Math 2
Unit 4 - Radical & Rational Functions
Lesson 6 → Types of Variations

Name________________________
Date____________________**Pd**_____

➢ **DIRECT VARIATION:** Linear function with a y-intercept of 0. In a direct variation, both of the quantities are either increasing or both are decreasing.

➢ There are two methods for solving a direct variation problem:

1) Equation of Variation: $y = kx$ where k is called the **constant of variation**

2) Proportion: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

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#1: The distance that a body near Earth's surface will fall from rest varies directly as the **square** of the number of seconds it has been falling. If a boulder falls from a cliff a distance of 122.5 m in 5 $seconds$, approximately how far will it fall in 8 $seconds$?

Method 1

$d = ks^2$

$122.5 = k(5)^2$

$\frac{122.5}{25} = \frac{25k}{25}$

$k = 4.9$

$d = (4.9)(8)^2$

$\boxed{313.6 \text{ meters}}$

Method 2

$\frac{d}{t^2}$:

$\frac{122.5}{5^2} = \frac{d}{8^2}$

$\frac{25d}{25} = \frac{7808}{25}$ $\boxed{313.6}$

➢ **JOINT VARIATION:** more than two quantities in a **direct variation** relationship

➢ Equation of Variation: $y = kxz$ where k is called the **constant of variaton**

#2: If y *varies jointly as x and z*, and $y = \frac{1}{2}$ when $x = 27$ and $z = \frac{-2}{3}$, find y when $x = 9$ and $z = 18$.

$y = kxz$

$\frac{1}{2} = k(27)(-\frac{2}{3})$ $\frac{\frac{1}{2}}{-18} = \frac{-18k}{-18}$

$k = -\frac{1}{36}$

$y = (-\frac{1}{36})(9)(18)$

$y = \boxed{-\frac{9}{2}}$ or -4.5

➢ **INVERSE VARIATION:** Rational function with vertical and horizontal asymptotes. In an inverse variation, one of the quantities is increasing while the second quantity is decreasing.

➢ Equation of Variation: $y = \frac{k}{x}$ where k is called the **constant of variation**

t = s

#3: The time of a trip varies inversely as the speed of the car. If a car being driven at 55 mph takes 2 $hours$ to get from Wake Forest to Greensboro, how fast is the car traveling if the trip takes 2.5 $hours$?

$t = \frac{k}{s}$

$\frac{2}{1} = \frac{k}{55}$

$\boxed{k = 110}$

$\frac{2.5}{1} = \frac{110}{s}$

$2.5s = 110$

2.5 2.5

$\boxed{S = 44 mph}$

- **COMPOUND VARIATION:** Both Inverse and Direct Variation in the same problem
- Equation of Variation: $y = \frac{kx}{z}$ where k is called the **constant of variation**

#4: The volume of gas varies directly with Kelvin temperature and inversely with pressure. If a certain gas has a volume of 342 *cubic meters* at a temperature of 300 *Kelvin degrees* under a pressure of 200 *KPa (kilopascals)*, what will be the volume of the same gas at a temperature of 320 *Kelvin degrees* under a pressure of 400 *kPA*?

V = t / P

$V = \frac{Kt}{P}$ $342 = \frac{K(300)}{(200)}$ $K = 228$ $V = \frac{(228)(320)}{400} =$ $\boxed{182.4 m^3}$

$\frac{342}{1} \times \frac{300K}{200}$

$\frac{68400}{300} = \frac{300K}{300}$

- **State whether each equation represents a direct, inverse, joint or compound variation. Then state the constant of variation.**

1. $y = \frac{9}{x}$	2. $z = 5xy$	3. $y = \frac{8x}{z}$	4. $y = 2x$	5. $xy = 12$
6. $z = \frac{xy}{15}$	7. $y = \frac{3}{4}xz$	8. $y = \frac{1}{3}x$	9. $z = \frac{x}{12y}$	10. $y = \frac{x}{5}$

- **Write a function for each variation relationship:**

11. W varies directly as the square of d.

12. V varies inversely as J.

13. V varies inversely as p and directly as T.

14. F varies jointly as A and the square of v.

15. L varies directly as the fourth power of d and inversely as the square root of h.

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Math 2
Unit 4 - Radical & Rational Functions
Lesson 5 → Types of Variations HOMEWORK

Name______________________________
Date________________________Pd______

Write an equation for each statement and then solve:

1. If y *varies directly as* x and $y = 15$ when $x = 3$, find y when $x = 12$. $y = kx$ $\frac{15}{3} = \frac{k(3)}{3}$ $k = 5$ $y = 5(12)$ $\boxed{y = 60}$	2. If y *varies directly as* x and $x = 36$ when $y = 4$, find x when $y = 24$.	3. If y *varies directly as* x^2 and $y = 12$ when $x = 4$, find y when $x = 6$.
4. If y *varies inversely as* x and $y = 2$ when $x = 8$, find x when $y = 14$.	5. *If* y *varies inversely as* x and $x = 7$ when $y = 21$, find y when $x = 42$.	6. If y *varies inversely as* x^3 and $y = 6$ when $x = \frac{-3}{4}$, find y when $x = 3$.
7. Suppose y *varies jointly with* x *and* z. If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 14$ and $z = 8$.	8. Suppose z *varies jointly with* x *and* y. If $x = 3$ and $y = 2$ when $z = 12$, find z when $x = 4$ and $y = 5$.	9. Suppose m *varies jointly* as n *and* p. If $n = 4$ and $p = 5$ when $m = 60$, find m when $n = 12$ and $p = 2$.
10. Suppose that y *varies directly as* x *and inversely as* z. If $y = 5$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.	11. Suppose y *varies directly as* $\sqrt{x}$ and *inversely as* z. If $y = 10$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 10$.	12. Suppose x *varies directly as* y^3 *and inversely as* $\sqrt{z}$. If $x = 7$ when $y = 2$ and $z = 4$, find x when $y = 3$ and $z = 9$.

Math 2 – Honors
Unit 4 - Radical & Rational Functions
Lesson 7 → Types of Variations HOMEWORK

Name____________________
Date__________________Pd_____

Write an equation for each statement and then solve:

1. If y *varies directly as* x and $y = 15$ when $x = 3$, find y when $x = 12$. $y = kx$ $\frac{15}{3} = \frac{3k}{3}$ $5 = k$ $y = 5(12)$ $y = 60$	2. If y *varies directly as* x and $x = 36$ when $y = 4$, find x when $y = 24$. $y = kx$ $\frac{4}{36} = \frac{36k}{36}$ $\frac{1}{9} = k$ $\frac{24}{1/9} = \frac{(1/9)(x)}{1/9}$ $216 = x$	3. If y *varies directly as* x^2 and $y = 12$ when $x = 4$, find y when $x = 6$. $y = kx^2$ $\frac{12}{16} = \frac{16k}{16}$ $3/4 = k$ $y = (3/4)6^2$ $y = 27$
4. If y *varies inversely as* x and $y = 2$ when $x = 8$, find x when $y = 14$. $y = \frac{k}{x}$ $2 = \frac{k}{8}$ $16 = k$ $14 = \frac{16}{x}$ $\frac{14x}{14} = \frac{16}{14}$ $x = 8/7$	5. *If* y *varies inversely as* x and $x = 7$ when $y = 21$, find y when $x = 42$. $y = \frac{k}{x}$ $21 = \frac{k}{7}$ $147 = k$ $y = \frac{147}{42}$ $\boxed{y = 3.5}$	6. If y *varies inversely as* x^3 and $y = 6$ when $x = \frac{-3}{4}$, find y when $x = 3$. $y = \frac{k}{x^3}$ $6 = \frac{k}{(-3/4)^3}$ $-81/32 = k$ $y = \frac{-81/32}{3^3}$ $y = -3/32$
7. Suppose y *varies jointly with* x *and* z. If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 14$ and $z = 8$. $y = kxz$ $20 = k(2)(5)$ $20 = 10k$ $k = 2$ $y = (2)(14)(8)$ $y = 224$	8. Suppose z *varies jointly with* x *and* y. If $x = 3$ and $y = 2$ when $z = 12$, find z when $x = 4$ and $y = 5$. $z = kxy$ $12 = k(3)(2)$ $\frac{12}{6} = \frac{6k}{6}$ $2 = k$ $z = (2)(4)(5)$ $z = 40$	9. Suppose m *varies jointly as* n *and* p. If n = 4 and p = 5 when m = 60, find m when n = 12 and p = 2. $m = knp$ $60 = k(4)(5)$ $60 = 20k$ $k = 3$ $m = (3)(12)(2)$ $m = 72$
10. Suppose that y *varies directly as* x *and inversely as* z. If $y = 5$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$. $y = \frac{kx}{z}$ $5 = \frac{k(3)}{4}$ $20 = 3k$ $k = 20/3$ $y = \frac{(20/3)(6)}{8}$ $y = 5$	11. Suppose y *varies directly as* $\sqrt{x}$ *and inversely as* z. If $y = 10$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 10$. $y = \frac{k\sqrt{x}}{z}$ $10 = \frac{k\sqrt{9}}{12}$ $10 = \frac{3k}{12}$ $120 = 3k$ $k = 40$ $y = \frac{40\sqrt{16}}{10}$ $y = \frac{160}{10}$ $y = 16$	12. Suppose x *varies directly as* y^3 *and inversely as* $\sqrt{z}$. If $x = 7$ when $y = 2$ and $z = 4$, find x when $y = 3$ and $z = 9$. $x = \frac{ky^3}{\sqrt{z}}$ $7 = \frac{k(2)^3}{\sqrt{4}}$ $7 = \frac{8k}{2}$ $\frac{14}{8} = \frac{8k}{8}$ $k = 7/4$ $x = \frac{(7/4)(3^3)}{\sqrt{9}}$ $x = \frac{47.25}{3}$ $\boxed{x = 15.75}$

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