QUIZ DATE: _____   TEST DATE: _____

Math 2   Name_____
Unit 1 – Geometric Transformations   Date_____Pd_____
Lesson 1 – Introduction to Transformations and Translations

Introduction to Transformations and Translations

➢ **Congruent figures:** _____.

 ✓ When two figures are congruent, you can move one figure on top of the other figure with
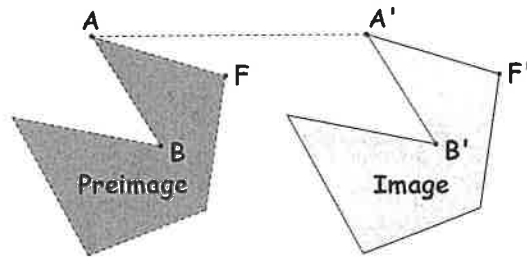
_____.

➢ **Transformation** of a geometric figure: change in its _____, _____, or _____.

➢ **Preimage** – _____ figure

 ✓ Notation: _____

➢ **Image** – _____ or _____ figure

 ✓ Notation: _____

➢ **Isometry** – transformation in which preimage and image are the _____ _____ and

_____ (also called: _____)

Examples: _____   _____   _____

➢ **Translation** – an isometry that maps all points the _____ _____ and the

_____ _____.

───

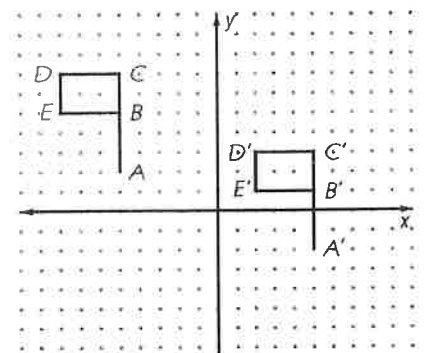❖ **Two ways to describe a translation** (using example shown right):
 ✓ Always **be specific** when completing **any** type of description!!

1) **Words:** Translation to the right 10 units and down 4 units.

2) **Algebraic rule** (motion rule):   T: $(x, y) \rightarrow (x + 10, y - 4)$
 ✓ **Right and Up are always** $(+)$

 ✓ **Left and Down are always** $(-)$

1

❖ **Example: Dot Paper Translations**

1) Use the dots to help you draw the image of the first figure so that A maps to A′.

2) Use the dots to help you draw the image of the second figure so that B maps to B′.

3) Use the dots to help you draw the image of the third figure so that C maps to C′.

4) Complete each of the following translation rules using your mappings from 1 – 3 above.

    a) For A, the translation **rule** is: $T: (x, y) \rightarrow ($ _____, _____ $)$

    b) For B, the translation **rule** is: $T: (x, y) \rightarrow ($ _____, _____ $)$

    c) For C, the translation **rule** is: $T: (x, y) \rightarrow ($ _____, _____ $)$
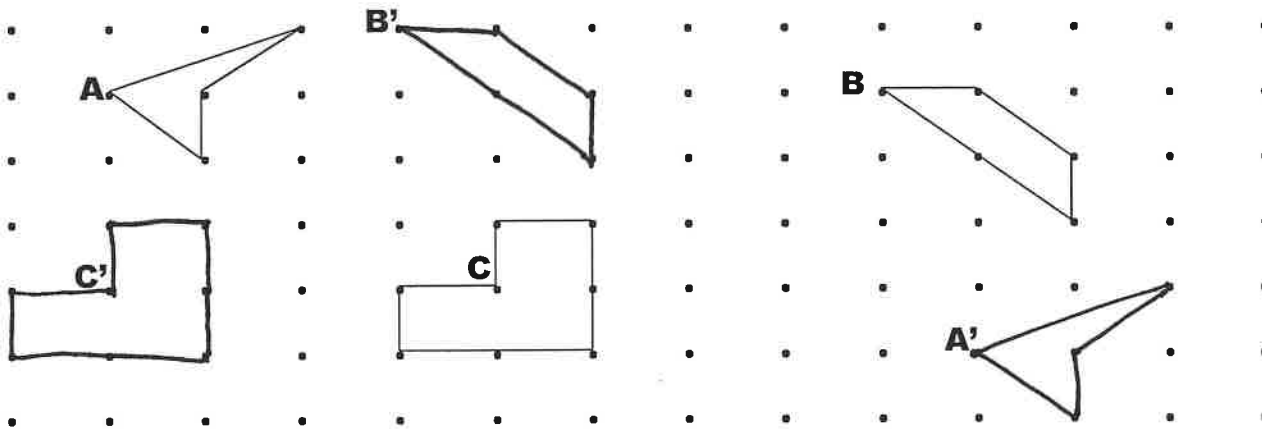
❖ **Example:** $\triangle GEO$ has coordinates $G(-2, 5)$, $E(-4, 1)$ $O(0, -2)$. A **translation** maps G to G′ $(3, 1)$.

1. The translation **rule** is $T: (x, y) \rightarrow ($ _____, _____ $)$

2. Describe the transformation in **words**: _____

3. Find the coordinates of: a) E′ (_____, _____)   b) O′ (_____, _____)

❖ **Example:** Given the translation from $\triangle ABC$ to $\triangle A'B'C'$, find the specified values for x and y.
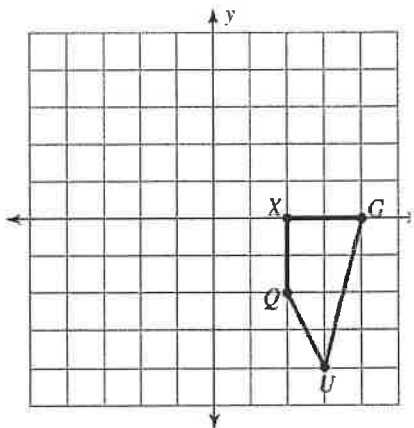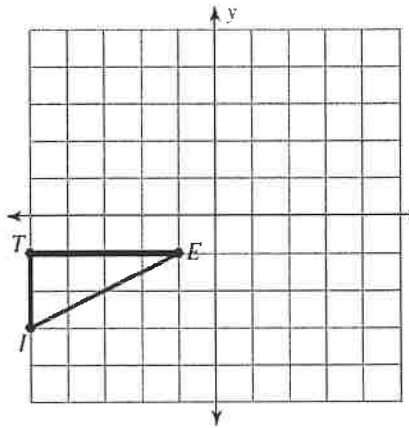Hint: $\triangle ABC \cong \triangle A'B'C'$
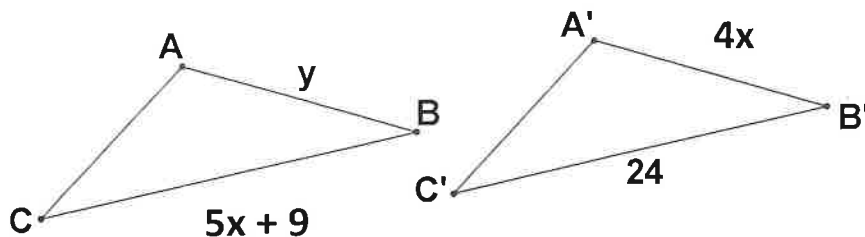
$x =$ _____

$y =$ _____

2

❖ **Graph the image of the figure using the transformation given write the algebraic rule and as requested write a specific verbal description or vector.**
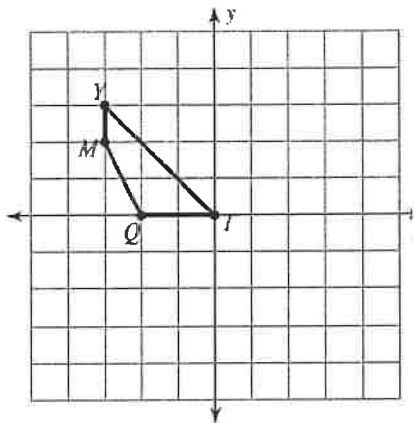
1) translation: 1 unit left

Algebraic
Rule:
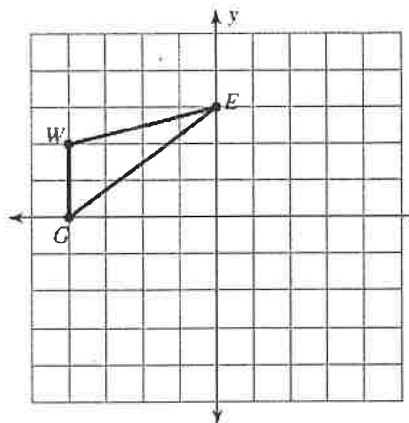
2) translation: 1 unit right and 2 units down

Algebraic
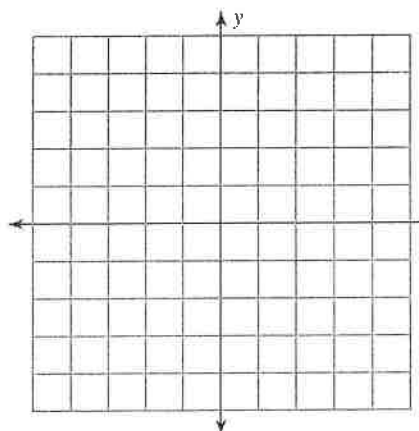Rule:

3) translation: 3 units right

Algebraic
Rule:

4) $T(x,y) \rightarrow (x+1, y-2)$
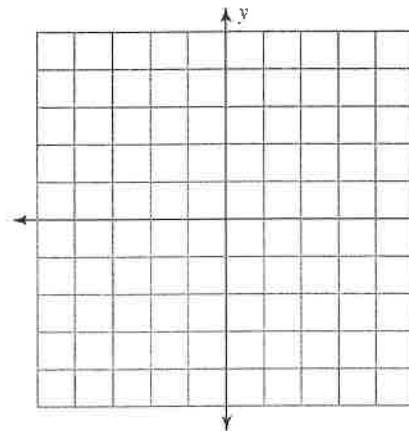

Description:

5) translation: 5 units up
$U(-3,-4), M(-1,-1), L(-2,-5)$

Description:

Algebraic
Rule:

6) $T(x,y) \rightarrow (x, y+3)$
$R(-4,-3), D(-4,0), L(0,0), F(0,-3)$

Description:

3

❖ **Find the coordinates of the vertices of each figure after the given transformation and write the algebraic rule.**

7) Translation: 2 units left and 1 unit down
$W(0, -1), F(-2, 2), H(2, 4), S(3, 0)$

    Vertices:

    Algebraic Rule:

8) Translation: 2 units down
$M(-4, 1), A(-2, 5), T(-1, 4), H(-1, 2)$

    Vertices:

    Algebraic Rule:

---

9) Translation: $T(x, y) \rightarrow (x - 4, y + 4)$
   $J(-1, -2), A(-1, 0), N(3, -3)$

    Vertices:

    Words:

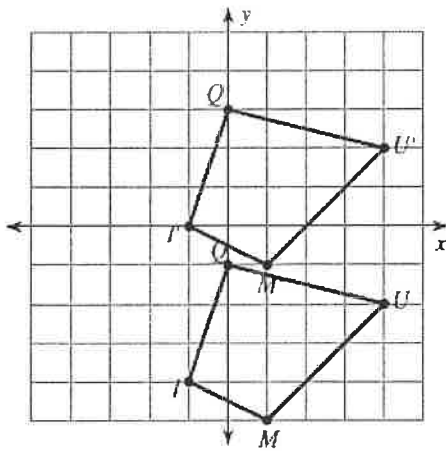10) Translation: $T(x, y) \rightarrow (x + 3, y)$
   $P(-4, -3), L(-2, -2), T(-2, -4)$

    Vertices:

    Words:

➤ **Write a specific description of each transformation and give the algebraic rule. Then use vector notation.**
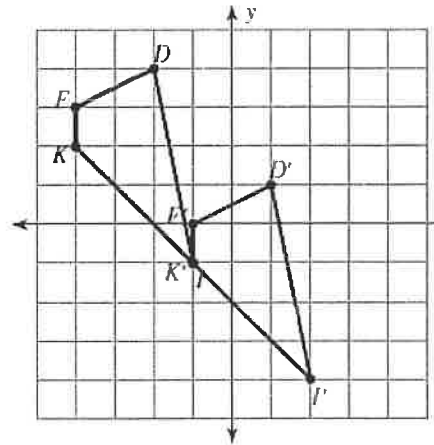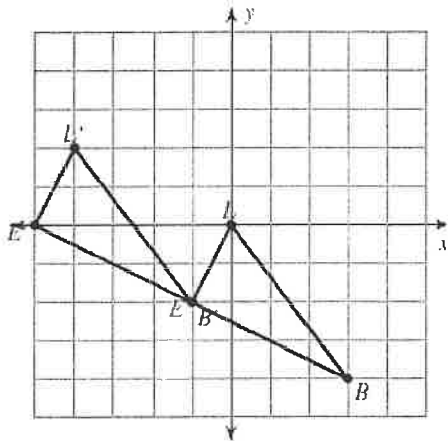
11)

Description:

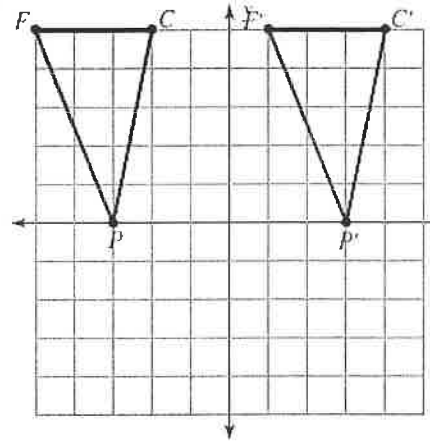Algebraic Rule:

12)

Description:

Algebraic Rule:

13)

Description:

Algebraic Rule:

14)

Description:

Algebraic Rule:

4

Math 2
Unit 1 – Geometric Transformations
Lesson 1 – Translations HOMEWORK

Name_____
Date_____Pd_____

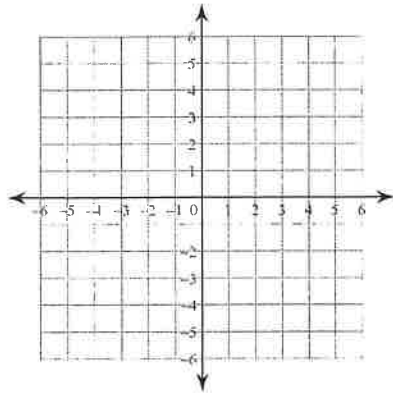**1.** Graph and label $\triangle ABC$ with vertices $A(-3,-1)$, $B(-1,4)$, and $C(2,2)$. Graph and label the image of $\triangle ABC$ under the translation $T:(x,y) \rightarrow (x+2, y-4)$.

A' _____
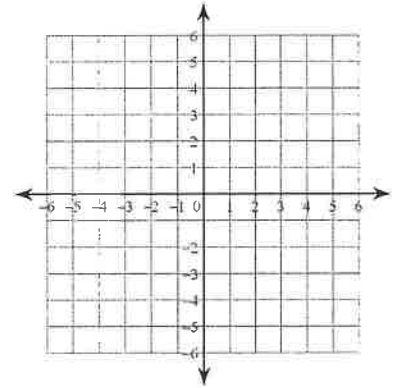
B' _____

C' _____

Describe the translation in words:

---

2. Graph and label quadrilateral DUCK with vertices $D(2,2), U(4,1), C(3,-2),$ and $K(0,-1)$. Graph and label the image of Quadrilateral DUCK when the Quadrilateral is **shifted left 4 and up 3.**
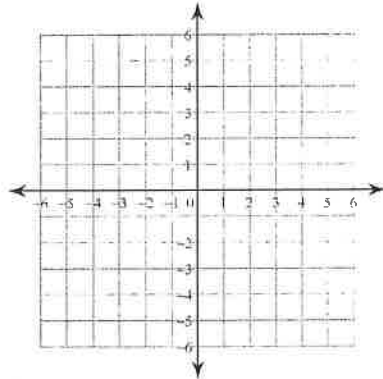
D' _____

U' _____

C' _____

K' _____

Write the rule in algebraic notation:
T:

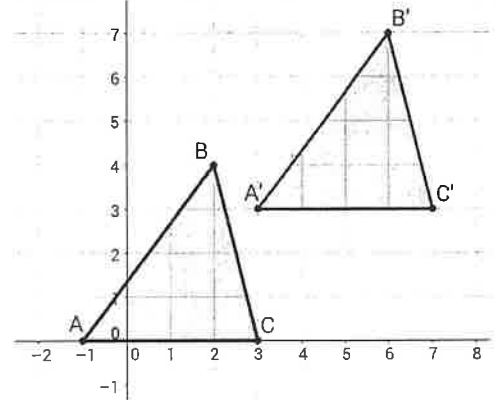**3.** Graph and label quadrilateral MATH with vertices $M(4,1)$, $A(2,4)$, $T(0,6)$, and $H(1,2)$. Graph and label the image of quad. MATH when the quadrilateral is shifted according to the rule $T:(x,y) \rightarrow (x-3, y-4)$.

M' _____

A' _____

T' _____

H' _____

Describe the translation in words:

---

4. Write the rule mapping the pre-image to the image.

Write the rule in algebraic notation:
T:

Describe the translation in words:

For # 5 – 6, Given $\triangle ABC$ translates to $\triangle A'B'C'$

5. Find x and y, given $m\angle A = y$, $m\angle A' = 2x+5$, $m\angle C = 3x+7$, $m\angle C' = 13$

$x = $ _____

$y = $ _____

6. Find x and y, given $BC = 4x-2y$, $B'C' = 11$, $AC = 3x$, and $A'C' = 27$.

$x = $ _____

$y = $ _____

5

Math 2
Unit 1 – Geometric Transformations
Lesson 2 – Reflections

Name_____
Date_____ Pd_____

Reflections:

- A reflection is a transformation in which the image is a mirror image of the preimage.

- A point on the line of reflection maps to _____ .

- Other points map to the _____ side of the reflection line so that the

  reflection line is the _____ of the segment joining a preimage and image point.

- Preimage and image points are **equidistant** from the line of _____ .

- Notation for reflections is $R_{Line\ of\ Reflection}$. Example: R_{x-axis} means reflection in or across the $x-axis$.

Reflections in the coordinate plane. Given ΔREF: $R(-3,1), E(0,4), F(2,-5)$

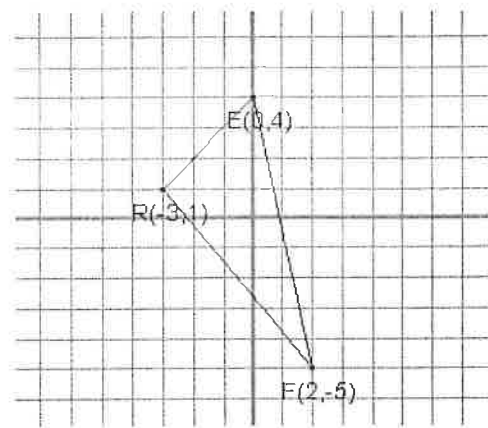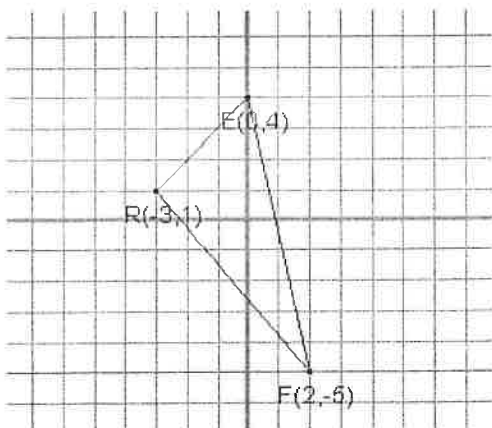1) On the first grid, draw the reflection of ΔREF in the $x-axis$.   Notation: _____
   Record the new coordinates: $R'($ ___ , ___ $)$, $E'($ ___ , ___ $)$, $F'($ ___ , ___ $)$

2) On the second grid, draw the reflection of ΔREF in the $y-axis$.   Notation: _____
   Record the new coordinates: $R'($ ___ , ___ $)$, $E'($ ___ , ___ $)$, $F'($ ___ , ___ $)$

E(0,4)   R(-3,1)   F(2,-5)

E(0,4)   R(-3,1)   F(2,-5)
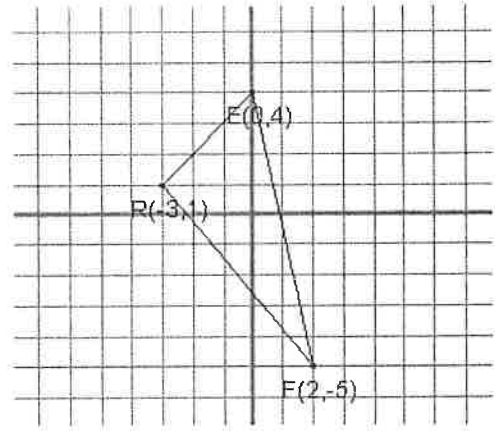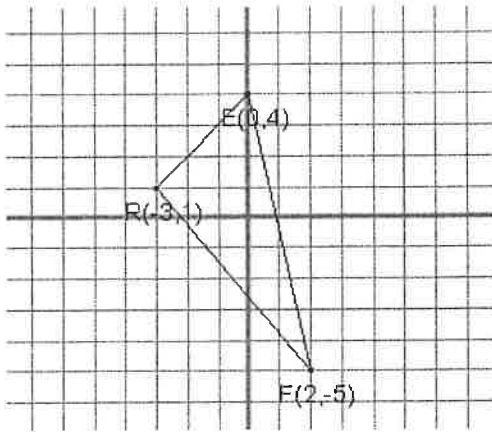
3) Graph the line $y = x$ on the third coordinate grid. Reflect the triangle in the line $y = x$.
   Record the new coordinates: $R'($ ___ , ___ $)$, $E'($ ___ , ___ $)$, $F'($ ___ , ___ $)$ Notation: _____

4) Graph the line $y = -x$ on the fourth coordinate grid paper. Reflect the triangle in the line $y = -x$.
   Record the new coordinates: $R'($ ___ , ___ $)$, $E'($ ___ , ___ $)$, $F'($ ___ , ___ $)$ Notation: _____

E(0,4)   R(-3,1)   F(2,-5)

E(0,4)   R(-3,1)   F(2,-5)

6

Look at the patterns and complete the rule. Then write the rule using proper notation.

1. Reflection in the $x - axis$ maps $(x, y) \rightarrow ($ _____ , _____ $)$ Notation: _____

2. Reflection in the $y - axis$ maps $(x, y) \rightarrow ($ _____ , _____ $)$ Notation: _____

3. Reflection in the line $y = x$ maps $(x, y) \rightarrow ($ _____ , _____ $)$ Notation: _____

4. Reflection in the line $y = -x$ maps $(x, y) \rightarrow ($ _____ , _____ $)$ Notation: _____

Reflections with Polygons

Reflection Symmetry

1. Given *Triangle ABC*.

   a. What is the equation of the line of reflection that maps *angle A* onto *angle B*? _____

   b. If we reflect *Triangle ABC* over the line of reflection found in part a, $\overline{AC}$ maps to _____.

   c. What can we conclude about the **measures** of $\angle A$ and $\angle B$?
      What can we conclude about the **lengths** of $\overline{AC}$ and $\overline{BC}$?

2. Given *Regular Hexagon ABCDEF*.

   a. List the three lines of symmetry drawn on the diagram at right: _____ , _____ , _____

   b. What is the image of point D when reflected across $\overleftrightarrow{BE}$?

   c. What is the image of $\angle OED$ when reflected across $\overleftrightarrow{FC}$?

   d. What conclusions can you make about these angles?

3. Given *Quadrilateral ABCD*.

   a. The slope of $\overleftrightarrow{BC}$ is _____. The slope of $\overleftrightarrow{AD}$ is _____.
      What kind of quadrilateral is ABCD? Explain how you know.

   b. Let line *m* be the equation of the reflection line mapping $\overline{CD}$ to $\overline{BA}$.
      Write the equation of line *m*.

   c. Reflect *Quadrilateral ABCD* over *line m*.

   $\angle A$ maps to _____ $\angle B$ maps to _____

Lesson 2 – Reflections Classwork

❖ **Graph the image using the transformation given, write the proper notation, and give the algebraic rule as requested.**
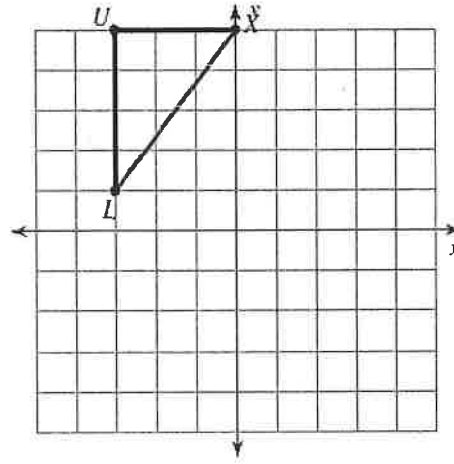
1) reflection across the $y - axis$
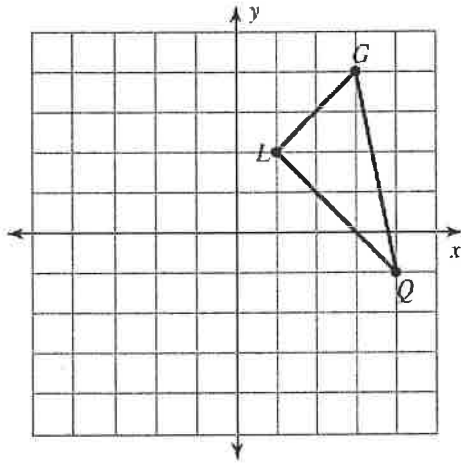
Notation:

Algebraic Rule:

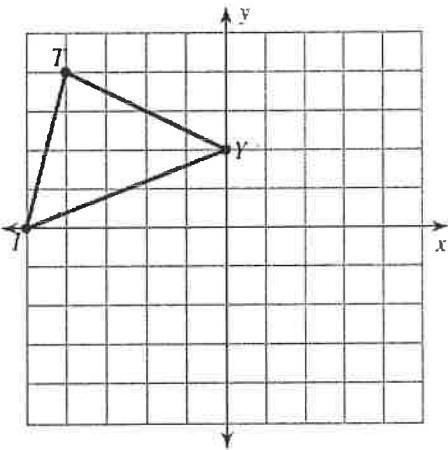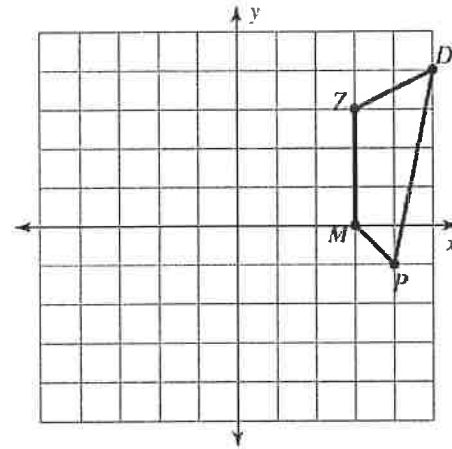2) reflection across $y = x$

Notation:

Algebraic Rule:

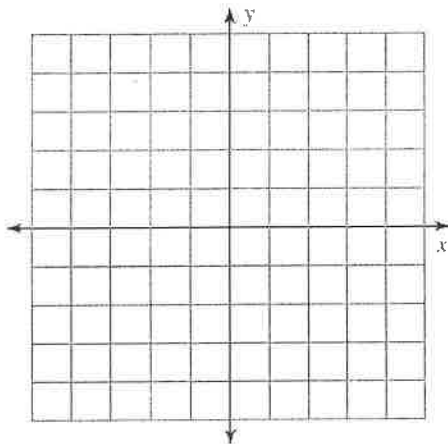3) reflection across $y = x$

Notation:

Algebraic Rule:

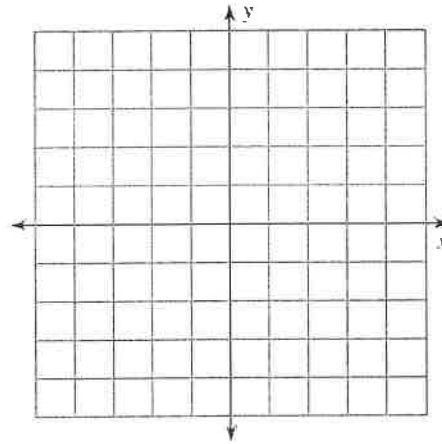4) reflection across the x-axis

Notation:

Algebraic Rule:

5) reflection across the x-axis
$T(2, 2), C(2, 5), Z(5, 4), F(5, 0)$

Notation:

Algebraic Rule:

6) reflection across $y = -x$
$H(-1, -5), M(-1, -4), B(1, -2), C(3, -3)$

Notation:

Algebraic Rule:

8

❖ **Find the coordinates of the vertices of each figure after the given transformation and give the algebraic rule and notation, as requested.**

7) Reflection across the $x - axis$
$K(1, -1), N(4, 0), Q(4, -4)$

Algebraic Rule:

Notation:

8) Reflection across $y = -x$
$R(-3, -5), N(-4, 0), V(-2, -1), E(0, -4)$

Algebraic Rule:

Notation:

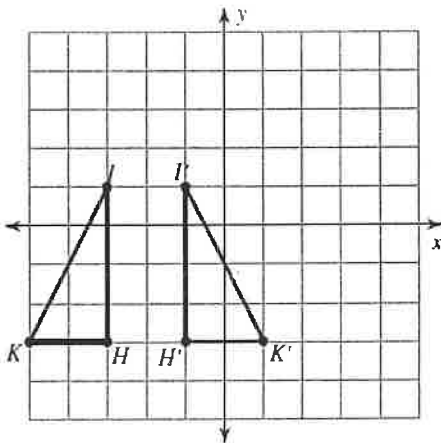9) Reflection across the $y - axis$
$F(2, 2), W(2, 5), K(3, 2)$

Algebraic Rule:

Notation:

10) Reflection across $y = x$
$V(-3, -1), Z(-3, 2), G(-1, 3), M(1, 1)$

Algebraic Rule:

Notation:

Write a specific description of each transformation and give the algebraic rule, as requested.

11)

Description:

Notation:

12)
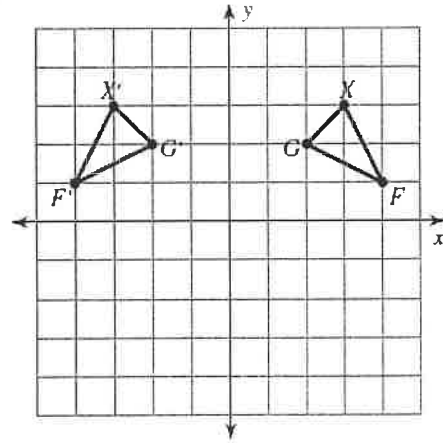
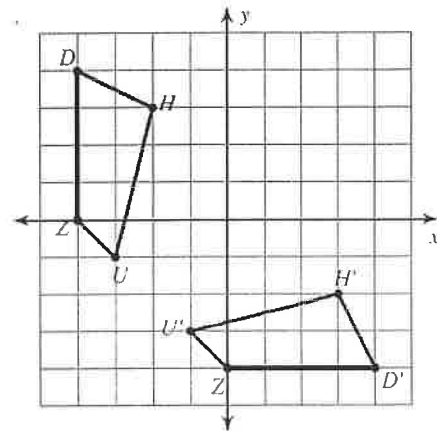Description:
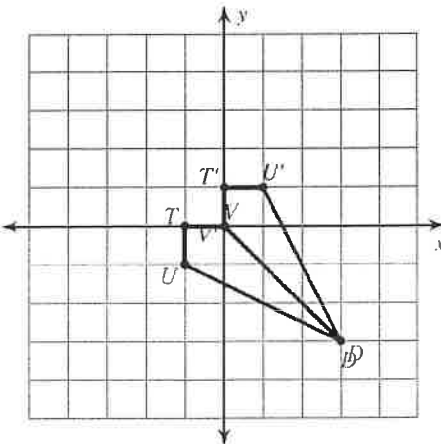
Algebraic Rule:

Notation:

13)

Description:

Algebraic Rule:

Notation:

14)

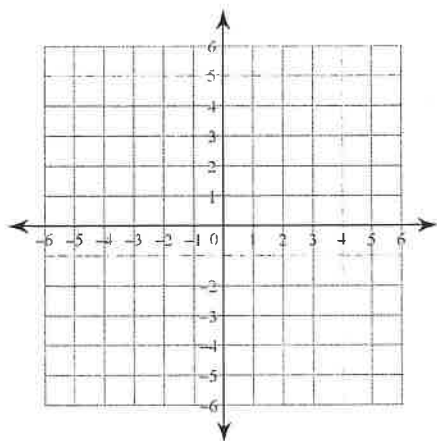

Description:

Algebraic Rule:

Notation:

9

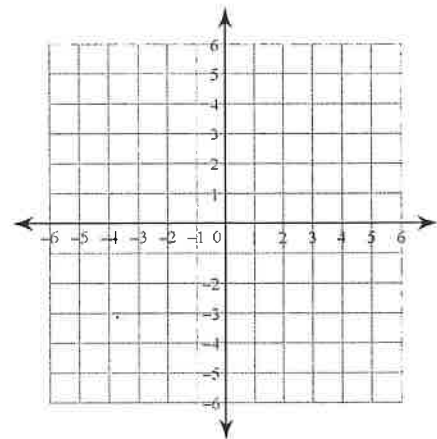1. ΔEFG if $E(-1,2), F(2,4)$ and $G(2,-4)$ reflected over the $y-axis$.

E'_____

F'_____

G'_____

Notation:

Rule:

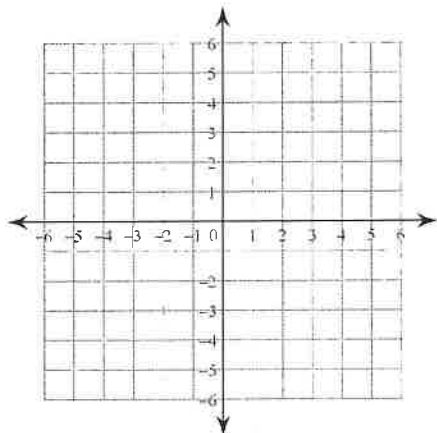2. ΔPQR if $P(-3,4), Q(4,4)$ and $R(2,-3)$ reflected over the $x-axis$.

P'_____

Q'_____

R'_____

Notation:

Rule:

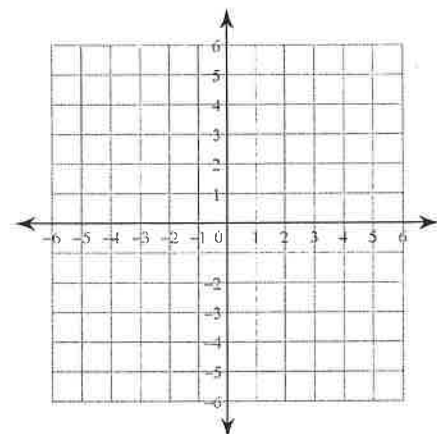3. Quadrilateral $VWXY$ if $V(0,-1), W(1,1),$ $X(4,-1),$ and $Y(1,-5)$ reflected over the line $y = x$.

V'_____

W'_____

X'_____

Y'_____

Notation:

Rule:

4. ΔBEL if $B(-2,3), E(2,4),$ and $L(3,1)$ reflected over the line $y=-x$.
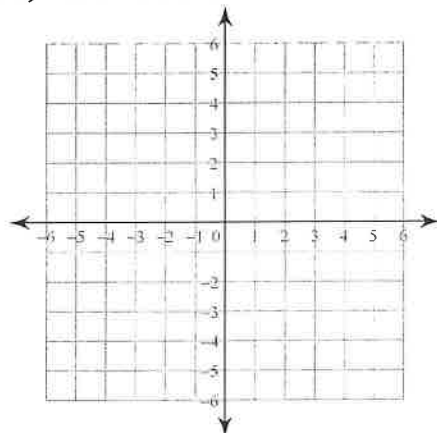
B'_____

E'_____

L'_____

Notation:

Rule: y-axis

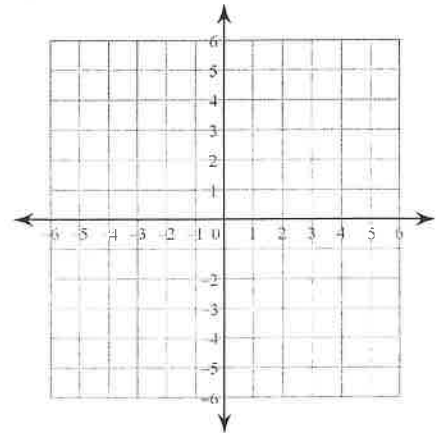5. Square $SQUR$ if $S(2,4), Q(4,0), U(0,-4),$ and $R(-2,2)$ reflected over the $x-axis$.

S'_____

Q'_____

U'_____

R'_____

Notation:

6. Quadrilateral $MATH$ if $M(1,4),$ $A(-1,2),$ $T(2,0)$ and $H(4,0)$ reflected over the $y-axis$.
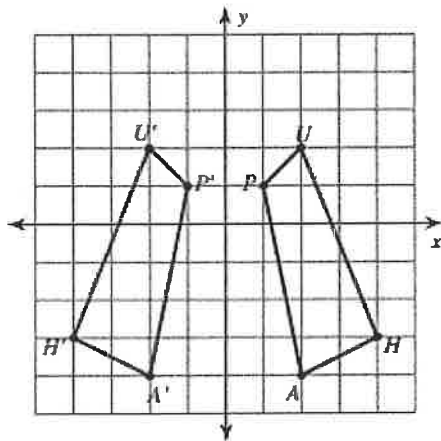
M'_____

A'_____

T'_____

H'_____

Notation:

10

Write a specific description of each transformation and give the algebraic rule, as requested.
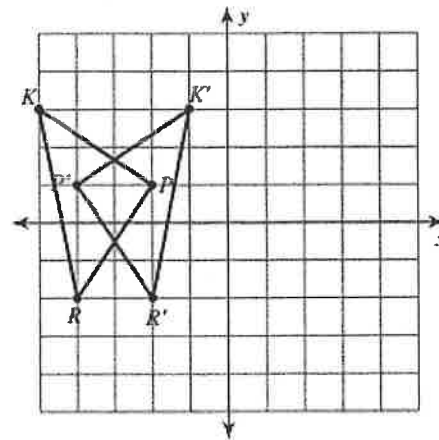
7.

Description:
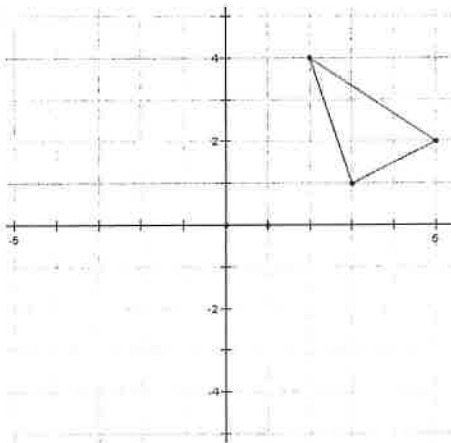
Algebraic Rule:

Notation:

8.

Description:

Notation:

Find the image of the following transformations and give a specific description.
Hint: If you get stuck, review the Checkpoints after today's activities. ☺
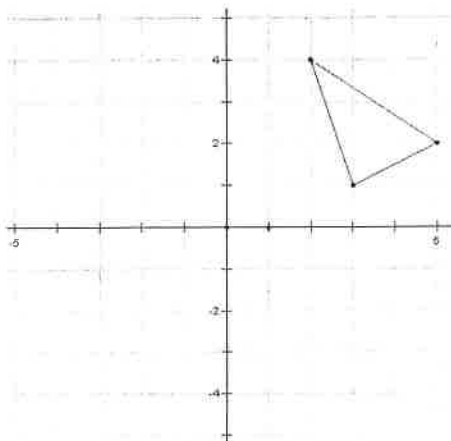
9. The points $(2,4), (3,1), (5,2)$ are reflected with the rule $(x,y) \rightarrow (x,-y)$

Description:

Notation:

10. The points $(2,4), (3,1), (5,2)$ are reflected with the rule $(x,y) \rightarrow (-x,y)$

Description:

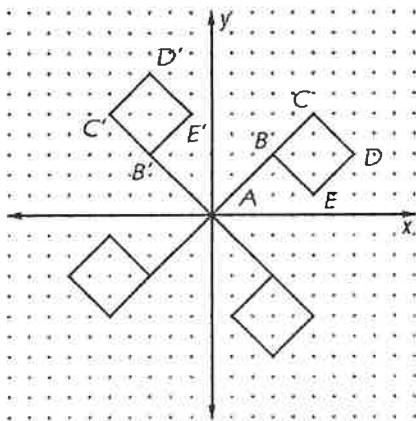Notation:

Rotations

Definition:

A **rotation** is a type of transformation which is a _____ in a given direction for a given number of

_____ around a fixed _____. To rotate an object, you must specify the

_____ of rotation, the _____ around which the rotation is to occur, and the direction.

➤ Rotations can be completed in two directions: counter-clockwise & clockwise

Rotations with a Coordinate Plane

➤ **Visualizing Rotations Centered About the Origin**

The flag shown below is rotated about the origin 90°, 180°, and 270°. Flag ABCDE is the **preimage**. Flag A'B'C'D'E' is a 90° counterclockwise rotation of ABCDE.

NOTE: Unless otherwise specified, the standard for rotations is **counterclockwise!**

➤ **Notation for Rotations:** $\mathcal{R}$ _____

➤ **Examples:** $\mathcal{R}_{90°}$ $\mathcal{R}_{270°CW}$
            $\mathcal{R}_{180°}$ $\mathcal{R}_{180°CW}$
            $\mathcal{R}_{270°}$ $\mathcal{R}_{90°CW}$

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➤ **Rotations on the Coordinate Plane Exploration:** Triangle ABC has coordinates $A(2, 0)$, $B(3, 4)$, $C(6, 4)$.

Trace the triangle and the $x-$ and $y-$ axes on patty paper.

1) Rotate *Triangle ABC* 90°, using the axes you traced to help you line it back up. Record the new coordinates.
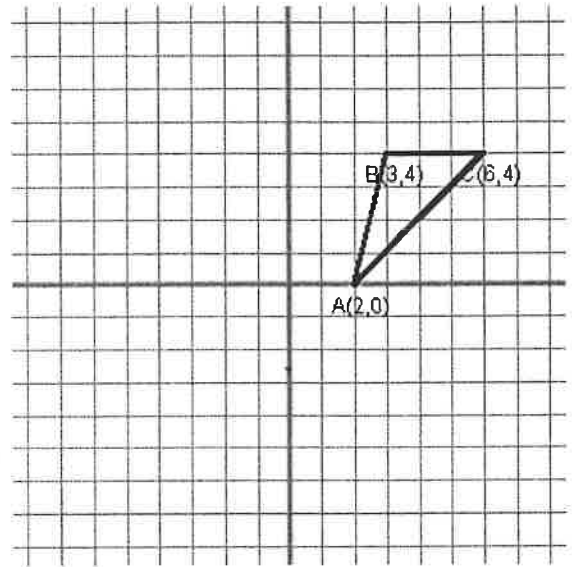
A'(_____ , _____), B'(_____ , _____), C'(_____ , _____)

2) Rotate *Triangle ABC* 270°, using the axes you traced to help you line it up. Record the new coordinates.

A'(_____ , _____), B'(_____ , _____), C'(_____ , _____)

3) Rotate *Triangle ABC* 180°, using the axes you traced to help you line it back up correctly. Record the new coordinates.

A'(_____ , _____), B'(_____ , _____), C'(_____ , _____)

➤ **Rotation Algebraic Rules:**
   ✓ **Look for patterns in the above examples to help complete the following rotation rules.**
   ✓ **Then write the rule using <u>proper notation</u> for 1 – 3.**

1. A 90° counter-clockwise rotation maps $(x, y) \rightarrow ($ _____ , _____). Notation: _____

2. A 270° counter-clockwise rotation maps $(x, y) \rightarrow ($ _____ , _____). Notation: _____

3. A 180° rotation maps $(x, y) \rightarrow ($ _____ , _____). Notation: _____

4. A rotation of 270° **clockwise** is equivalent to a rotation of _____.

5. A rotation of 270° **counterclockwise** is equivalent to a rotation of _____.

6. A rotation of 180° **counterclockwise** is equivalent to a rotation of _____.

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➢ **Graph the image of the figure using the transformation given. Also, give the coordinates of the image, the algebraic rule, and the proper notation for the transformation.**
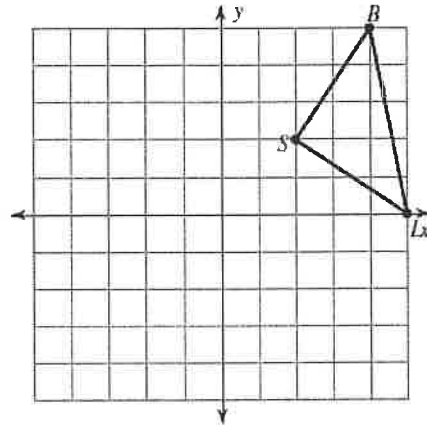
1) rotation 180° about the origin
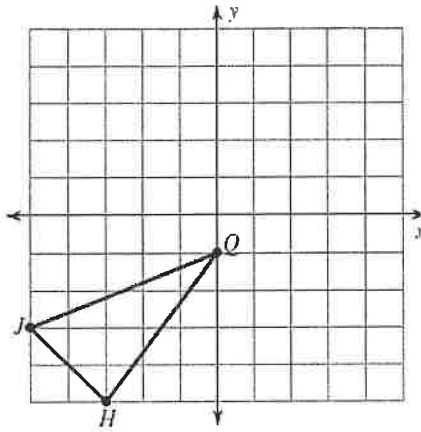
Coordinates:

Algebraic Rule:

Notation:

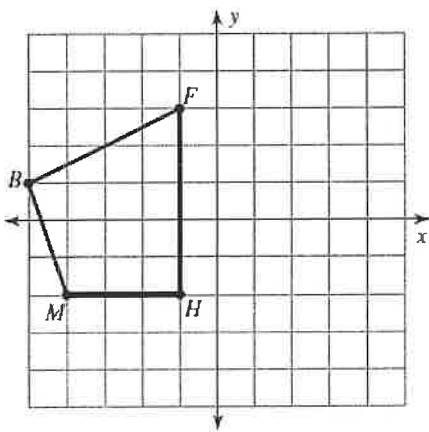2) rotation 90° about the origin

Coordinates:

Algebraic Rule:

Notation:

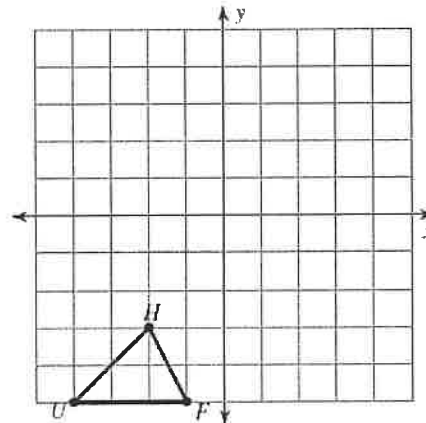3) rotation 270° about the origin

Coordinates:

Algebraic Rule:

Notation:

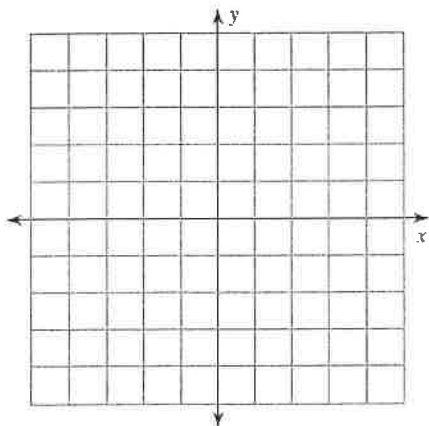4) rotation 180° about the origin

Coordinates:

Algebraic Rule:

Notation:

5) rotation 90° CCW about the origin
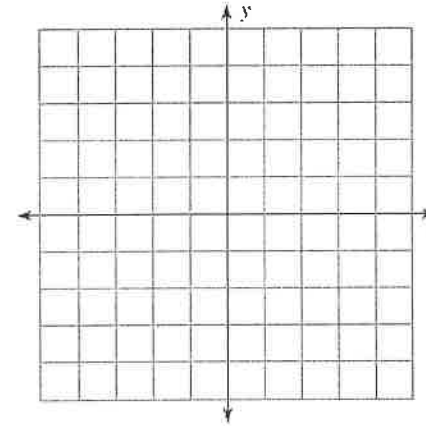$U(1, -2)$, $W(0, 2)$, $K(3, 2)$, $G(3, -3)$

Coordinates:

Algebraic Rule:

Notation:

6) rotation 180° about the origin
$V(2, 0)$, $S(1, 3)$, $G(5, 0)$

Coordinates:

Algebraic Rule:

Notation:

14

> Identify the coordinates of the vertices for each figure after the given transformation. Also, give the algebraic rule and correct notation for each transformation.

7) rotation 180° about the origin

$Z(-1,-5), K(-1,0), C(1,1), N(3,-2)$

Vertices:

Algebraic Rule:

Notation:

8) rotation 180° about the origin

$L(1,3), Z(5,5), F(4,2)$

Vertices:

Algebraic Rule:

Notation:

9) rotation 90° about the origin

$S(1,-4), W(1,0), J(3,-4)$

Vertices:

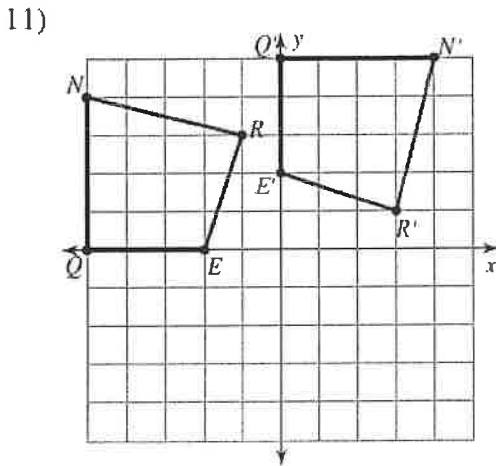**Algebraic Rule:**

Notation:

10) rotation 270° about the origin

$W(-5,-3), A(-3,1), G(0,-3)$

Vertices:

Algebraic Rule:

Notation:
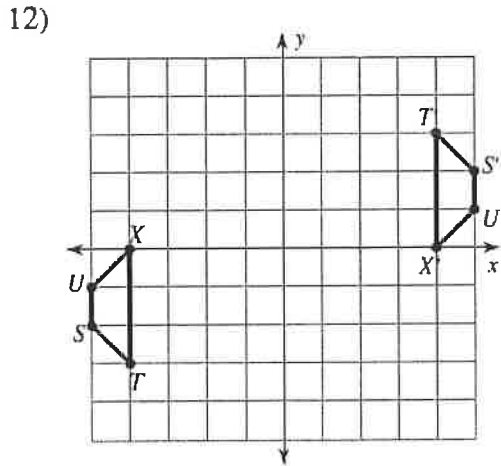
> Write a specific description of each transformation AND give the algebraic rule and notation.
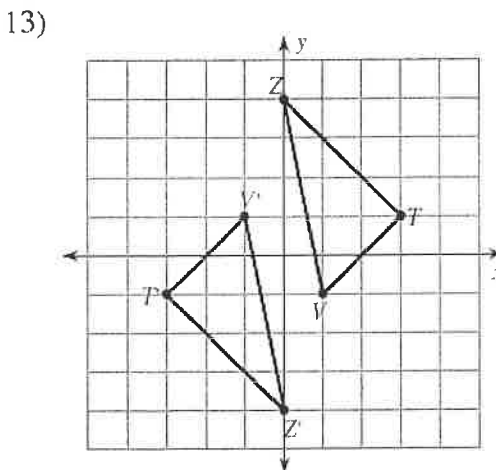
11)

Description:

Algebraic Rule:

Notation:

12)

Description:

Algebraic Rule:
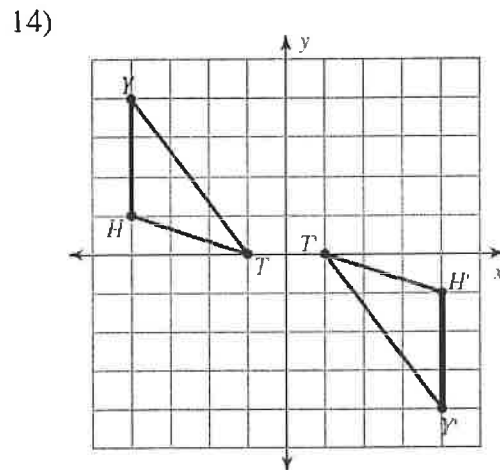
Notation:

13)

Description:

Algebraic Rule:
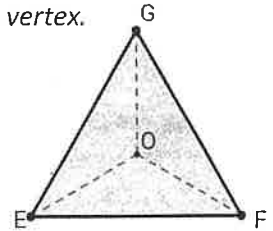
Notation:

14)

Description:

Algebraic Rule:

Notation:

15

Part 1 – Regular Polygons and Rotational Symmetry

*A **regular polygon** is a **polygon** that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).   In the case of **regular polygons** the **center** is the point that is equidistant from each vertex.*

1. Given *Regular Triangle EFG* with center O.
   a. F is rotated about O. If the image of F is G, what is the angle of rotation?

   b. $\overline{FG}$ is rotated 120° about O. What is the image of $\overline{FG}$?

   > General Rule: The regular triangle has rotation symmetry with respect to the center of the polygon
   >
   > and angles of rotation that measure _____ , _____ and _____.

Side note: A regular triangle is also called an _____ triangle or an _____ triangle.

2. Given *Regular Quadrilateral EFGH* with center O.
   a. F is rotated about O. If the image of F is G, what is the angle of rotation?

   b. F is rotated about O. If the image of F is H, what is the angle of rotation?

   c. $\overline{FG}$ is rotated 270° about O. What is the image of $\overline{FG}$?

   > General Rule: The regular quadrilateral has rotation symmetry with respect to the center of the polygon
   >
   > and angles of rotation that measure _____, _____,_____ and _____.

Side note: A regular quadrilateral is often called a _____.

3. Given *Regular Pentagon ABCDE* with center O.

   a. C is rotated about O. If the image of C is D, what is the angle of rotation?

   b. C is rotated about O. If the image of C is E, what is the angle of rotation?

   c. C is rotated about O. If the image of C is A, what is the angle of rotation?

   d. $\overline{DC}$ is rotated 288° about O, what is the image of $\overline{DC}$?
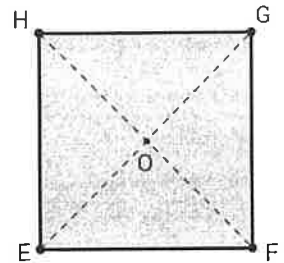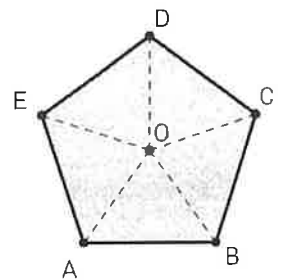
   > General Rule: The regular pentagon has rotation symmetry with respect to the center of the polygon and angles of
   >
   > rotation that measure _____ , _____ , _____ , _____ and _____.

4. Given *Regular Hexagon ABCDEF* with center O.

   a. C is rotated 60° about O, what is the image of C?

   b. C is rotated 120° about O, what is the image of C?

   c. C is rotated 180° about O, what is the image of C?

   d. $\overline{DC}$ is rotated 240° about O, what is the image of $\overline{DC}$?
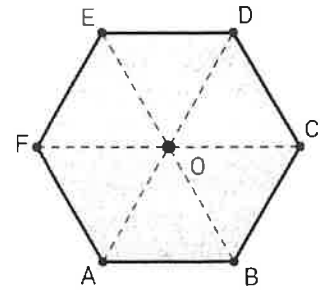
General Rule: The regular hexagon has rotation symmetry with respect to the center of the polygon

and angles of rotation that measure _____, _____, _____, _____, _____ and _____.

---

5. Given *Regular Octagon ABCDEFGH* with center O.

    a. When point C is rotated about O, the image of point C is point D. Describe the rotation (be sure to include degree).

    b. When point C is rotated about O, the image of point C is point F. Describe the rotation (be sure to include degree).
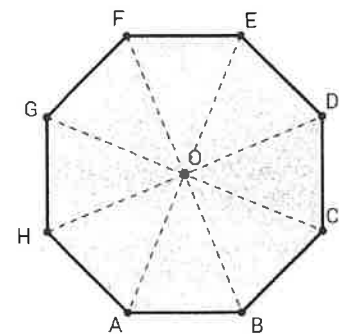

A regular polygon can be mapped onto itself if we rotate in multiples of the central angle measure.

The central angle of a regular polygon is found by _____


Part 2 – Parallelograms and Rotational Symmetry

6. Given *Parallelogram ABCD*, there is a center of rotation, O, that will map point A onto point C.

    a. What are the coordinates of O?

    b. What degree of rotation mapped C onto A using the center O?

    c. If we rotate the parallelogram around center O using the degree measure found in part b, $\angle D$ maps to _____.

    d. If $\angle A$ maps to $\angle C$, then $\angle A$ and $\angle C$ are _____.

    e. If $\angle D$ maps to _____, then $\angle D$ and _____ are _____.
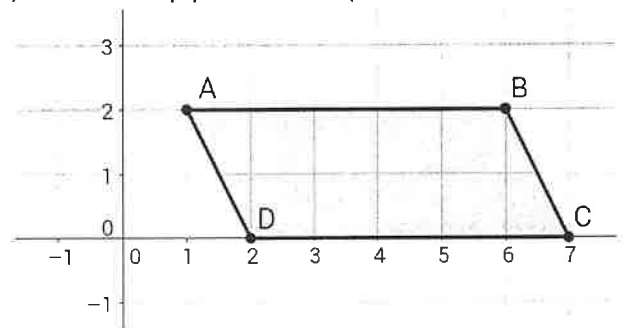
❖ **Graph the preimage and image. List the coordinates of the image. Then write the rule and proper notation.**

1) ΔRST: $R(2,-1), S(4,0),$ and $T(1,3)$
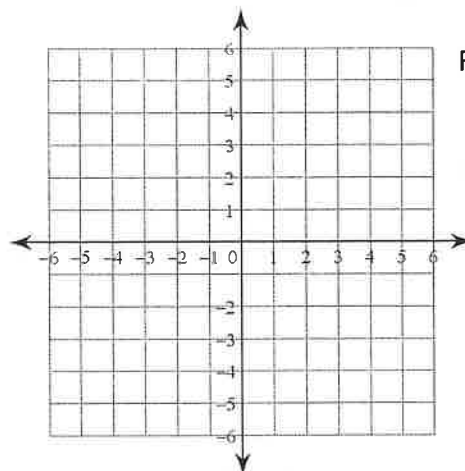 90° *counterclockwise* about the origin.
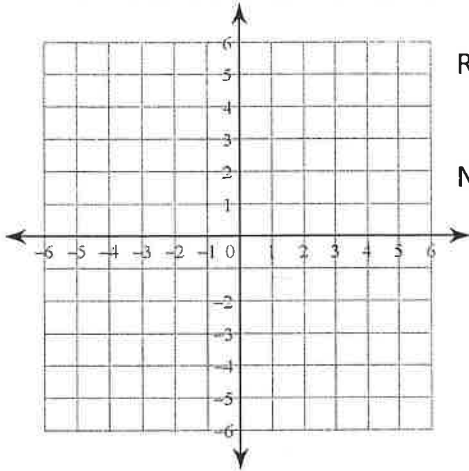
Rule:

Notation:

R' (___ , ___)   S' (___ , ___)   T' (___ , ___)

2) ΔFUN: $F(-4,-1), U(-1,3),$ and $N(-1,1)$
 180° *clockwise* about the origin.

Rule:

Notation:

F' (___ , ___)   U' (___ , ___)   N'(___ , ___)

3) ΔTRL: $T(2,-1), R(4,0),$ and $L(1,3)$
 90° *clockwise* about the origin.
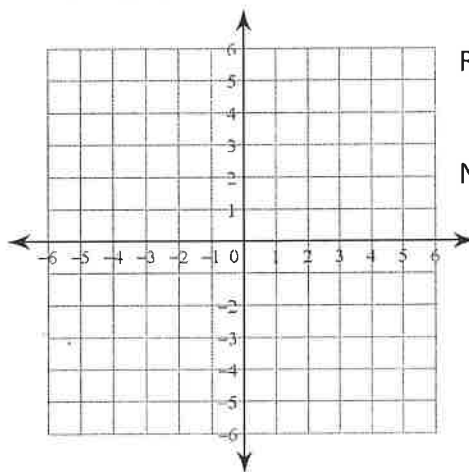
Rule:

Notation:

T' (___ , ___)   R'(___ , ___)   L'(___ , ___)

4) ΔCDY: $C(-4,2), D(-1,2),$ and $Y(-1,-1)$
 180° *counter clockwise* about the origin.
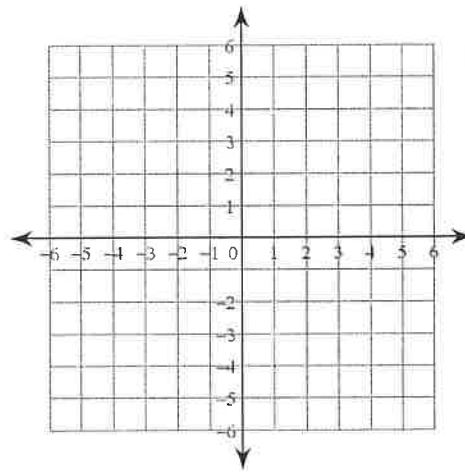
Rule:

Notation:

C' (___ , ___)   D' (___ , ___)   Y' (___ , ___)

5) Application

$ABCDE$ is a regular pentagon with center X.

a. Name the image of point E for a *counterclockwise* 72° rotation about X.

b. Given the image for a *clockwise* 216° rotation about X is $\overline{CB}$. What was its preimage?

c. Describe 2 rotations with a preimage of point D and image of B.

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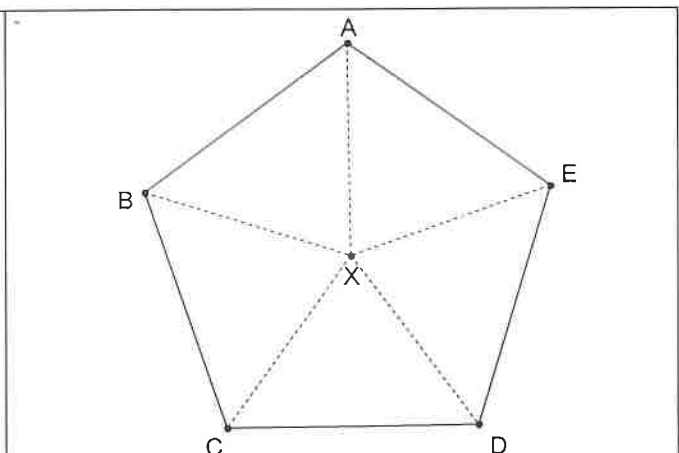
Practice: Rotations with Coordinates

For each problem graph the image points. Specifically describe in words the rotation that occurred. Then, write the Algebraic Rule and the proper notation for the rotation.
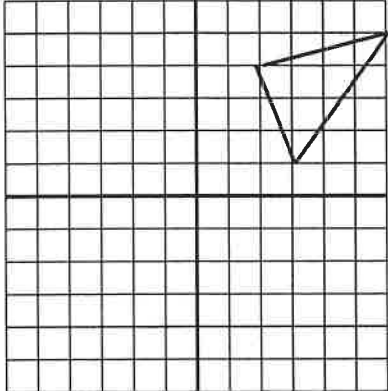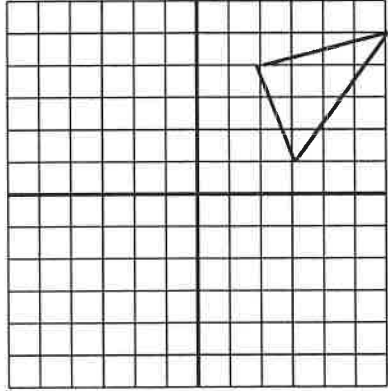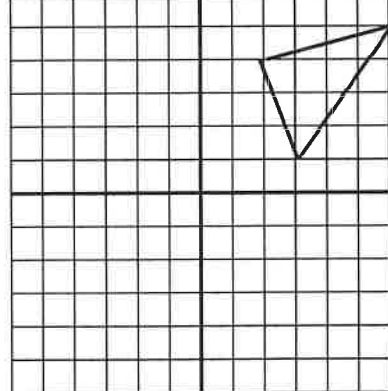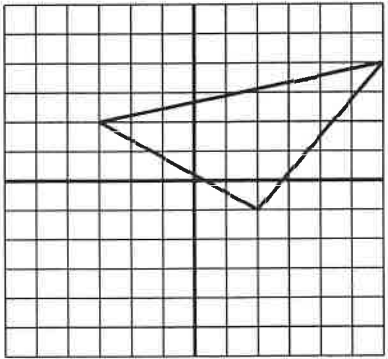
1) The coordinates of $\triangle ABC$ are $A(3,1)$, $B(6,5)$ and $C(2,4)$. The coordinates of A'B'C' are $A'(-1,3)$, $B'(-5,6)$, and $C'(-4,2)$.

Description:

Algebraic Rule:

Notation:

2) The coordinates of $\triangle ABC$ are $A(3,1)$, $B(6,5)$ and $C(2,4)$. The coordinates of A'B'C' are $A'(1,-3)$, $B'(5,-6)$, and $C'(4,-2)$.

Description:

Algebraic Rule:

Notation:

3) The coordinates of $\triangle ABC$ABC are $A(3,1)$, $B(6,5)$ and $C(2,4)$. The coordinates of A'B'C' are $A'(-3,-1)$, $B'(-6,-5)$, and $C'(-2,-4)$.

Description:

Algebraic Rule:

Notation:

4) The coordinates of $\triangle ABC$ are $A(2,-1)$, $B(6,4)$ and $C(-3,2)$. The coordinates of A'B'C' are $A'(-1,-2)$, $B'(4,-6)$, and $C'(2,3)$.

Description:

Algebraic Rule:

Notation:

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1. Pre-image: $A(0,0)$, $B(8,1)$, $C(5,5)$

Rotate the figure 180°	
Reflect the figure over the $x-axis$	
Translate the figure according to $(x,y) \rightarrow (x+6, y-1)$	

2. Pre-image: $D(-12,6)$, $E(-4,6)$, $F(-6,9)$

Translate the figure according to $(x,y) \rightarrow (x+1, y-6)$	
Reflect the figure over the $x-axis$	
Reflect the figure over the $y-axis$	

3. Pre-image: $G(2,2)$, $H(-2,2)$, $I(-2,-2)$

Rotate the figure 90° clockwise	
Translate the figure according to $(x,y) \rightarrow (x+2, y+2)$	
Reflect the figure over the line $y = x$	

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4. Pre-image: $J(7,2)$, $K(0,9)$, $L(-6,-5)$

Reflect the figure over the $y-axis$	
Reflect the figure over the $x-axis$	
Rotate the figure 90° counbter-clockwise about the origin	

5. Pre-image: $M(0,0)$, $N(-13,0)$, $O(0,12)$

Rotate the figure 180° about the origin	
Translate the figure according to $(x,y) \rightarrow (x+5, y+5)$	
Reflect the figure over the line $y = x$	

6. Pre-image: $P(6,-3)$, $Q(8,-5)$, $R(7,-7)$

Translate the figure according to $(x,y) \rightarrow (x-4, y+3)$	
Reflect the figure over the line $y = -x$	
Rotate the figure 180°	

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Math 2
Unit 1 – Geometric Transformations
Lesson 5 – Dilations

Name_____
Date_____ Pd_____

Alice in Wonderland

In the story, Alice's Adventures in Wonderland, Alice changes size many times during her adventures. The changes occur when she drinks a potion or eats a cake. Problems occur throughout her adventures because Alice does not know when she will grow larger or smaller.

Part 1

As Alice goes through her adventure, she encounters the following potions and cakes:

Red potion – shrink by $\dfrac{1}{9}$ Chocolate cake – grow by 12 times

Blue potion – shrink by $\dfrac{1}{36}$ Red velvet cake – grow by 18 times

Green potion – shrink by $\dfrac{1}{15}$ Carrot cake – grow by 9 times

Yellow potion – shrink by $\dfrac{1}{4}$ Lemon cake – grow by 10 times

Find Alice's height after she drinks each potion or eats each bite of cake. **If everything goes correctly, Alice will return to her normal height by the end.**

Starting Height	Alice Eats or Drinks	Scale factor from above	New Height
54 inches	Red potion	$\dfrac{1}{9}$	6 inches
6 inches	Chocolate cake		
	Yellow potion		
	Carrot cake		
	Blue potion		
	Lemon cake		
	Green potion		
	Red velvet cake		**54 inches**

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Part 2

A) The graph below shows Alice at her normal height.

B) Plot point A' such that it is twice as far from the origin as point A. Do the same with all of the other points. Connect the points to show Alice after she has grown.
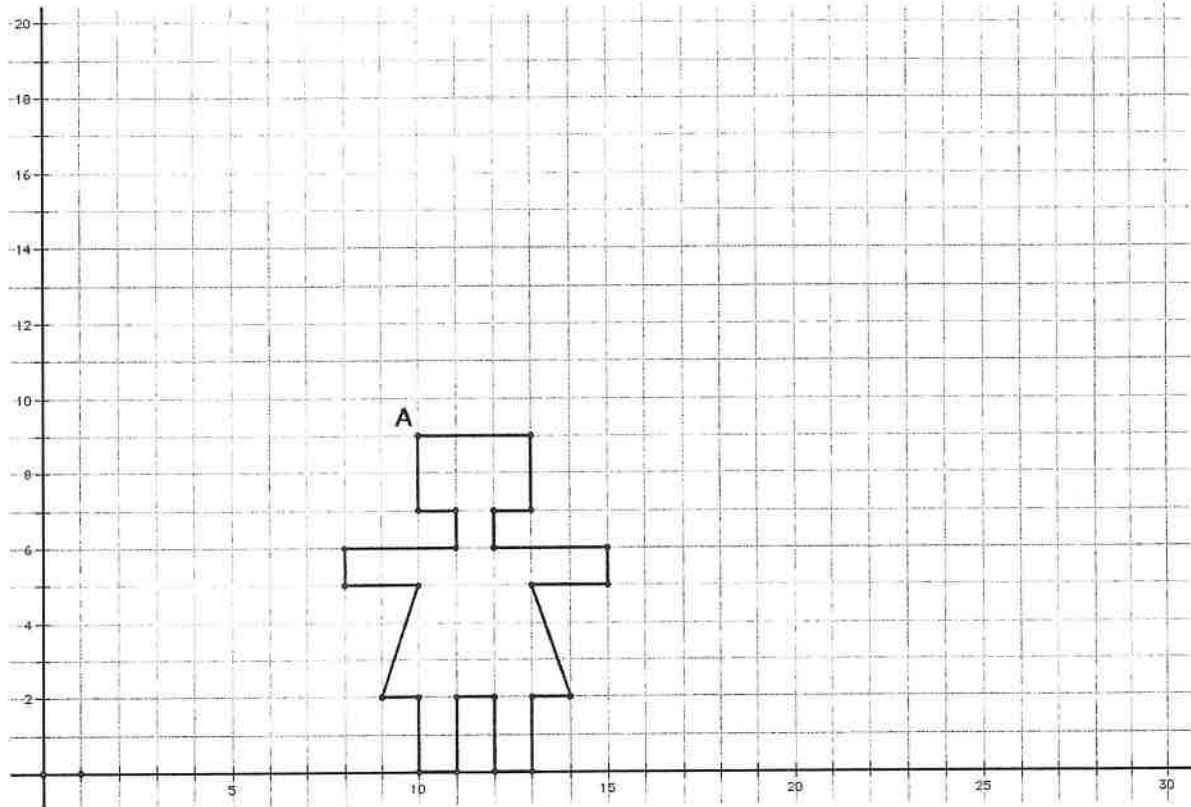
C) Answer the following questions:

1. How many times larger is the new Alice? _____

2. How much farther away from the origin is the new Alice? _____

3. What are the coordinates for point A? _____ Point A'? _____

4. What arithmetic operation do you think happened to the coordinates of A?

5. Write your conclusion as an Algebraic Rule $(x, y) \rightarrow (\quad , \quad)$

6. What arithmetic operation on the coordinates do you think would shrink Alice in half?

7. Write your conclusion as an algebraic rule.

8. If Alice shrinks in half, how far away from the origin will her image be from her preimage?

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➤ A **DILATION** stretches or shrinks the original figure.

- The description of a dilation should include the _____ _____, the _____ of the dilation, and whether the dilation is an _____ or a _____.

- The amount by which the image grows or shrinks is called the "_____ _____."

- The _____ of dilation is a fixed point in the plane about which all points are expanded or contracted.

- A dilation is an enlargement of the pre-image if the _____ _____ is _____.

- A dilation is a reduction of the pre-image if the _____ _____ is _____.

- If the scale factor is 1, then the pre-image and image are _____.

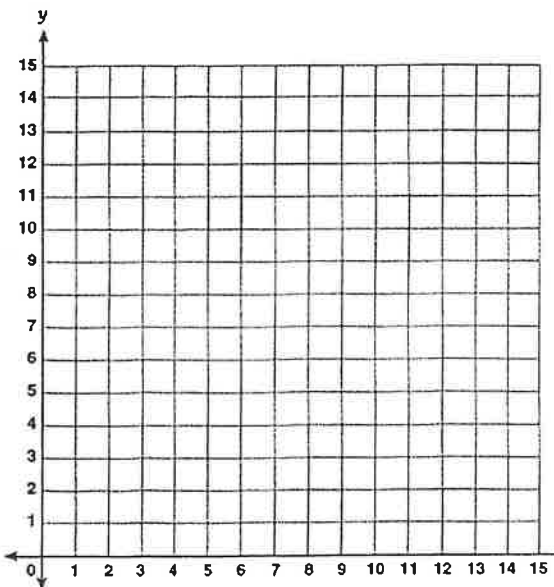❖ **Algebraic Rule: (x, y) → (ax, ay)**

If a > 1 then the dilation is __ _____

If 0 < a < 1 then the dilation is __ _____

The distance between the center of a dilation and any point on the pre-image is equal to the _____ _____ multiplied by the distance between the dilation center and the corresponding point on the image.
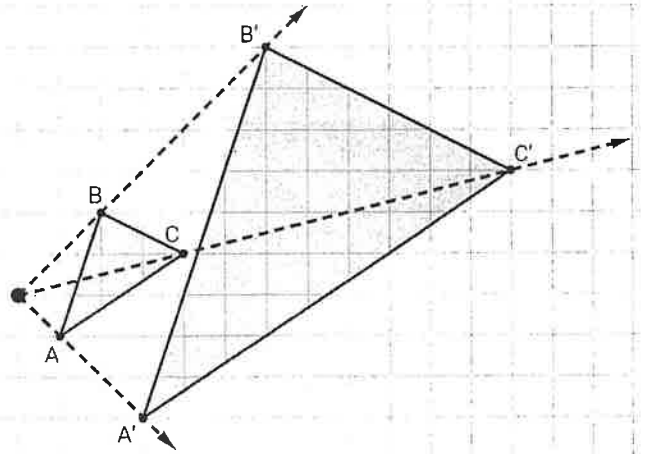
1. Graph and connect these points: $(2, 2)$ $(4, 6)$ $(6, 2)$ $(6, 6)$.
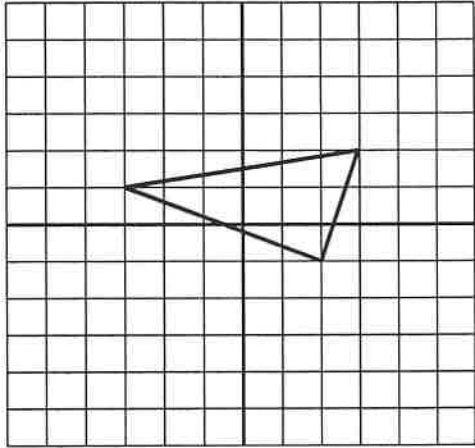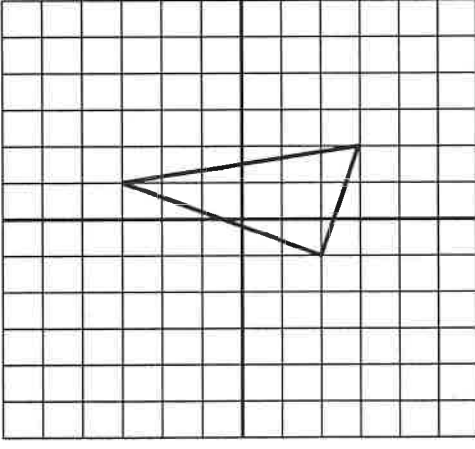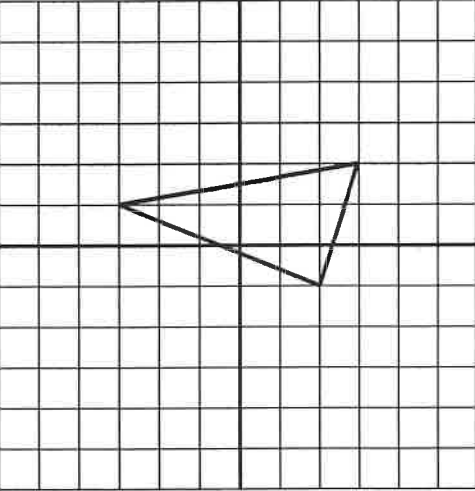

2. Graph the image on the same coordinate plane by applying a scale factor of 2.

Write the rule: _____

3. Graph the image on the same coordinate plane by applying a scale factor of $\frac{1}{2}$.

Write the rule: _____

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For each problem, graph the image points, and describe the transformation that occurred. Specify if the transformation is an **enlargement or reduction** and by what **scale factor**. Then, examine the coordinates to create an Algebraic Rule.

1) The coordinates of $\triangle ABC$ are
   $A(2,-1), B(3,2)$ *and* $C(-3,1)$.
   The coordinates of A'B'C' are
   $A'(1,\frac{-1}{2}), B'\left(\frac{3}{2},1\right)$ *and* $C'(\frac{-3}{2},\frac{1}{2})$.

   Transformation:

   Scale Factor:

   Algebraic Rule:

2) The coordinates of $\triangle ABC$ are
   $A(2,-1), B(3,2)$ *and* $C(-3,1)$.
   The coordinates of A'B'C' are
   $A'(4,-2), B'(6,4),$ *and* $C'(-6,2)$.

   Transformation:

   Scale Factor:

   Algebraic Rule:

3) The coordinates of $\triangle ABC$ are
   $A(2,-1), B(3,2)$ *and* $C(-3,1)$.
   The coordinates of A'B'C' are
   $A'(3,\frac{-3}{2}), B'(\frac{9}{2},3),$ *and* $C'(\frac{-9}{2},\frac{3}{2})$.

   Transformation:

   Scale Factor:

   Algebraic Rule:

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Math 2
Unit 1 – Geometric Transformations
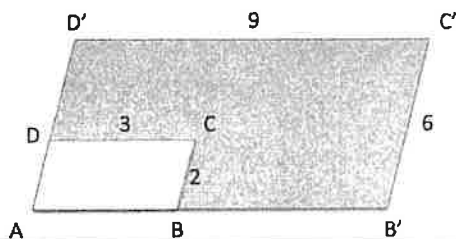**Lesson 5 – Dilations HOMEWORK**

Name_____
Date_____Pd_____

1. Describe the transformation given by rule $(x, y) \rightarrow (3x, 3y)$. Is it an "Isometry"? Why or why not?

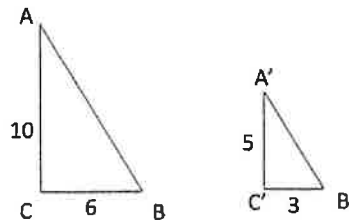2. Write an algebraic rule for the dilation:

      A. by a factor of 3

      B. by a factor of $\frac{1}{2}$.

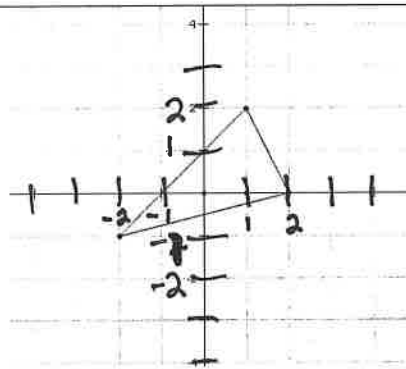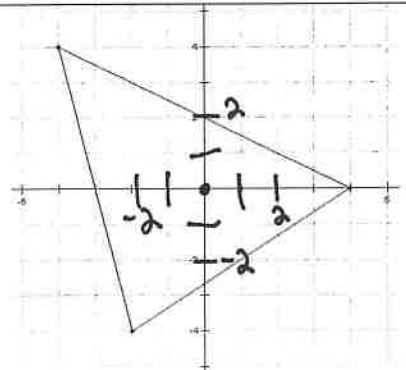| 3. Find the scale factor of the dilation that maps ABCD to A'B'C'D'.	4. Find the scale factor of the dilation that maps ABC to A'B'C'.

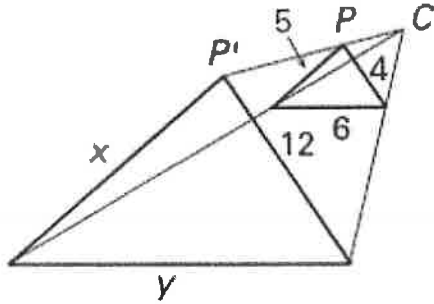| 5. Graph the dilation of the object shown using a scale factor of 2. Algebraic Rule:	
6. Graph the dilation of the object shown using a scale factor of ½. Algebraic Rule:	

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Find the scale factor. Tell whether the dilation is an enlargement or a reduction. Then find the values of the variables.
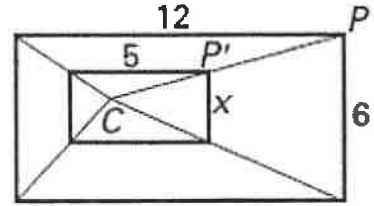
7.

SF = _____   _____

x = _____   y = _____

8.

SF = _____   _____

x = _____

Determine if the following scale factor would create an enlargement, a reduction, or an isometric figure. *Explain your reasoning using the scale factor.*

9. 3.5

10. $^2/_5$

11. 0.6

12. 1

13. $^4/_3$

14. $^5/_8$

Given the point and its image, determine the scale factor.

15. $A(3,6)$ $A'(4.5,9)$

16. $G'(3,6)$ $G(1.5,3)$

17. $B(2,5)$ $B'(1,2.5)$

18. The sides of one right triangle are 6, 8, and 10. The sides of another right triangle are 10, 24, and 26.

   Determine if the triangles are similar. If so, what is the ratio of corresponding sides?

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Math 2
Unit 1 – Geometric Transformations
QUIZ REVIEW HOMEWORK

Name_____

Date_____Pd_____

❖ **For each of the following, graph and label the image for each transformation using proper prime notation.**
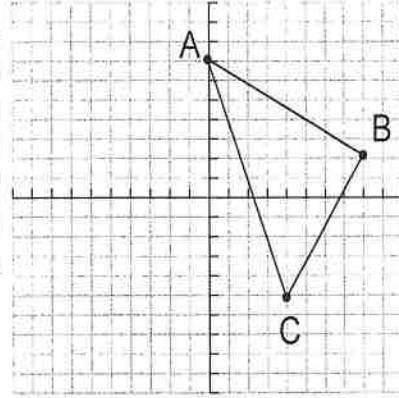
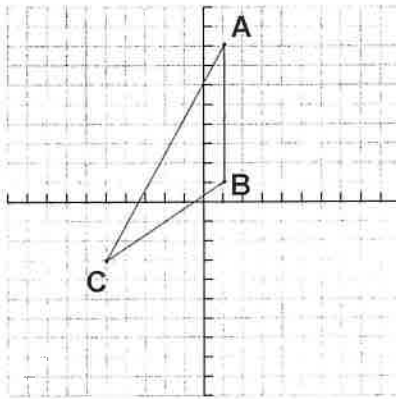1. Reflect over the $y - axis$
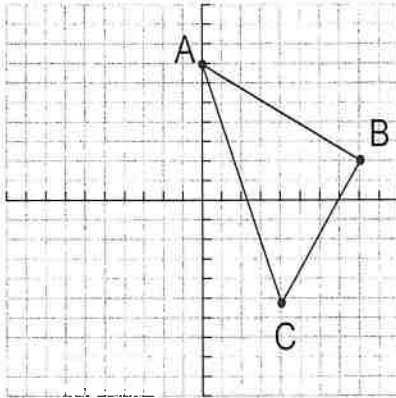
Algebraic Rule:

Notation:

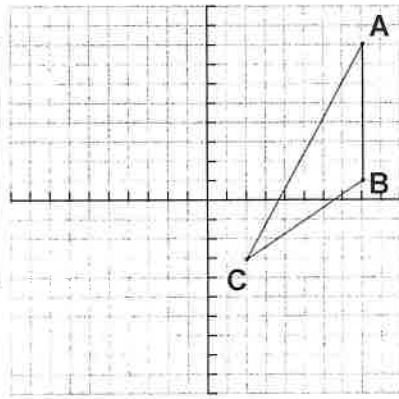2. Dilate with a scale factor $r = \frac{1}{2}$

Algebraic Rule:

3. Rotate about the origin 90°

Algebraic Rule:

Notation:

4. Translate: $(x, y) \rightarrow (x - 5, y + 2)$

Words:

❖ **Perform each of the transformations for # 5 – 10 using the ordered pairs below.**
❖ **Write each answer as ordered pairs.**

$$(1, -5), (-2, 4), (3, 0)$$

5. Reflect over the $x - axis$	6. Reflect over the line $y = x$	7. Rotate 90°
8. Rotate 180°	9. Dilate with a scale factor of 3	10. $T: (x, y) \rightarrow (x + 3, y - 4)$

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Math 2
Unit 1 – Geometric Transformations
QUIZ REVIEW HOMEWORK

Name_____

Date_____ Pd_____

❖ **State whether the isosceles triangle mapped to the other triangle is by a reflection, translation, or rotation.**

11. Triangle 1 to Triangle 5   _____

12. Triangle 5 to Triangle 2   _____

13. Triangle 2 to Triangle 4   _____

14. Triangle 3 to Triangle 4   _____

15. Triangle 1 to Triangle 4   _____
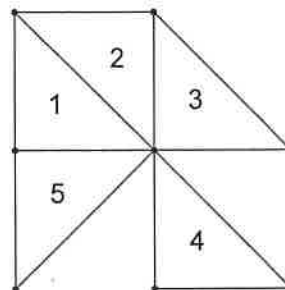
❖ **Answer each of the following.**

16. Describe the translation that maps all points down 7 units and right 12 units.

   a) **Algebraic Rule:**_____

17. If the translation $(-1, 7) \rightarrow (5, -2)$, then $(0, 5) \rightarrow ($ _____ , _____ $)$

18. If $T: (x, y) \rightarrow (x - 2, \ y + 6)$, and D = $(8, -1)$, find point D'._____

19. W is reflected over the $y - axis$. If W is $(3, -8)$, find W'._____
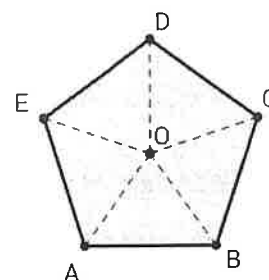
20. M is dilated with a scale factor $r = \frac{3}{4}$. If M is $(9, -3)$, find M'._____

21. Given $Regular \ Pentagon \ ABCDE$ with center O.

   a) A is rotated about O. If the image of A is C, what is the angle of rotation?

   b) E is rotated about O. If the image of E is A, what is the angle of rotation?

   c) $\overline{BC}$ is rotated 288° about O. What is the image of $\overline{BC}$?

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Math 2
Unit 1 – Geometric Transformations
Lesson 6 – Compositions of Transformations

Name_____

Date_____Pd_____

A **composition** is a sequence of _____.

An example of a composition is a **glide reflection** since it is the composition of a _____

and a _____.

➢ **Composition of Motions with Algebraic Rules**

Using your algebraic rules, write a new rule after both transformations have taken place.

1) Translate a triangle 4 units right and 2 units up, and then reflect the triangle over the line $y = x$.

2) Rotate a triangle 90 degrees counterclockwise, and then dilate the figure by a scale factor of 3.

3) Translate a triangle 4 units left and 2 units down, and then reflect the triangle over the $y - axis$.

4) Rotate a triangle 90 degrees clockwise, and then dilate the figure by a scale factor of $^1/_3$.

5) Translate a triangle 4 units right and 2 units down, and then reflect the triangle over the $x - axis$.

6) Rotate a triangle 180 degrees counterclockwise, and then dilate the figure by a scale factor of 2.

7) Translate a triangle 4 units left and 2 units up, and then reflect the triangle over the line $y = x$.

8) Rotate a triangle 180 degrees clockwise, and then dilate the figure by a scale factor of $^1/_2$.

---

9) **a.** On a coordinate grid, draw a triangle using $A(-9, -2), B(-6, -1), C(-6, -3)$ to represent a duck foot.
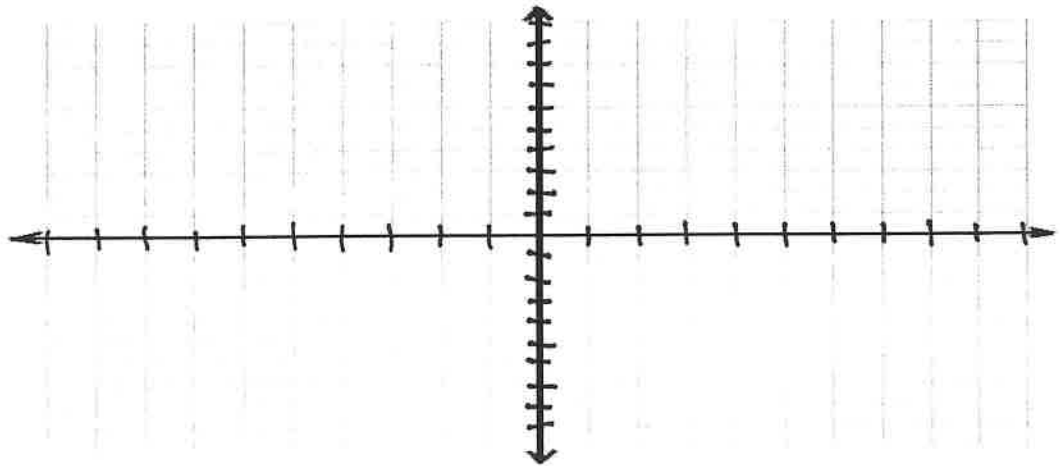
b. Transform ΔABC using R_{x-axis}, followed by $T: (x, y) \rightarrow (x + 5, y)$. Label the final image ΔA'B'C'.

c. Write a coordinate rule for this composite transformation.

d. Now apply the coordinate rule you gave in Part c two more times to ΔA'B'C'.

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Math 2
Unit 1 – Geometric Transformations
Lesson 6 – Compositions of Transformations – CLASSWORK
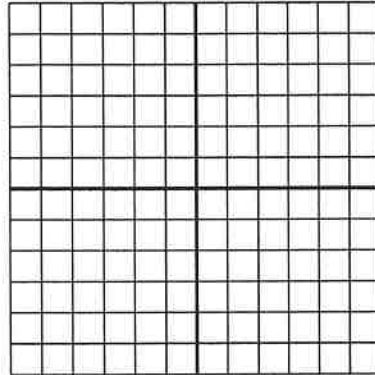
Name_____
Date_____Pd_____

Part 1: Given the description, write an algebraic rule to represent the transformation. Then graph the pre-image and image on the graph below. Use $\triangle ABC$ with $A(2,-2), B(3,1), and\ C(1,2)$.
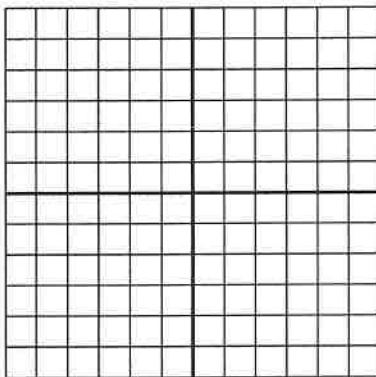
1) $\triangle ABC$ is dilated by 2.
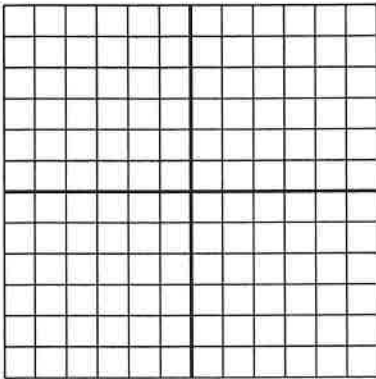
Algebraic Rule: _____

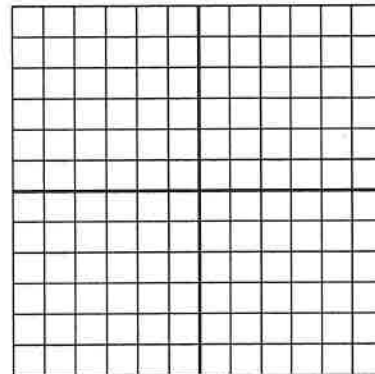2) $\triangle ABC$ is moved up 4 and 2 to the right

Algebraic Rule: _____

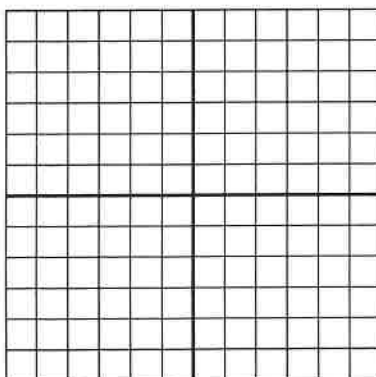3) $\triangle ABC$ is rotated 180° then dilated by a factor of 2.

Algebraic Rule: _____

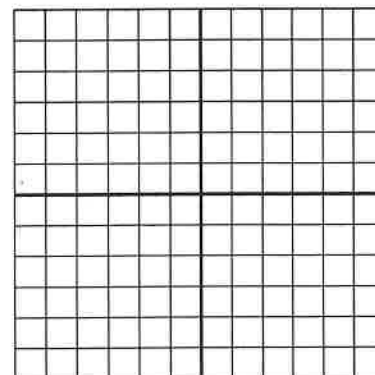4) $\triangle ABC$ is reflected over the $y-axis$ then dilated by a factor of 2

Algebraic Rule: _____

5) $\triangle ABC$ is reflected over $y=-x$ and moved moved up 2.

Algebraic Rule: _____

6) $\triangle ABC$ is reflected over the x-axis, then dilated by ½ then moved down 2 and left 1.

Algebraic Rule: _____

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Name_____
Date_____Pd_____

Interpreting Functions

Kim and Jim are twins and live at the same home. They each walk to school along the same path at exactly the same speed. However, Jim likes to arrive at school early and Kim is happy to arrive 7 minutes later, just as the bell rings. Pictured at right is a graph of Jim's distance from school over time.
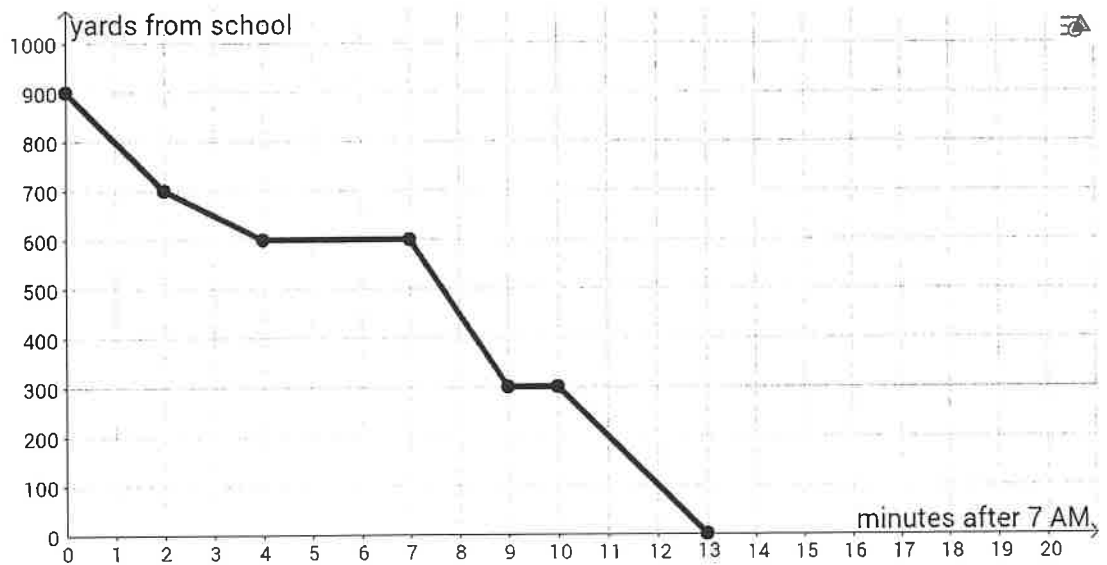
1. Use a dotted line to sketch Kim's graph of distance from school over time (once she leaves for school).

2. How many minutes after 7AM does Jim leave for school? _____

3. How many minutes after 7AM does Jim arrive at school? _____

4. How many minutes after 7AM does Kim leave for school? _____

5. How many minutes after 7AM does Kim arrive at school? _____

6. What is Jim's farthest distance from school? _____

7. What is Jim's closest distance to school? _____

8. What is Kim's farthest distance from school? _____

9. What is Kim's closest distance to school? _____

➢ **Use your answers to the above questions to fill in the following:**

10. Jim's domain: _____ $\leq x \leq$ _____
    (where x represents time after 7AM)

11. Kim's domain: _____ $\leq x \leq$ _____
    (where x represents time after 7AM)

12. Jim's range: _____ $\leq y \leq$ _____
    (where y represents distance from school)

13. Kim's range: _____ $\leq y \leq$ _____
    (where y represents distance from school)

➢ Inequalities can also be written in **interval notation**. Parentheses and/or brackets are used to show whether the endpoints are excluded or included. For example, [3, 8) is the **interval** of real numbers between 3 and 8, **including** 3 and **excluding** 8. Another example, [4, ∞) is the interval of real numbers greater than or equal to 4.

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Domain and Range in Translations

➤ Quick review: The **domain** is the set of all possible x-values on the graph. The **range** is the set of all possible y-values on the graph.
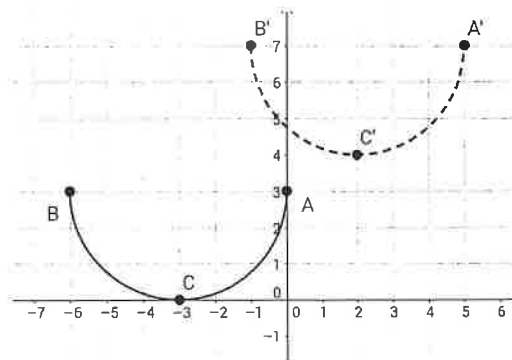
1. Describe the translation(s) from the pre-image to the image.

  a. Given the following graph, state the domain and range of the pre-image in interval notation:

   Domain: _____ Range: _____.

  b. State the domain and range of the image in interval notation:
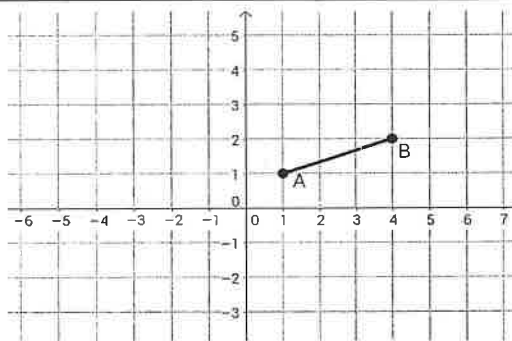
   Domain: _____ Range: _____.

2. Draw and label the image of $\overline{AB}$ translated left 2 and down 3.

  a. State the domain and range of the pre-image:

   Domain: _____ Range: _____.

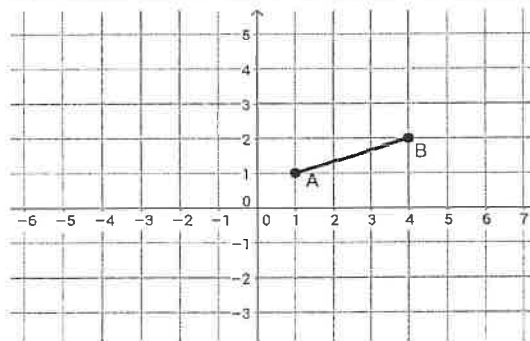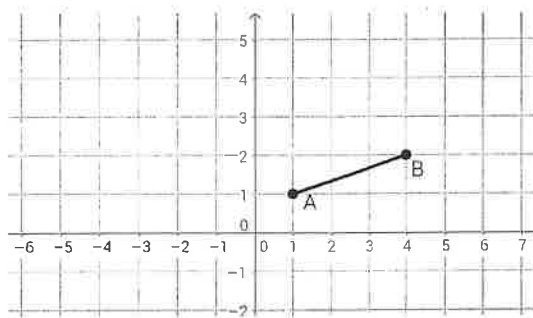 b. State the domain and range of the image:

   Domain: _____ Range: _____.

3. Draw and label the image of $\overline{AB}$ reflected over the x-axis.

  a. State the domain and range of the pre-image:

   Domain: _____ Range: _____.

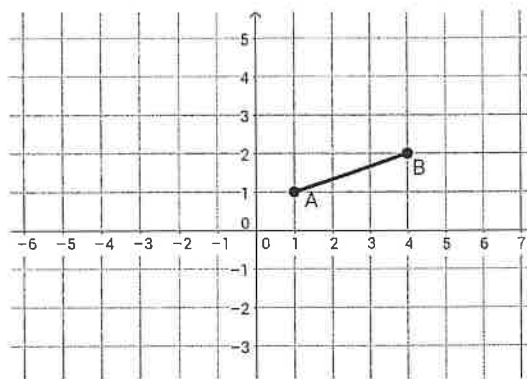 b. State the domain and range of the image:

   Domain: _____ Range: _____.

4. Draw and label the image of $\overline{AB}$ reflected over the y-axis.

  a. State the domain and range of the pre-image:

   Domain: _____ Range: _____.

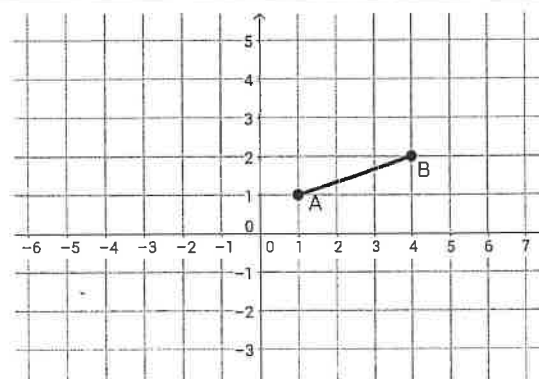 b. State the domain and range of the image:

   Domain: _____ Range: _____.

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5. **Draw and label the image of $\overline{AB}$ reflected over the line $y = x$.**

   a. State the domain and range of the pre-image:

      Domain: _____ Range: _____.

   b. State the domain and range of the image:

      Domain: _____ Range: _____.

6. **Draw and label the image of $\overline{AB}$ rotated 90°.**

   a. State the domain and range of the pre-image:

      Domain: _____ Range: _____.

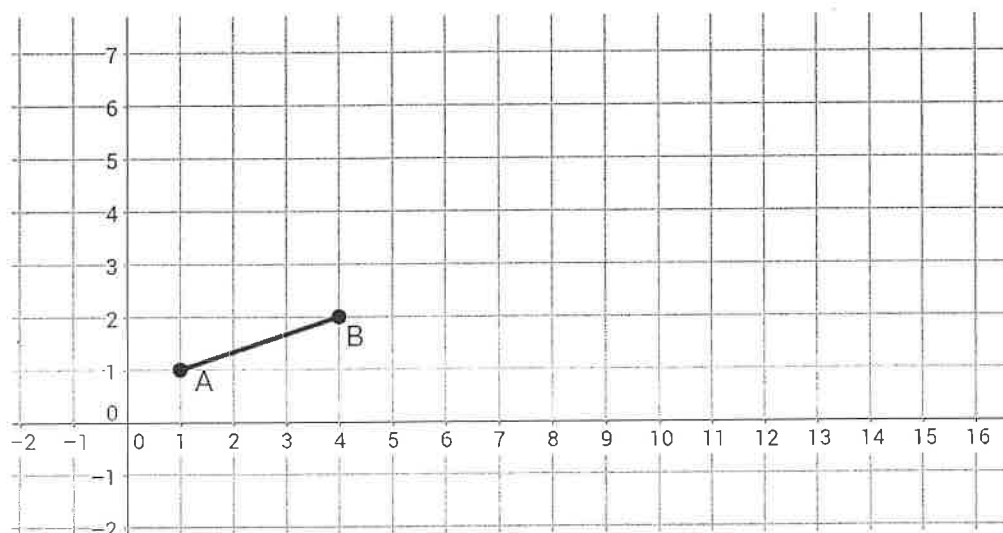   b. State the domain and range of the image:

      Domain: _____ Range: _____.

7. **Draw and label the image of $\overline{AB}$ dilated by a scale factor of 3 with a center of (0,0).**

   a. State the domain and range of the pre-image:
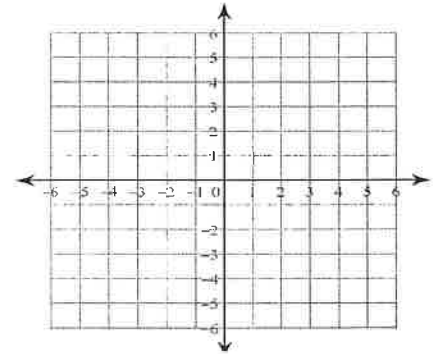
      Domain: _____ Range: _____.

   b. State the domain and range of the image:

      Domain: _____ Range: _____.

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Classwork: Given the patterns seen above, can you predict the domain/range of an image given a pre-image domain/range? Let's try:

1. Given a relation composed of points $A(2, 5)$, $B(1, -1)$, and $C(4, 2)$.

   A) State the domain and range of the relation as an interval:

   D: _____   R: _____

   B) State the domain and range of the image.

   a) **Translated right 2 and down 3:**

   D: _____   R: _____

   b) **Reflected in the $x - axis$:**

   D: _____   R: _____

   c) **Reflected in the $y - axis$:**

   D: _____   R: _____

   d) **Reflected in the line $y = x$:**

   D: _____   R: _____

   e) **Rotated 90°:**

   D: _____   R: _____

   f) **Dilated by a factor of 7 with C(0,0)**

   D: _____   R: _____

2. Given a **line segment** with endpoints $(0, 4)$ and $(3, 0)$

   A) State the domain and range of the segment.   D: _____$\le x \le$_____   R: _____$\le y \le$_____

   B) State the domain and range of the image **interval notation** when the relation is:

   a) **Translated right 2 and down 3:**

   D: _____

   R: _____

   b) **Reflected in the $x - axis$:**

   D: _____

   R: _____

   c) **Reflected in the $y - axis$:**

   D: _____

   R: _____

   d) **Reflected in the line $y = x$:**

   D: _____

   R: _____

   e) **Rotated 90°:**

   D: _____

   R: _____

   f) **Dilated by a factor of 7 with a center of $(0, 0)$:**

   D: _____

   R: _____

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KNOW FOR TEST!!!!!

Transformation Rules:

Translation: $T: (x, y) \rightarrow (x \pm a, y \pm b)$		Remember translations can also be described in words.
Reflection: $\begin{aligned} (x, y) &\rightarrow (x, -y) \\ (x, y) &\rightarrow (-x, y) \\ (x, y) &\rightarrow (y, x) \\ (x, y) &\rightarrow (-y, -x) \end{aligned}$	**Reflection Notation:** $\begin{aligned} &R_{x-axis} \\ &R_{y-axis} \\ &R_{y=x} \\ &R_{y=-x} \end{aligned}$	Remember reflection can occur over other lines on the coordinate plane.
Rotation: $\begin{aligned} (x, y) &\rightarrow (-y, x) \\ (x, y) &\rightarrow (y, -x) \\ (x, y) &\rightarrow (-x, -y) \end{aligned}$	**Rotation Notation:** $\begin{aligned} &\mathcal{R}_{90°} \\ \mathcal{R}_{90°CW} \ &or \ \mathcal{R}_{270°} \\ &\mathcal{R}_{180°} \end{aligned}$	Remember to always rotate counter-clockwise (left) unless otherwise specified.
Dilation: $(x, y) \rightarrow (ax, ay)$		Remember a dilation can be an enlargement or a reduction

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- For each transformation, state the coordinates for each:

	Image of (x, y)	Image of $(1,4)$	Image of $(-2, 7)$
1. Reflect over $y - axis$			
2. Reflect over $x - axis$			
3. Reflect over $y = x$			
4. Reflect over $y = -x$			
5. Rotate 90° *clockwise* about the origin			
6. Rotate 90° *counterclockwise* about the origin			
7. Rotate 180° about the origin			
8. Rotate 270° about the origin			

- For each of the following, graph and label the image for each transformation described.
- Then write using the correct notation.
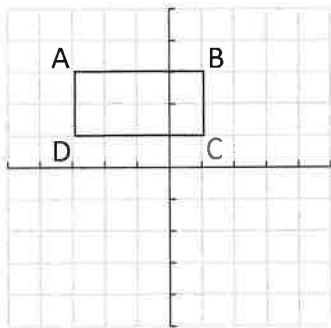
9. Reflect over the $x - axis$ 10. Rotate 180° about the origin   11. Translate right 4 units & down 3 units
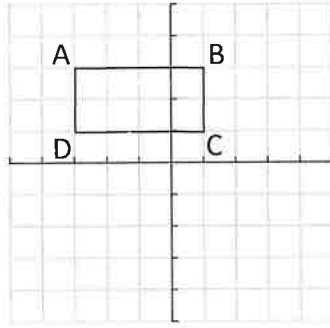
_____   _____   _____
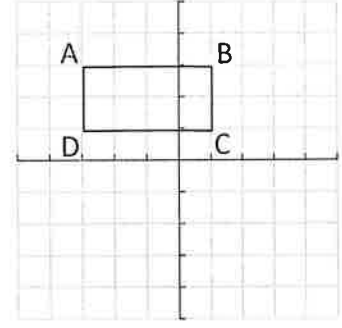
- State whether the specified pentagon is mapped to the other pentagon by a **reflection, translation, or rotation**

12. Pentagon 1 to Pentagon 3   _____

13. Pentagon 5 to Pentagon 6   _____

14. Pentagon 2 to Pentagon 5   _____

15. Pentagon 1 to Pentagon 2   _____

16. Pentagon 4 to Pentagon 6   _____
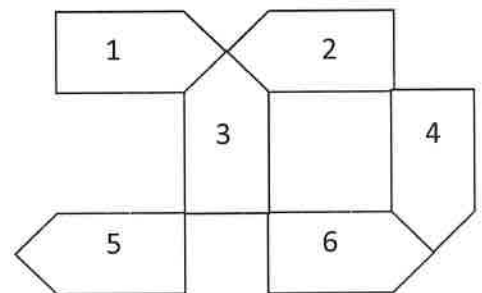
- Perform each of the transformations using the points below for #16-19. .

$$(7,-4) \quad (0,6) \quad (-2,3)$$

17. Reflect over the $y - axis$	19. Rotate 90° counter-clockwise
18. Reflect over the line $y = -x$	20. Dilate by a scale factor r = ½

- Answer each of the following.

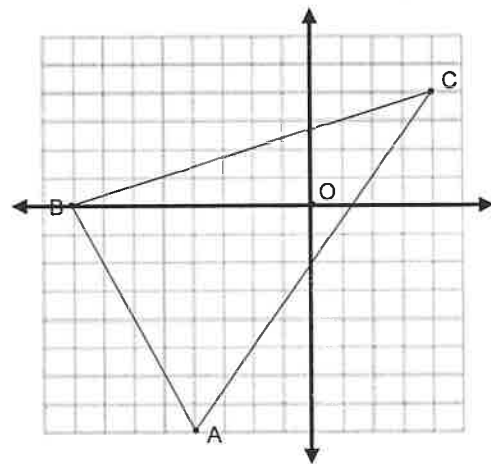21. $\triangle ABC$ has vertices $A(5,-2), \ B(-4,0), \ C(7,1)$.

Find the coordinates of the image of the triangle if it is dilated by a scale factor r = 3.

A'(_____ , _____)

B'(_____ , _____)

C'(_____ , _____)

21. Dilate $\triangle ABC$ using a scale factor r = $\frac{1}{4}$.

22. For each problem, there is a composition of motions. Using your algebraic rules, come up with a new rule after both transformations have taken place.   ** *Don't forget to distribute the* $(-)$

a. Translate a triangle 5 units left and 3 units up, and then reflect the triangle over the $x - axis$.

b. Translate a triangle 2 units right and 7 units down, and then rotate 90° clockwise.

c. Rotate a triangle 90 degrees counterclockwise, and then reflect in the line $y = x$.

d. Reflect in the line $y = -x$, and then translate right 4 units and down 2 units.

23. The diagonals of *Regular Hexagon ABCDEF* form six equilateral triangles as shown.
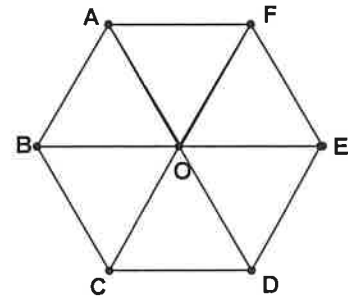
Fill in the correct **letter** after the given transformation:

a. Rotate 60° clockwise: $E \rightarrow$ _____

b. Rotate 60° counter-clockwise: $D \rightarrow$ _____

c. Rotate 120° clockwise: $F \rightarrow$ _____

d. Rotate 300° counter-clockwise: $E \rightarrow$ _____

e. Rotate 60° clockwise: _____ $\rightarrow B$

24. Given a **line segment** with endpoints $(1, -2)$ **and** $(4, 5)$

    A) State the **domain and range** of the pre − image segment.   D: _[_____]_   R: _[_____]_

    B) State the domain and range of the image **interval notation** when the relation is:

        a) **Translated right 1 and up 4:**

            D: _____

            R: _____

        d) **Reflected in the line $y = x$:**

            D: _____

            R: _____

        b) **Reflected in the $x - axis$:**

            D: _____

            R: _____

        e) **Rotated 90°:**

            D: _____

            R: _____

        c) **Reflected in the $y - axis$:**

            D: _____

            R: _____

        f) **Dilated by a factor of 5 with a center of $(0, 0)$:**

            D: _____

            R: _____

1. Rotate the rectangle about the origin through angles of:

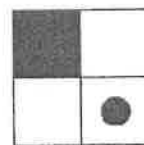   a. 90 degrees

   b. 180 degrees
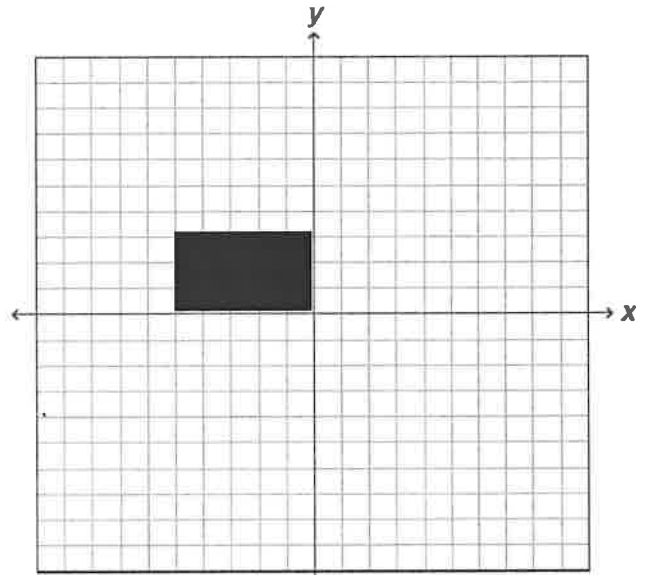
   c. 270 degrees

2. Rotate the triangle about the origin, using these angles:

   a. 90 degrees

   b. 180 degrees
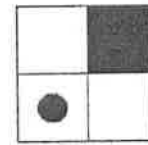
   c. 270 degrees
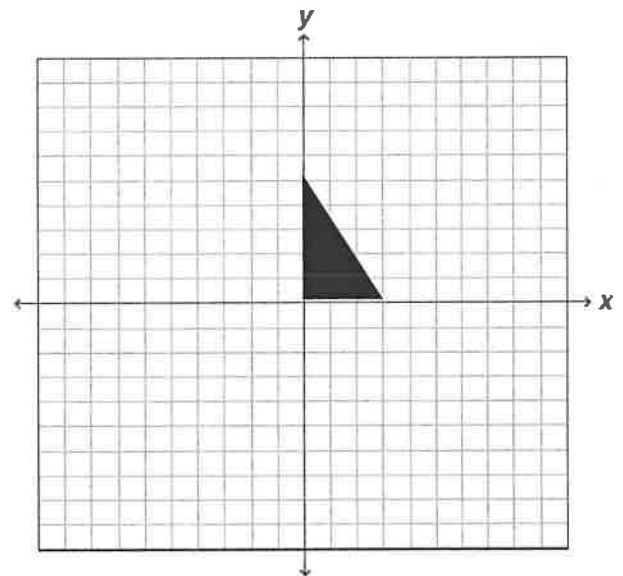
3. Study Figures I and II.
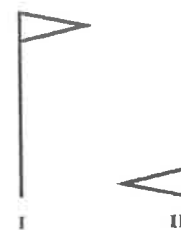Which transformation of Figure I is shown in Figure II?

I   II

4. Study figures I and II.
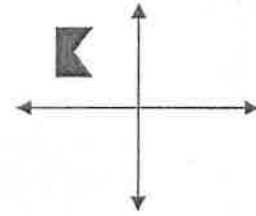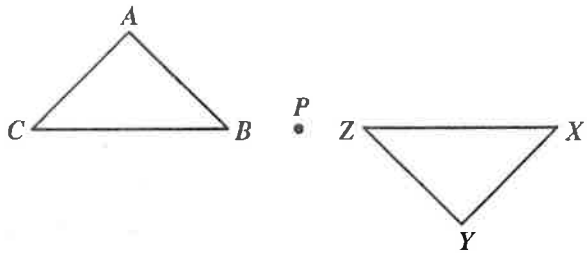Which transformation, if any, of Figure I is shown in Figure II?
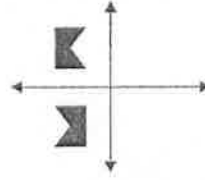
I   II

5. Which represents a translation of the figure →

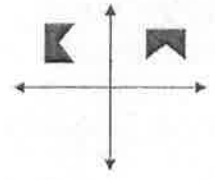6. $\triangle XYZ$ was obtained from $\triangle ABC$ by a rotation about the point P.

Write the correspondence of the vertices:

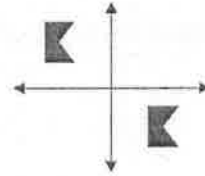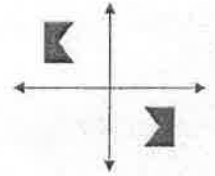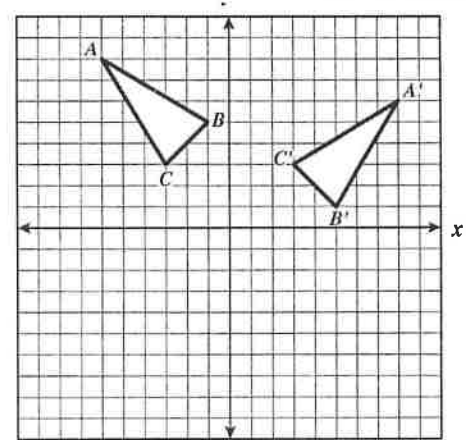$A \rightarrow$ _____ $B \rightarrow$ _____ $C \rightarrow$ _____

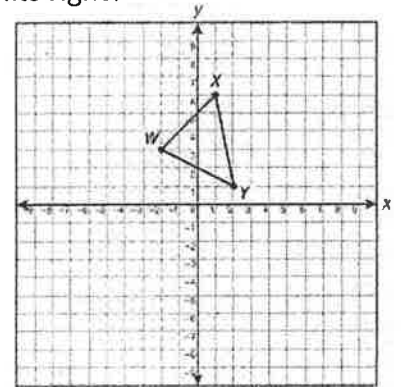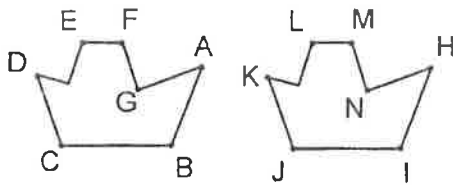7. Triangle $A'B'C'$ is apparently →

O A.   A translation of triangle ABC across the x-axis
O B.   A 90° clockwise rotation of triangle ABC about the origin
O C.   A reflection of triangle ABC across the y-axis
O D.   A reflection of triangle ABC across the x-axis

8. Name the image of **X** when triangle WXY is translated 2 units down and 5 units right?

9. Which point is a horizontal translation of E?

10.   Write a rule for the composition if a point is

   A. Rotated 270° , then reflected across $y = -x$ and finally translated 5 left and 7 up.

   B. Rotated 270° , then translated 5 left and 7 up and finally reflected across $y = -x$.