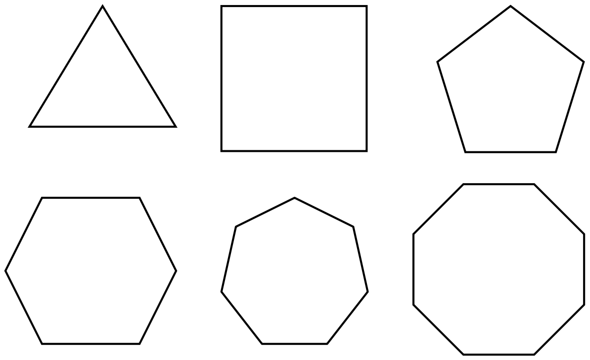
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



SET

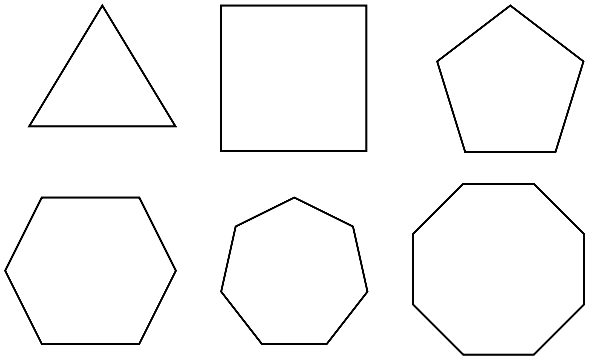
Topic: Finding angles of rotational symmetry for regular polygons, lines of symmetry and diagonals

4. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.



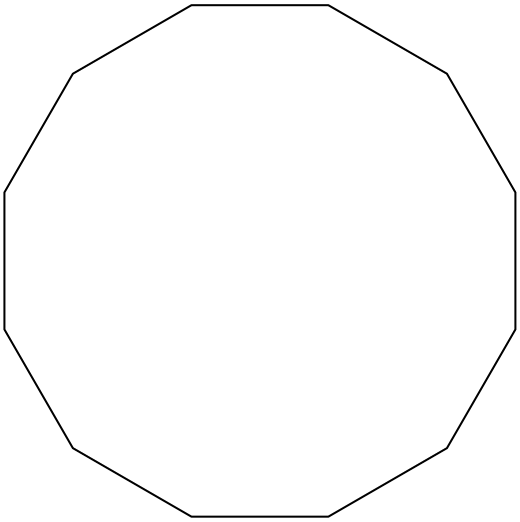
Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	

5. Draw all of the diagonals in each regular polygon. Fill in the table and find a pattern, is it linear, exponential or neither? How do you know? Attempt to find an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

6. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



7. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

8. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

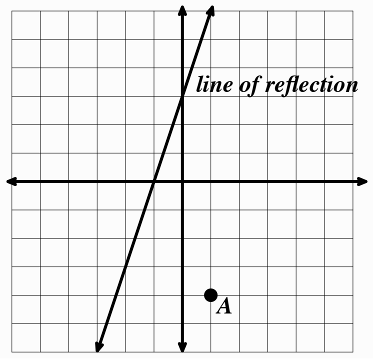
9. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

10. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

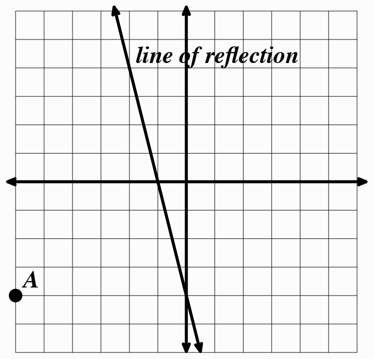
GO

Topic: Reflecting and rotating points on the coordinate plane.
(The coordinate grid, compass, ruler and other tools may be helpful in doing this work.)

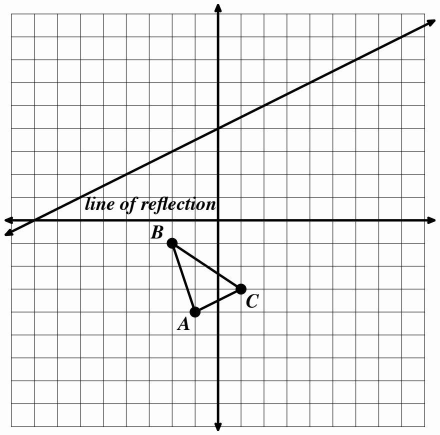
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



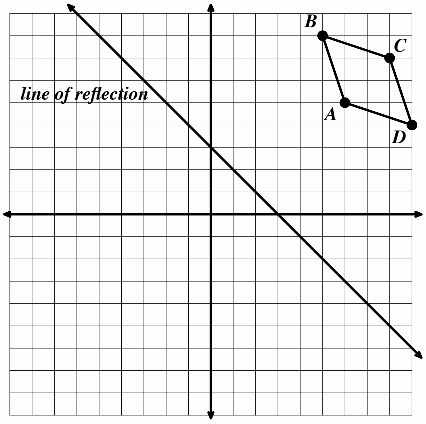
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



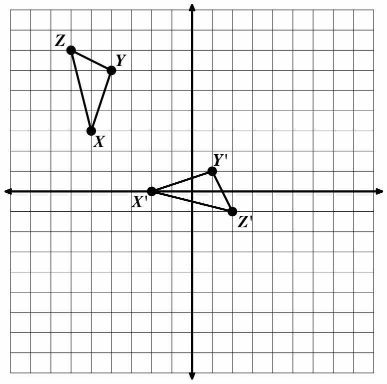
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



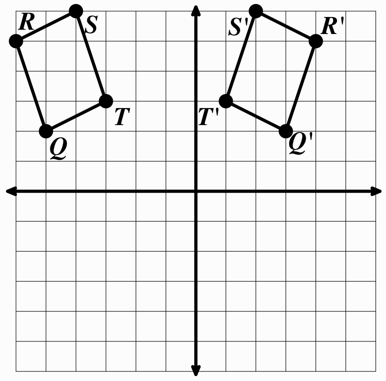
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



6.7 Quadrilaterals—Beyond Definition

A Practice Understanding Task

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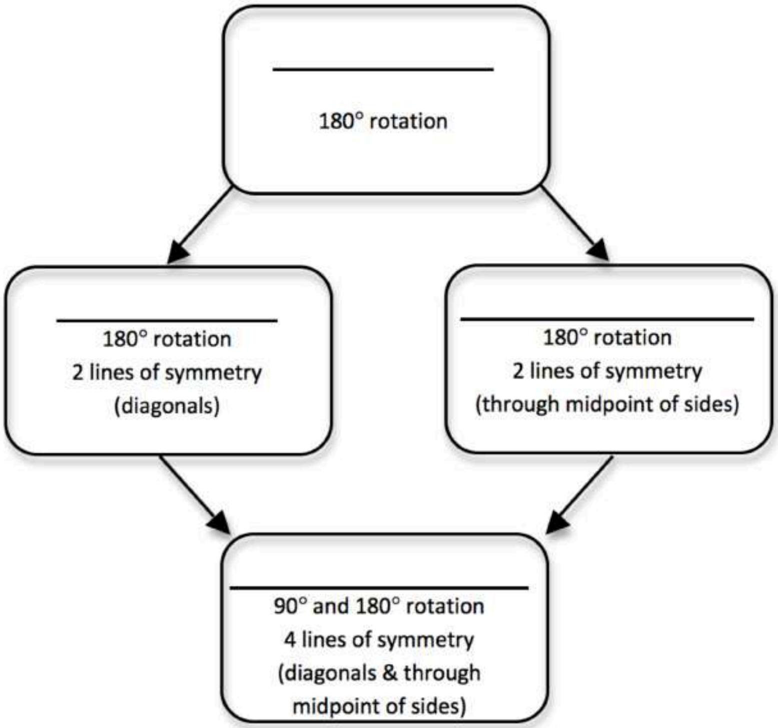
We have found that many different quadrilaterals possess lines of symmetry and/or rotational symmetry. In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

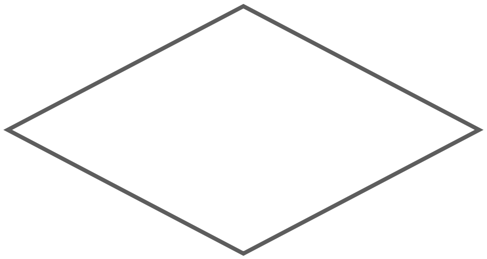
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that "all four angles are right angles?" Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals. Then justify why the properties would be true using the transformational symmetry.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

Based on what you know about transformations, what else can we say about parallelograms besides the defining property that "opposite sides of a parallelogram are parallel?" Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.

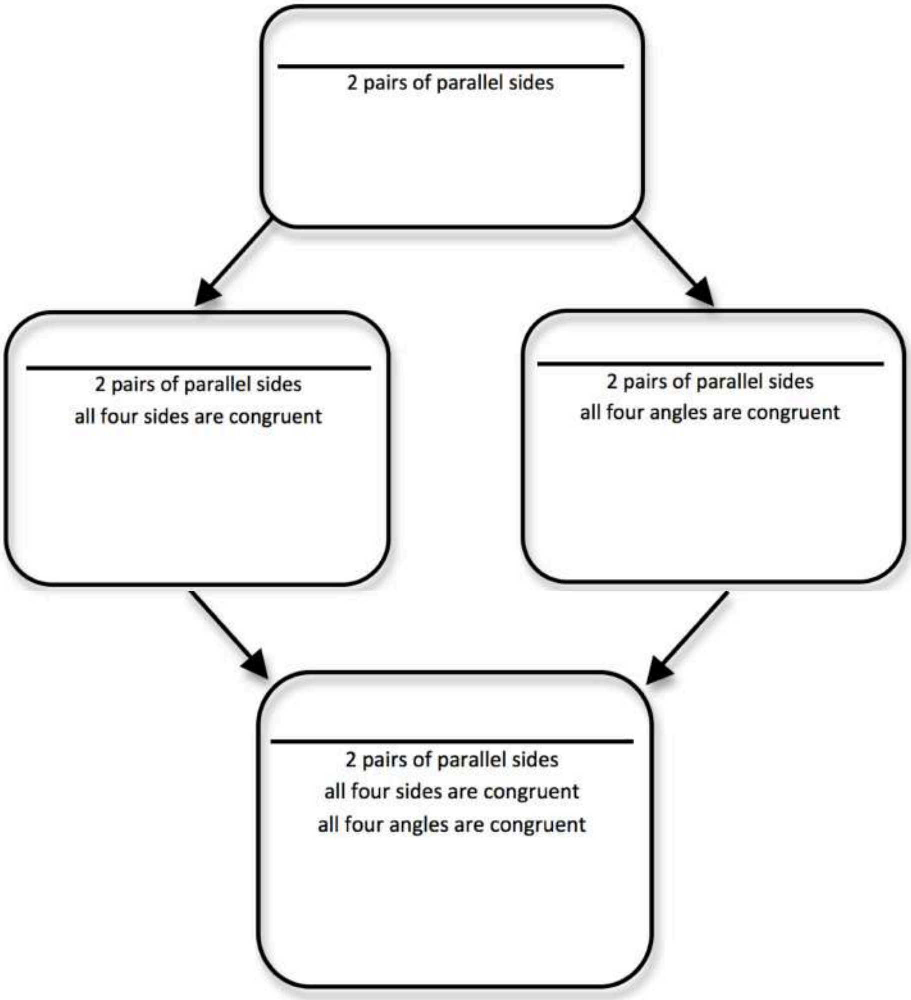


Based on what you know about transformations, what else can we say about a rhombus besides the defining property that "all sides are congruent?" Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

4. A **square** is both a rectangle and a rhombus.

Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals. Then justify why the properties would be true using the transformational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

READY, SET, GO! Name Period Date

READY

Topic: Defining congruence and similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

SET

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

GO

Topic: Slope and distance.

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. Distances should be provided in the most exact form.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

a. Slope: b. Distance:

S