

➤ A rational function is a function that can be written as the ratio of two polynomials where the denominator does not equal zero.

➤ $f(x) = \frac{p(x)}{q(x)}$ where $q(x) \neq 0$

❖ Steps to graph a rational function:

$$y = \frac{n}{x-h} + k$$

1) Determine the location of the asymptotes based on the transformations:

- A) Vertical asymptotes are placed based on the **horizontal translation**: $x = h$
- B) Horizontal asymptotes are placed based on the **vertical translation**: $y = k$

2) **Vertical Stretch or Compression**: n tells us how far the branches have been stretched from the asymptotes. We can use it to help us find out corner points to start our branches.

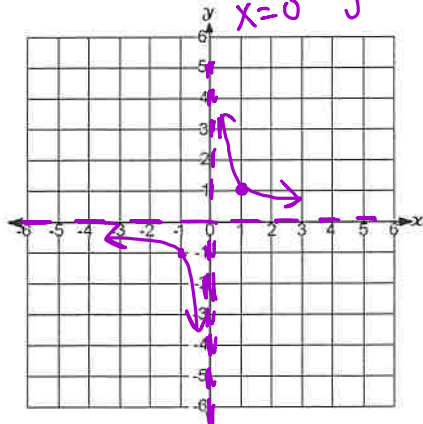
$y = 2(x-1)^2 - 1$

Distance from asymptotes = \sqrt{n}

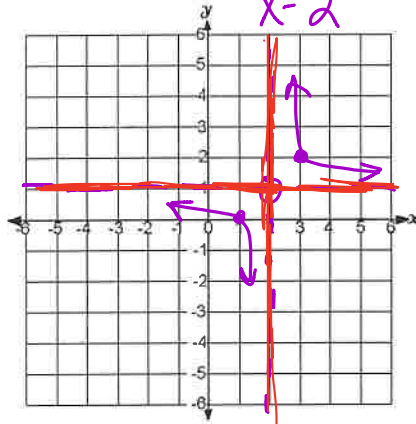
3) Look at the table on the calculator for other points and then sketch the two branches.

❖ Graph each of the following examples:

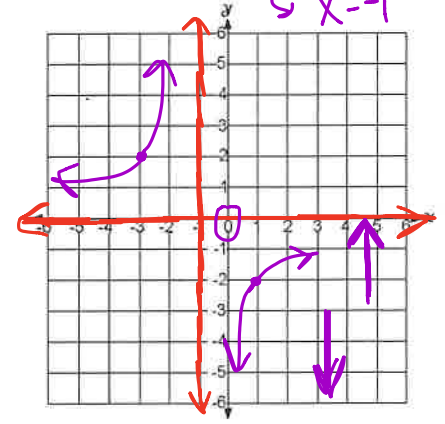
1. $y = \frac{1}{x+0} + 0$
 $x=0$ $y=0$



2. $y = \frac{1}{x-2} + 1$
 $x=2$



3. $y = -\frac{4}{x+1}$
 Reflect 2 away
 $x=-1$ $y=0$



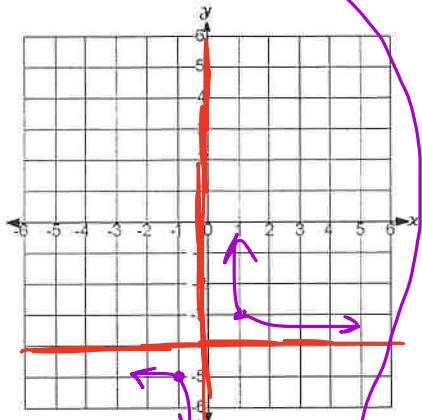
Equation of VA: $x=0$
 Equation of HA: $y=0$
 Describe translations: \emptyset
 Domain: $(-\infty, 0) \cup (0, \infty)$
 Range: $(-\infty, 0) \cup (0, \infty)$

Equation of VA: $x=2$
 Equation of HA: $y=1$
 Describe translations: $k=2, v=1$
 Domain: $(-\infty, 2) \cup (2, \infty)$
 Range: $(-\infty, 1) \cup (1, \infty)$

Equation of VA: $x=-1$
 Equation of HA: $y=0$
 Describe translations: h x-axis
 Domain: $x \neq -1$ $l=1, s$ by 4
 Range: $(-\infty, -1) \cup (-1, \infty)$ s by 2

$(-\infty, 0) \cup (0, \infty)$
 $y \neq \emptyset$
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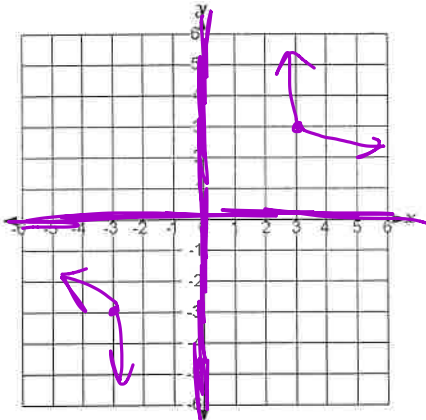
4. $y = \frac{1}{x} - 4$



Equation of VA: $x = 0$
 Equation of HA: $y = -4$
 Describe translations: 0H, 4V
 Domain: $(-\infty, 0) \cup (0, \infty)$
 Range: $(-\infty, -4) \cup (-4, \infty)$

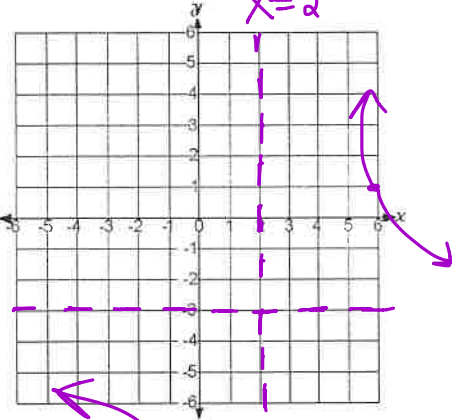
$y = \frac{1}{x}$

5. $xy = 9$



Equation of VA: $x = 0$
 Equation of HA: $y = 0$
 Describe translations: 5 by 9
 Domain: $(-\infty, 0) \cup (0, \infty)$
 Range: $(-\infty, 0) \cup (0, \infty)$

6. $y = \frac{16}{x-2} - 3$



Equation of VA: $x = 2$
 Equation of HA: $y = -3$
 Describe translations: R2, D3, 5 by 16
 Domain: $(-\infty, 2) \cup (2, \infty)$
 Range: $(-\infty, -3) \cup (-3, \infty)$

7. Describe each graph as compared to the parent graph $y = \frac{1}{x}$.

$y = \frac{-2}{x-7} + 5$

The graph of this _____ function has been translated _____ seven units and translated _____ units _____. It has been vertically stretched by a factor of _____ and _____ across the x-axis. The graph is increasing from _____ to _____. The function has a domain of _____ and a range of _____.

$y = \frac{7}{x+2} - 4$

The graph of this _____ function has been translated _____ two units and translated _____ units _____. It has been vertically stretched by a factor of _____. The graph is _____ from left to right. The function has a domain of _____ and a range of _____.

8. Write the equation of a rational function $y = \frac{1}{x}$ with following transformations:

A. Right 4 and Down 5
 $y = \frac{1}{x-4} - 5$

B. Left 3 and Up 2 and Reflected across x -axis.
 $y = -\frac{1}{x+3} + 2$

C. Left 6 and Vertically Stretched by a factor of 4.
 $y = \frac{4}{x+6}$ OR $\frac{16}{x+6}$

D. Right 5 and graph will be in II & IV quadrants
 $y = \frac{16}{x-5} + 4$
 R5, U4, R x-axis, 4 units away from cross